

REAL TIME MATHEMATICAL MODELING OF THE AERODYNAMIC CHARACTERISTICS OF AN AIRCRAFT ENCOUNTERING VORTEX WAKES

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Abstract

The work describes formation and evolution of vortex street turbulence behind an aircraft in a turbulent atmosphere. It suggests the approach to creation of a mathematical model of aircraft aerodynamics in the in-flight refueling mode based on the use of artificial neural networks. Accuracy appraisal and response time of program modules of the flight simulator software assessment when simulating aerial refueling have been conducted.

The main purpose of this work is to develop program modules of software of an aviation training device or a flight simulator when the aerial refueling is simulated. IL-78 is considered as an aerial tanker and Il-76 as a receiver aircraft. During the in-flight refueling (IFR) the receiver aircraft is subjected to the nonhomogeneous field of velocities caused by the jet vortex wake of the aerial tanker, in which case additional aerodynamic forces and moments acting on the receiver do emerge. The paper developed a mathematical model of aerodynamic characteristics of aircraft in jet vortex wake based on the set of aircraft flow-around calculation programs and wake flows. This technique can be used to model a flight in case one aircraft inadvertently enters the other aircraft's wake.

1 Problem setting

The created mathematical model of generating aircraft implemented flow without separation with position of flight controls and devices corresponding to the cruising mode. The geometrical parameters of the aircraft are the following: length – 47.5 m, wingspan $l =$

51.6 m, mean aerodynamic chord (MAC) of the wing $l_{CAX} = 7.164$ m, wing setting angle $\alpha_0 = 3^\circ$, $S_{\text{крыла}} = 300$ m.

The range of tanker flight modes: from 159 m/s to 197 m/s by velocity which corresponds to M 0.5-0.62 at 6000 m. Flight weight ranges from 126 to 186 tonnes.

1.1 Calculation method

The panel method determined the distribution of circulation and the shape of vortex sheet immediately behind the generating aircraft [1]. To define the near wake flow parameters (up to 420 meters), a method utilizing 2D Navier-Stokes transient equations averaged by Reynolds has been used.

Far wake flow parameters were determined by means of a simplified evolution model of two vortices dissipating in a turbulent atmosphere. To find additional forces and moments acting on the aircraft in the wake, panel method calculations were made in quasistationary setting. The problem partition is shown in Figure 1.

2 Problem solvation

2.1 Near wake parameters calculation

A model [2] – [4] has been specifically developed to calculate the diffusion of jet vortex wake based on the invariant modeling method developed under the supervision of Donaldson in the University of Princeton [5]. The equations for calculation of turbulent flow in a vortex wake have been established in the work [3] on the assumption that a parameter, comprising the

