A SIMULATION-BASED STUDY OF INVENTORY DISTRIBUTION NETWORK

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Abstract

This paper presents a simulation approach to analyze an inventory distribution network and compares various simulation results of modeling methods. The results indicate that the performance of an inventory network depends on the distribution of customer demands.

1 Introduction

Most service centers are organized into networks of distribution sites that procure parts, process, and distribute them to customers. Most of these networks are "multi-echelon" or "multi-level" distribution networks, within which a part may move more than one step before reaching final customers.

Inventories exist throughout the distribution network for various reasons. The end customers create demands on service centers' inventories, and customers' uncertain demands, combined with uncertain production and/or shipping times (lead time), largely determine the inventory at a given site. On the other hand, having inventories is very costly, especially for the airline logistics support industry, where the demands for most spare parts are very sporadic, and the lead times of replenishing parts from suppliers are long, and the costs of holding inventories are very expensive.

Most inventory managements have the goal of minimizing inventory while achieving the desired service levels, and simulation technique becomes necessary to find out that at what inventory cost, we could achieve the desired service levels and how the inventory network reacts to customer demands and how inventory policies perform.

The aim of this paper is to use a simulation approach to analyze an inventory distribution network and to compare simulation results under various modeling of customer demands and intermittent intervals between two consecutive customer orders for targeted service levels in the airline logistic support industry.

2 Problem Formulation

Consider the following multi-echelon distribution network shown in Fig 1.

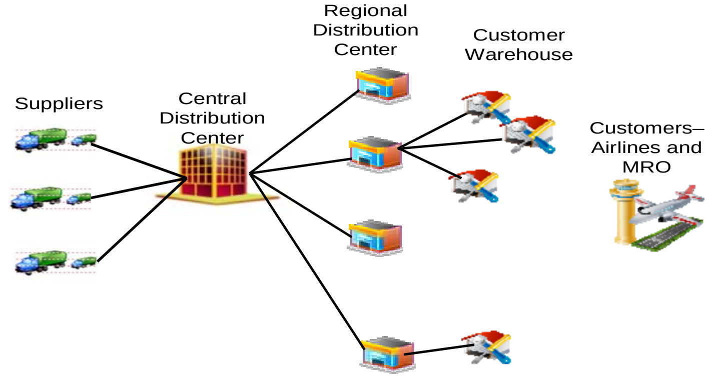

Figure 1 Inventory Distribution Network

In this network structure, the network issues part replenishment orders to suppliers and the suppliers ship the parts to central distribution center for quality and quantity checks. The regional distribution centers get replenishment from the central distribution center, and the customer warehouses get replenishment from their regional centers or the central distribution center. The network owns the customer warehouses for their major or loyal customers. The customers, which are either airlines or MROs or distributors, place and get their orders from a closest regional distribution center or the central distribution center or the customer warehouse which stays at the customer's maintenance center.

Each center and warehouse has its own (R, Q) as inventory policies, where R is the reorder point (ROP), and Q is the reorder quality (ROQ), and service levels. However the inventory decisions and policies at regional distribution centers and customer warehouses are made by the central distribution center. The entire network has its own ROP, ROQ, and service levels. Figure 2 shows typical inventory cycles of a part at a distribution center.

Classic inventory theory models the distribution of a part's demands parametrically, making particular assumptions about the variability of demands and lead times. When one sets up inventory policies such as reorder point and safety stock for an inventory network, the distribution of customer demands generally is assumed to be known such as normal distribution $(N - distribution)$ or negative binomial distributions to allow simple calculation of the demand's variance. However, in the aircraft spare parts market, demands of some spare parts fall other distributions such as Gamma distribution $(\Gamma - distribution)$ and Chi-Square distribution ($\chi^2 - distribution$), or some other parts may not have an analytical probability distribution function. Therefore, if we use normal distribution or negative binomial distribution to model the demands of parts while they have other distributions, there would be difference between targeted (or calculated) inventory policies and actual inventory operations. In recent years, with rapid development in advanced computing technology, authors [1, 8, 10] have been using bootstrap techniques to empirically calculate the demand during the lead time and the percentile of demand for the lead time window.