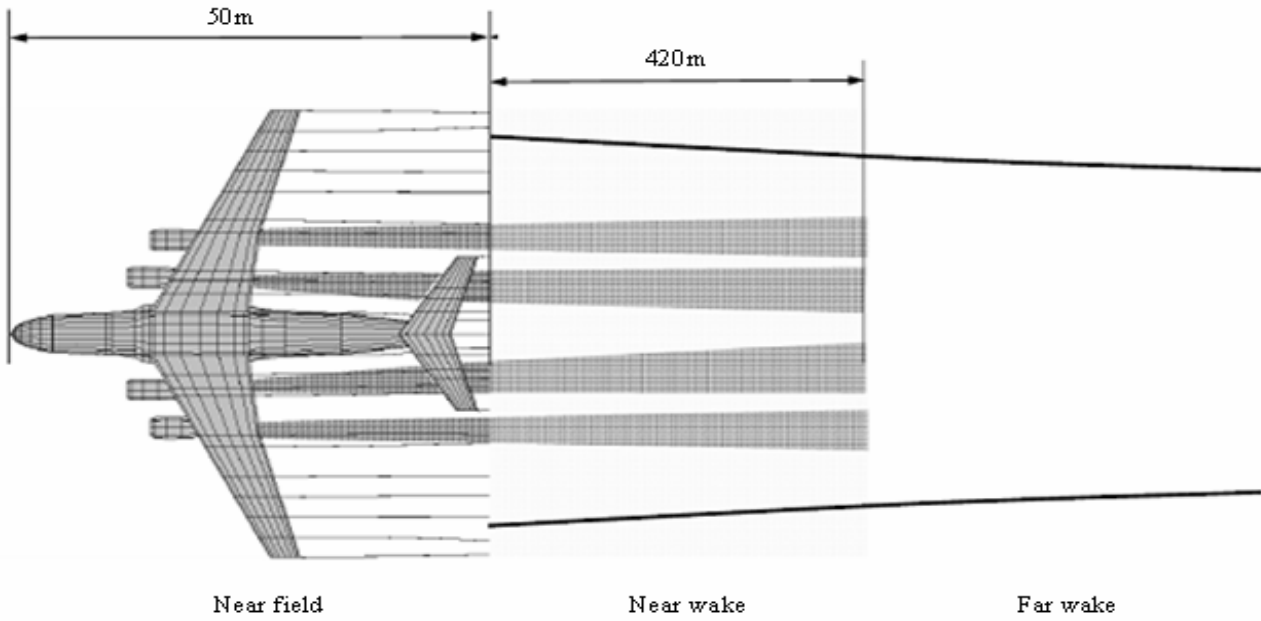


Fig. 1 Problem fragmentation plan

equations and responsible for a specific magnitude of typical size of large turbulent vortices, is constant within the entire flow field. This parameter is called macroscale of turbulent flow. Its value for wake vortex according to [4]:

$$\Lambda_0 / b = 0.015 \quad (1)$$

where b is a distance between the centers of vortices.

Attempts to apply the model [3], [4] to the calculation of the wake vortex lead to a heavy flow diffusion in vortex cores [6]. To eliminate this drawback, it is assumed in this work that macroscale Λ depends on the characteristics of longitudinal vorticity fields. In this case, the system of equations for calculation of the averaged turbulent flow functions can be recorded as follows:

$$\frac{D_u}{D_t} = \frac{1}{4} L_u + \frac{\Lambda_q}{4} \Delta_u \quad (2)$$

$$\begin{aligned} \frac{D_\omega}{D_t} = & \frac{1}{2} L_\omega - \frac{\partial^2(\Lambda_q)}{\partial y \partial z} \frac{\partial^2 \psi}{\partial y \partial z} + \\ & + \frac{1}{4} \left(\frac{\partial^2(\Lambda q)}{\partial y^2} - \frac{\partial^2(\Lambda q)}{\partial z^2} \right) \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial y^2} \right) + \\ & + \frac{\Lambda q}{4} \Delta \omega - \frac{g}{T_\infty} \frac{\partial T}{\partial z} \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{Dq}{Dt} = & \frac{\Lambda}{4} \left(\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial q}{\partial z} \right)^2 + 4 \left(\frac{\partial^2 \psi}{\partial y \partial z} \right)^2 + \left(\frac{\partial^2 \psi}{\partial z^2} - \frac{\partial^2 \psi}{\partial y^2} \right)^2 \right) \\ & + 0.3 \Lambda \left(\left(\frac{\partial q}{\partial y} \right)^2 + \left(\frac{\partial q}{\partial z} \right)^2 + q \Delta q \right) \frac{q^2}{8 \Lambda} + 0.3 L q \frac{\Lambda g}{3 T_\infty} \frac{\partial T}{\partial y} \end{aligned} \quad (4)$$

$$\frac{DT}{Dt} = \frac{1}{3} L T + \frac{\Lambda q}{3} \Delta T \quad (5)$$

$$\Delta \psi = -\omega \quad (6)$$

Here x, y, z is a Cartesian system of coordinates connected with the aircraft. X-axis is directed along the velocity vector of the approach flow; y-axis is directed upward; t – time; u_∞ – flow velocity; u – longitudinal velocity; ψ – flow function of cross current; ω – longitudinal component of vorticity; T_∞ – potential temperature in Kelvin ($T_\infty = T_{*\infty} + gy/c_p$), where $T_{*\infty}$ – absolute temperature); T – potential temperature deviation from the equilibrium value; g – gravitational acceleration; $q = \sqrt{u'^2 + v'^2 + w'^2}$ – level of turbulent velocity fluctuations; differentiation operators:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{\partial \psi}{\partial z} \frac{\partial}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial}{\partial z}$$

$$L = \frac{\partial(\Lambda q)}{\partial y} \frac{\partial}{\partial y} + \frac{\partial(\Lambda q)}{\partial z} \frac{\partial}{\partial z}$$

$$\Delta = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

Calculations by formulas (2) – (6) are supposed to be made starting from a certain section $x = x_*$, where longitudinal velocity slightly differs from the inflow velocity and the temperature from that of the ambient air. In this work $x_* = 50\text{m}$. According to the transient similarity, the solution of a system of 2D transient wake equations, obtained at the point in time t , is equivalent to the solution for a 3D steady-flow wake in section $x = u_\infty t$.

The above equation system is not closed. To close it, the Λ should be defined in the entire flow field. Λ value measures the effective turbulent viscosity. The higher is the Λ value, the faster are heterogeneities diffused in the vorticity fields, axial velocity and temperature. We have already supposed that Λ depends on the flow swirling. Flow relaminarization is even observed in vortex cores, i.e. drastic reduction of turbulent viscosity and therefore Λ value. The question is to select the magnitude depending on the vorticity distribution.

In this work it is assumed that the Λ parameter, which is proportional to the scale of turbulence, satisfies relation (1) far from the vortex center (1) whereas close to the vortex core, and is proportional to the distance from the core. A composite magnitude is built in the entire flow field. For this purpose a model and close to experimentally observed vorticity profile in the core of the turbulent vortex $\omega(r) = \alpha e^{-\beta r^2}$ was used, where r is the distance from the vortex core, α and β are dimensional constants responsible for intensiveness and size of the vortex. The solution for magnitude of macroscale Λ is sought in the form of:

$$\frac{\Lambda}{\Lambda_0} = \left\{ \frac{(\omega_{\max}^2 - \omega^2)^2}{C^4 \Lambda_0^4 (\Delta \omega)^2 \omega_{\max}^2 + (\omega_{\max}^2 - \omega^2)^2} \right\}^{1/4} \quad (7)$$

where Λ_0 is determined in accordance with formula (1), ω_{\max} – maximum vorticity value, C – a dimensionless constant unknown in advance. Away from the vortex cores $|\omega| \ll \omega_{\max}$, therefore $\Lambda \approx \Lambda_0$. At $r \rightarrow 0$ we have $\Lambda \rightarrow r/(\sqrt{2}C)$. The numeric value of the unknown constant is found from the comparison of the experimental and calculated data:

$$C = 5 \quad (8)$$

Relations (7), (8) close the system of turbulent flow equations.

Equations (2) – (8) have been solved in rectangular area $y_1 \leq y \leq y_2, -z_1 \leq z \leq z_1$. Let us establish the boundary conditions. Vorticity fields, temperatures and longitudinal velocities and intensiveness have been determined from the problem solution in the near field. It was believed that in sections $x \geq x_*$ at boundaries $y = y_1, z = -z_1, z = z_1$ the vorticity is equal to zero while the temperature is equal to that of the ambient air. At the upper boundary $y = y_2$ the following boundary conditions were established: $\partial \omega / \partial y = 0, \partial T / \partial y = 0$ and $\partial q / \partial y = 0$.

Let us turn to boundary conditions for q function. At the boundaries of the calculated area for q function, instead of the typical value q_L , characterizing the intensity of turbulent disturbances with L scale, q_{Λ_0} value was given, which corresponds to intensiveness of turbulent pulsations of Λ_0 scale. According to [7]:

$$\frac{q_{\Lambda_0}}{q_L} = \left(\frac{\Lambda_0}{L} \right)^{1/3}$$

The boundary condition for flow function is defined from relation [8]:

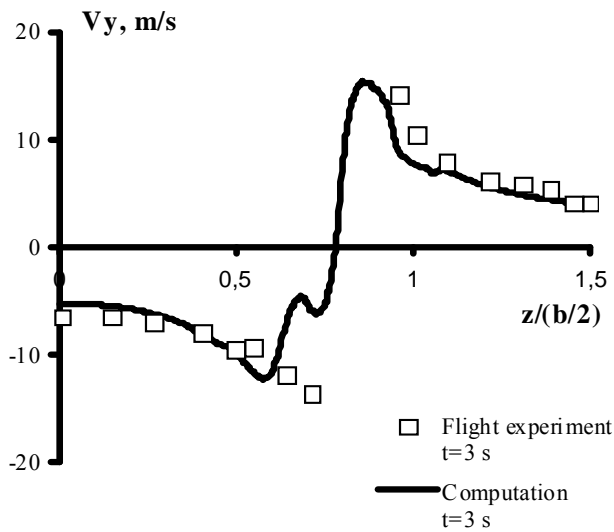
$$\psi(y, z) = -\frac{1}{4} \int \omega(y', z') \ln((y - y')^2 + (z - z')^2) dS(y', z'),$$

where dS is the cross-section area element.