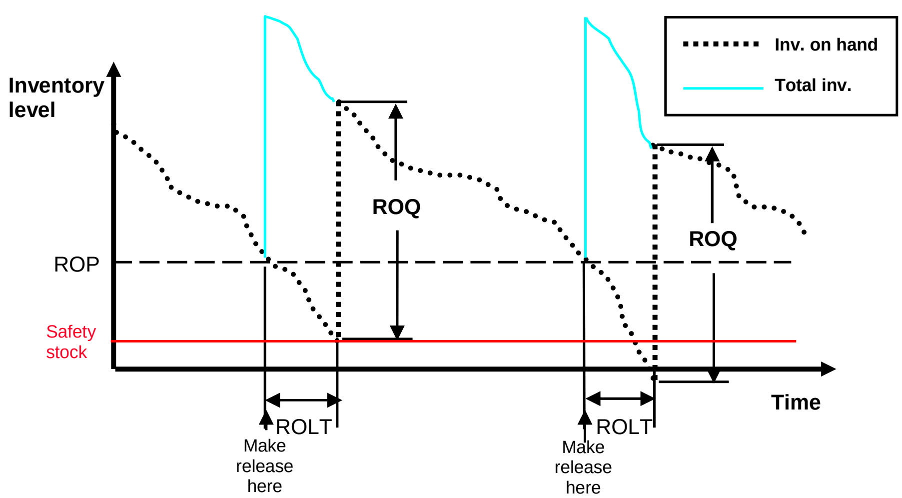

Figure 2 Inventory Cycles

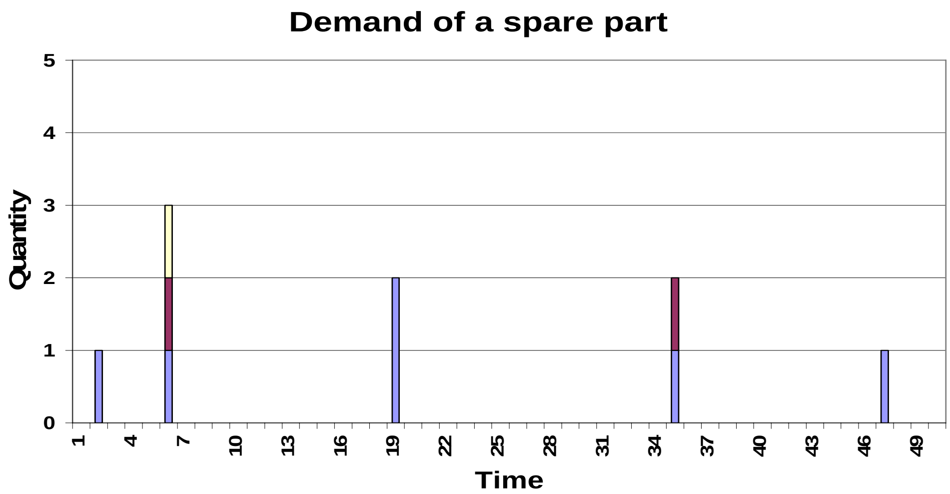

Figure 3 A Demand Example

Consider the demand of a spare part over time shown in Fig. 3.

Each bar represents the customers' demands at time. At the 6th and 35th time unit, there were three and two customer orders respectively, and each order had quantity one, and at other time units, there was either one customer order or no customer demand.

In the above example, the mean of demands is 1.13, and standard deviation is 0.35. The mean of intermittent intervals of customers' orders is 7.5, and the standard deviation is 7.04.

For an inventory distribution network, before the inventory policies, such as ROP and ROQ and service levels, are implemented, we would like to know if the designed inventory policies at distribution centers satisfy the actual inventory operation and customer demands, and what the minimum inventory cost is in order to achieve the targeted service levels from the operation point of view.

Inventory simulation is an excellent tool to get answers for above questions, and it would help us to make adjustments on inventory policies in order to achieve business goals. The following variables need to be considered while we simulate a distribution network:

- Demands of customer orders
- Time that customers place their orders, or the intermittent interval between two consecutive customer orders
- Re-order lead time

In this paper, in order to simplify the problem, we assume that the re-order lead time is constant and analyze the other two variables in the distribution network. We will study the impact of modeling other two variables and simulate the operation of the above distribution network.

3 Methods of modeling variables

3.1 Distributions

- Normal Distribution $(N-distribution)$

Normal distribution, $X \sim N(\mu,\sigma)$, is a commonly used method if the simulated variable is evenly distributed. The following chart (Fig. 4) shows the probability density function with various mean μ and variance σ.