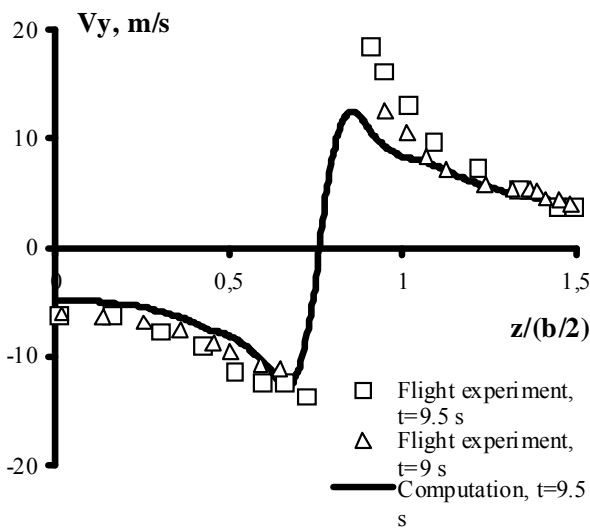
Finite-difference equations were recorded by means of the implicit scheme of alternating direction method of the second-order accuracy [9]. To verify the equations and the calculation

program, the estimated and experimental data have been compared.

The work [10] presents the measured data of velocity profile in the wake behind a high-flying A321 aircraft. Figure 2 present measured and estimated profiles of vertical velocity along the line passing through the centers of vortices. The inflow velocity was $u_{\infty}=60$ m/s. Transverse distance z is attributed to semispan $z_1 = z/(b/2)$. During the flight experiment the measurements were made in $t = 3; 9$ and 9.5 s after the aircraft passage.



a)



b)

Fig. 2 Measured and estimated profiles of vertical velocity along the line passing through the centers of vortices ($t=9$ s; $t=9.5$ s)

Also the vorticity fields and longitudinal velocity measured at TsAGI were used. Tests

were made using a mock aircraft at the inflow velocity of 25 m/s. The fields of velocity measured in cross-section at the distance of 0.6 span from the wing were used as the initial data in numerical computation. It was accepted that the intensity of atmospheric turbulence q equals 0.1 m/s.

By means of equations (2) – (8) the near wake of Il-78 was calculated. The near wake area started in the section of 50 m from the IL-78 aircraft nose and represented a parallelepiped 420 m long, 100 m high and 100 m wide (see Figure 1). The design area ranged from $y = -75$ m to $y = +25$ m altitudinally. The design area is located symmetrically relative to the aircraft's plane of symmetry from $z = -50$ m to $z = +50$ m latitudinally. The flow parameters of the near wake were calculated for the initial data corresponding to a basic flight mode: flight altitude 6000 m, flight speed 178 m/s, aircraft weight 156 t.

Mathematical simulation data allow to obtain a detailed picture of the field of velocities in the aircraft's wake. The obtained fields of disturbance velocities are used to identify the loads acting on the receiver.

2.2 Receiver aircraft model

The mathematical model of the receiver aircraft in panel representation is given in Figure 3.

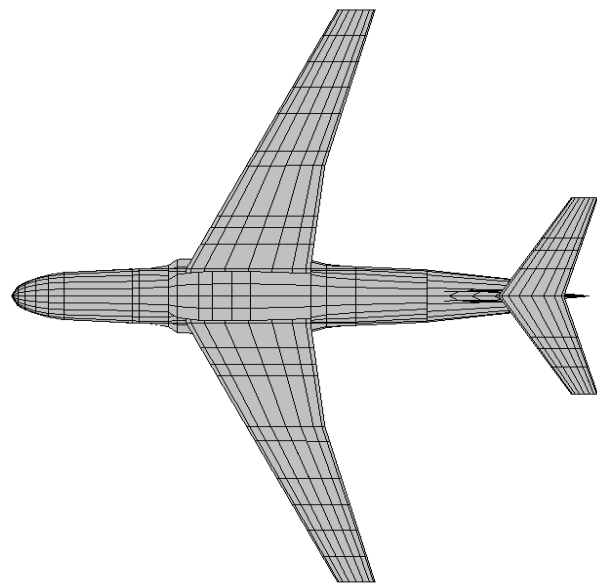


Fig. 3 Mathematical model of the receiver aircraft

The total number of panels for the description of the aircraft model is about 1200.

Aerodynamic forces and moments acting on the receiver aircraft are calculated in the following ranges of aircraft attitude and angular position relative to the aerial refueler:

- longitudinal position from 53 to 10000 meters from the aircraft nose;
- vertical position from –300 to 100 meters;
- lateral position from –150 to 150 meters from the plane of symmetry;
- pitch angle from 2° to 6° (by fuselage water line);
- yaw angle from -2° to 2° ;
- bank angle from -4° to 4° .

Flight speed coincides with that of the tanker by magnitude and direction.

The additional forces and moments caused by the influence vortex wake on the receiver aircraft were determined subject to the following algorithm:

1. Parameters of relative and angular position of the aircraft in the wake were set.
2. Aerodynamic forces and moments in the homogenous flow were determined by means of the panel method.
3. Velocities disturbed from jet-vortex wake in control points of the panels were found using interpolation.
4. Flow-field analysis with regard to additional velocities is made; aerodynamic forces and moments are computed.

Additional forces and moments were computed by subtracting the values received at Stage 2 from those obtained at Stage 4.

2.3 Neural network approximation of additional forces and moments

The evaluation of additional forces and moments acting on the aircraft in the wake vortex takes about 10 seconds per one reference point using an up-to-date computer. When using these data for modeling of refueling dynamics at a flight training device in real-time mode, the time for aerodynamics definition should be reduced to 0.001 s, which is determined by a temporal integration step of aircraft equations. To solve this problem, the approximation technique of the obtained array of aerodynamic

characteristics with the help of artificial neural networks was used in this work. With such approach, the pre-taught artificial neural networks were used as program modules of aircraft aerodynamics in the software of the flight training device. This allowed to drastically reduce the calculation time of aerodynamic characteristics of the following aircraft (up to 0.0001 s) with insignificant degradation of accuracy in their determination.

Neural networks of a multilayer perceptron type with two hidden layers were utilized [11]. The number of neurons on the first and second layer is 11 and 5 respectively. Input vector of neural networks contained the values of three coordinates of the receiver aircraft relative to the tanker, three angles determining angular position of the aircraft, the tanker's flight speed and weight, while the output was aerodynamic force and moment increment caused by the tanker' wake. Altogether 6 neural networks for approximating three force coefficients (c_x , c_y , c_z) and three moment coefficients (m_x , m_y , m_z) were taught.

To generate a set of patterns, the neural networks were taught and tested by a total of 200,000 calculations made at random values of the receiver position, flight speed and flight weight of the tanker. The ranges of the calculated parameters are given above.

Upon teaching the neural networks, the accuracy assessments of force and moment increment caused by the aircraft's getting in wake vortex were carried out to compare them with the characteristics calculated on the basis of the panel program.

Overall assessment of accuracy in determining additional forces and moments was made. Mean square deviations of approximation error were $\sigma(\delta c_x) = 0.0005$, $\sigma(\delta c_y) = 0.0067$, $\sigma(\delta c_z) = 0.0027$, $\sigma(\delta m_x) = 0.0013$, $\sigma(\delta m_y) = 0.0009$, $\sigma(\delta m_z) = 0.0066$. Here δ means the difference between an increment of aerodynamic coefficients obtained by calculation and approximation by neural networks.

The distribution of probability density f of approximation errors of δc_x , δc_y , δm_x and δm_z increments, obtained throughout the whole range of patterns, is given in Figure 4.