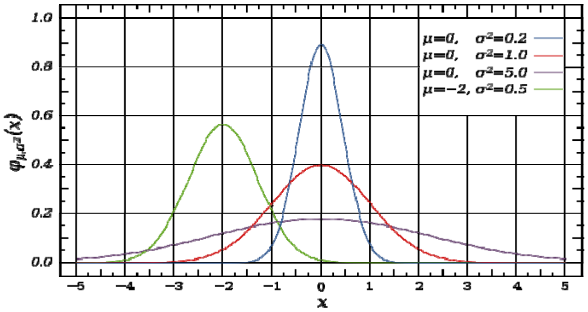

Figure 4 Normal Distribution

- Gamma Distribution ($\Gamma - distribution$)

Gamma distribution, $X \sim \Gamma(k,\theta)$, where k is called the shape parameter, and θ is called scale parameter, is an another commonly used method to model a random variable. Unlike normal distribution, the shape of probability density function of a Gamma distribution has a long tail.

The probability density function of x is:

$$f(x;k,\theta) = x^{k-1}\frac{e^{-x/\theta}}{\theta^k \Gamma(k)} \quad \text{for } X>0 \text{ and } k,\theta > 0$$

The following chart (Fig. 5) shows the probability density function with various values of k and θ.

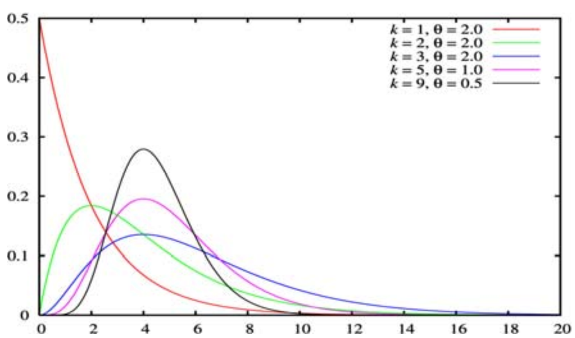

Figure 5 Gamma Distribution

- Chi-Square Distribution($\chi^2 - distribution$)

If $x_i \sim N(\mu,\sigma)$ for i=1,..,k, then $Q = \sum_{i=1}^{k} x^2{}_i$ has a chi-square distribution with k degrees of freedom (Q~ $\chi^2(k)$). The probability density function of x is:

$$f(x,k) = \frac{1}{2^{k/2}\Gamma(k/2)} x^{\frac{k}{2}-1} e^{-x/2} \quad \text{for } x \geq 0$$

The following chart (Figure 6) shows the various shapes of probability density function of chi-square distribution with different k. Similar to Gamma distribution, Chi-Square distribution has a long tail as well.

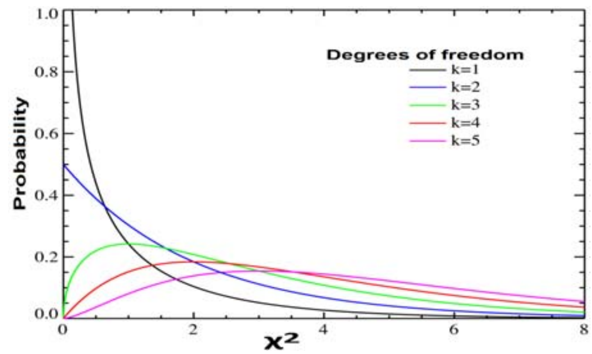

Figure 6 Chi-Square Distribution

- Exponential Distribution

The exponential distribution occurs naturally when describing the lengths of the inter-arrival times in a homogeneous Poisson process.

Let x be a random variable with exponential distribution with a rate of λ. Then the probability density function of x is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, x \geq 0 \\ 0, x < 0 \end{cases}$$

The following chart (Figure 7) shows the various shapes of probability density function of exponential distribution with different λ values

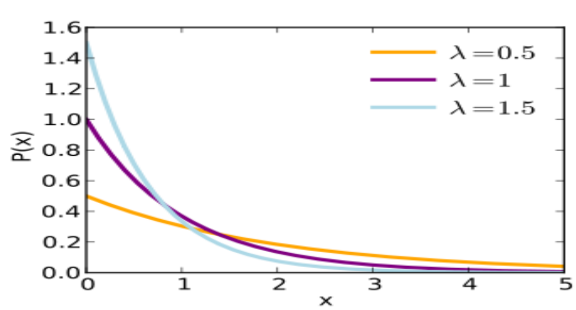

Figure 7 Exponential Distribution

3.2 Modeling Demand Variable

There are two ways to model the demand variable

- Use historical customers' demands

One way to model the demand of future customers' orders is to empirically use the historical customer orders and use random number to generate the future order sequence and generate random demand arrival times.

In the above example, the sequence of customer orders (the historical customer orders) is (1, 1, 1, 1, 2, 1, 1, 1) and there were 8 orders. We can use a random number generator between 1 and 8 to generate a random number sequence for the future customer orders. Depending on the length of the simulation, we can continue to use this historical order data to generate the time series of customer orders.