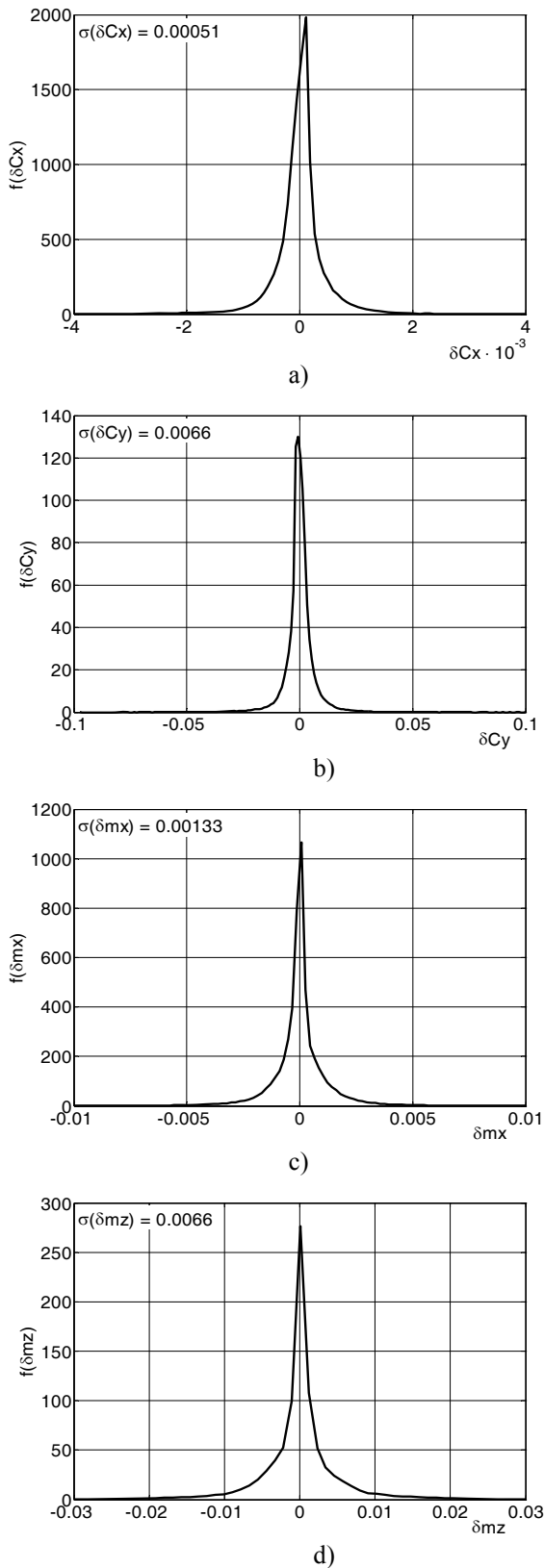


Fig. 4 The distribution of probability density f of approximation errors of a) δc_x , b) δc_y , c) δm_x , d) δm_z increments, obtained throughout the whole range of patterns

To test the calculated modules and analyze the results, isolines of additional forces and moments acting on the receiver aircraft in several sections $x=\text{const}$ in the wake of the tanker aircraft were computed. Figures 5 – 8 show the images of isolines of the resistance coefficient, lift coefficient, roll and longitudinal moments of the receiver aircraft while situated in section $x=80$ m from the aircraft nose obtained by means of neural network approximators. The flight speed and weight of aerial refueler were 178 m/s and 156 t respectively. The receiver aircraft's pitch angle was 2° , while roll and yaw angles were equal to zero.

The diagrams show that the most significant loss of lifting force is observed when the receiver aircraft stays in the Il-78 tanker's plane of symmetry. An additional nose-up moment occurs in this zone, too. It is seen that the maximum lift change and longitudinal moment are spaced by height.

It should be noted that vortex wake exercises the biggest influence on the change of roll moment (Figure 8). In general, the rolling and turning moments directed to the plane of symmetry of the tanker act on the following receiver aircraft.

3 Conclusion

Program modules for calculation of corrections to the aircraft's aerodynamic coefficients in the vortex wake behind the Il-78 aerial refueler may be used as part of the software of flight simulators to imitate in-flight refueling mode. The application of artificial neural networks for approximation of additional forces and moments provides the necessary speed for modeling in real time mode. Calculation time is about 0.0001 s on a modern computer. These modules can be used both for modeling a flight in inadvertent getting of one plane in the wake of the other, however in this case the difference in velocity vectors of the two aircrafts and characteristics of the turbulent atmosphere should be taken into account.