The advantage of this method is that it is close to what customers order. Sometimes, we may not be able to get an analytical probability distribution function (or exact distribution of the demands) for the demand variable of a part, and this empirical method generates future demands without using a distribution function. However if the historical time series is not longer enough, this data may not represent the demands in the market for the part and what would happen in the future.

- Use random variable distribution methods

If the distribution of demand of a part is known in advance, it would be useful to use the distribution to generate the customer demands and simulate the operation of the inventory network. We will demonstrate using various distributions such as normal distribution, gamma distribution, and chi-square distribution to simulate demands (or use these distributions to approximate the variable) and compare the results from those distributions under various service levels.

3.3 Modeling the intermittent interval (or inter-arrival) variable

We will use exponential distribution to simulate the intermittent interval between two customer orders.

4. Simulation

Step 1: Customers place their orders at either a regional center or the central distribution center. If the customer has a warehouse of the network at their site, the customer could place an order at the warehouse as well.

Step 2: If the customer warehouse or the regional center or the central distribution center has the part with the demanded quantity depending on where the order was placed, then the order will be shipped out from the regional center or the customer warehouse. In addition, if the inventory level is less than ROP at the warehouse or the regional distribution center or the central distribution center after the shipment, a replenishment order will be issued.

Step 3: If the customer warehouse doesn't have the part or doesn't have the right quantity to satisfy the customer's order, then a replenishment order will be issued to its regional distribution center to get the warehouse's ROQ.

Step 4: If the regional center doesn't have the part or doesn't have the right quantity to satisfy the customer's order, then a replenishment order will be issued to the central distribution center to get the regional distribution center's ROQ.

Step 5: If both the customer warehouse and the regional center have the part but either of locations has the right quantity to satisfy the customer's demand and the sum of quantities at both locations is greater than or equal to the customer's demand, then the order will be split , and parts will be shipped from both locations. Replenishment orders will be issued.

Step 6: If the inventory level at the central distribution center is less than ROP, then a replenishment order will be issued to suppliers.

Step 7: The lead-time at the central distribution center is from the time when it places an order to a supplier or suppliers to the time when it receives the order.

Step 8: The lead-time at a regional center is the shipment time from the central distribution center to the regional distribution center if the central distribution center has demanded part and quantity. Otherwise, the lead-time is the shipment time plus the lead-time at the central center

Step 9: The lead-time at a customer warehouse is the shipment time from the regional distribution center to the warehouse. Otherwise it is the shipment time plus the lead-time at the regional center.

In the following, we will look at a particular spare part and its inventory policies and simulate these policies under various modeling methods for customer demands. In addition, we will compare the results and modeling methods and the impacts to the inventory policies. Furthermore, we assume that the reorder lead time is fixed, and the intermittent arrival has an exponential distribution $x \sim Exp(1/\lambda)$. We use the exponential distribution to generate the intermittent arrival time variable that a customer order arrives, and use one of modeling methods to generate demands.

Table 1 shows that the Normal Distribution has the highest service level, but the Gamma distribution has the closest service level to the targeted one.

Table 2 shows that the Chi-Square Distribution has the highest service level, but the normal distribution has the closest service level to the targeted one.

Table 3 shows that the Chi-Square Distribution has the closest service level to the targeted one.

Table 1. Simulations for inventory policies with 70% service level

Modeling Type	Targeted SL	Avg. Simulation SL	ROP	Safety Stock	ROQ
Normal Distribution	70%	76.2%	19	1.28	30
Gamma Distribution	70%	71.0%	19	1.28	30
Chi-Square Distribution	70%	73.4%	19	1.28	30

Table 2. Simulations for inventory policies with 90% service level

Modeling Type	Targeted SL	Avg. Simulation SL	ROP	Safety Stock	ROQ
Normal Distribution	90%	89.3%	21	3.14	30
Gamma Distribution	90%	87.6%	21	3.14	30
Chi-Square Distribution	90%	92.1%	21	3.14	30

Table 3. Simulations for inventory policies with 99% service level

Modeling Type	Targeted SL	Avg. Simulation SL	ROP	Safety Stock	ROQ
Normal Distribution	99%	97.2%	24	5.7	30
Gamma Distribution	99%	95.7%	24	5.7	30
Chi-Square Distribution	99%	98.5%	24	5.7	30

5 Conclusions

This paper has presented a simulation approach to analyze an inventory distribution network by using various modeling methods to customer demands and compare the results of those methods. How the distribution network reacts to the inventory policies depends on what distribution the variable of customer demands is, and in order to achieve inventory management goals, the targeted inventory policies may have to be adjusted accordingly.

6 Contact Author

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