The technique suggested in this article has been approved when modeling dynamics of the

aircraft flight while getting into the vortex wake at the flight simulator PSPK-102 TsAGI [12] and flight training device MIPT (Moscow Institute of Physics and Technology).

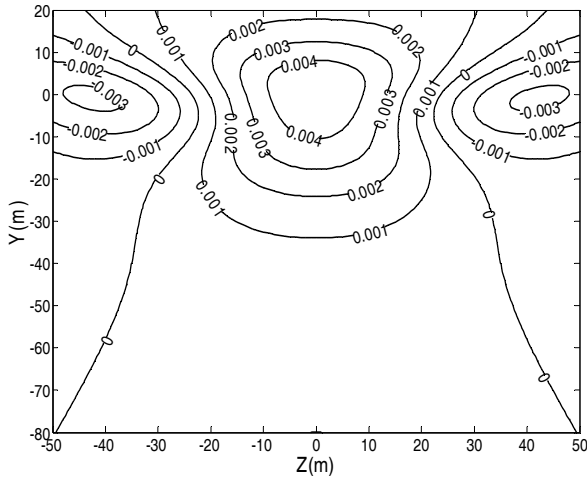


Fig. 5 Drag coefficient increment isolines

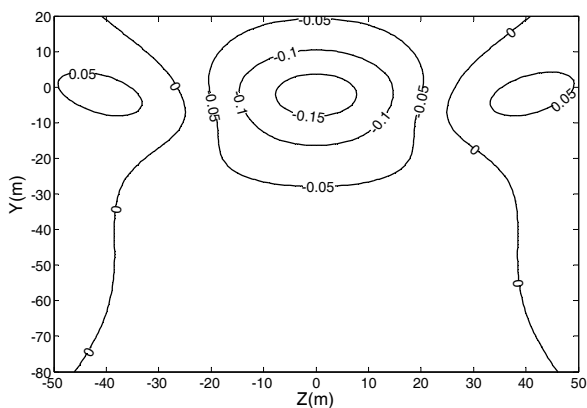


Fig. 6 Lift force coefficient increment isolines

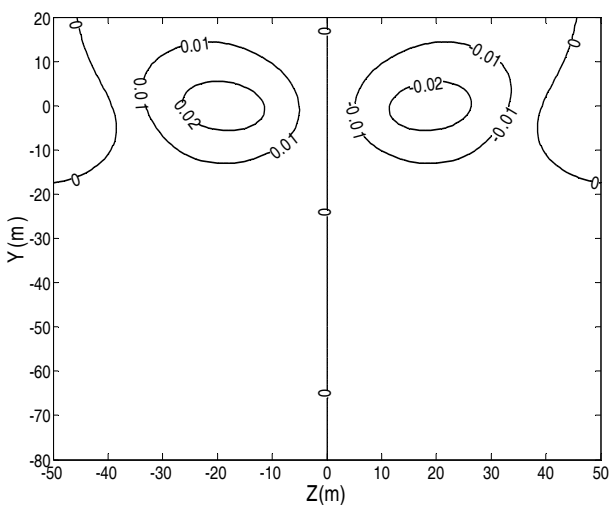


Fig. 7 Rolling moment coefficient increment isolines

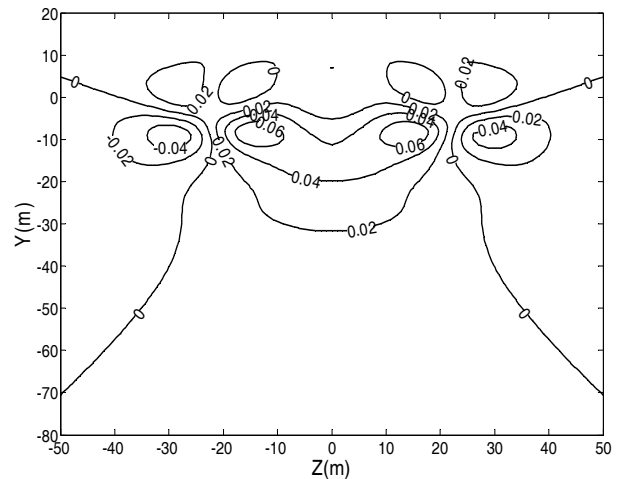


Fig. 8 Pitching moment coefficient increment isolines

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