DYNAMIC CONTROL OF PLATE WITH EMBEDDED SHAPE MEMORY ALLOY WIRES

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Abstract

Smart materials such as shape memory alloys (SMAs) and piezoelectric ceramics are increasingly being used in vibration control applications. In this paper, the alteration in the natural frequency of a composite structure with embedded NiTiNOL-based shape memory alloy wires in it will be presented.

The governing equations will be solved analytically and numerically to show the effects of recovery stresses produced in wires and their subsequent effects on stiffness of the plate and hence its natural frequency. The results show that the natural frequencies of the composite plate change slightly at a temperature above the austenite finish temperature of the embedded pre-strained SMA wires. Different SMA wires arrangements are studied in order to optimize the control strategy.

1 Introduction

The technology of smart materials and structures has given a new face of development to the fields of aerospace, robotics and structural engineering due to which the demand for less weight stand-alone systems is growing. Smart materials [1] represent a group of functional materials, which can sense and respond to changes in their environment in a predefined manner. They also have the flexibility to go back to their initial stage once the changes in the environment are vanished.

Shape Memory Alloys (SMA's) are smart materials which have the ability to return to a predetermined shape when heated. When an SMA is cold, or below its transformation temperature, it has a very low yield strength and can be deformed quite easily into any new shape, which it will retain. However, when the material is heated above its transformation temperature it undergoes a change in crystal structure which causes it to return to its original shape. Depending on the temperature of the system, the large induced strain (order of 10%) is recovered either in a hysteresis loop upon unloading or upon heating the material. If the SMA encounters any resistance during this transformation, it can generate extremely large forces. This phenomenon provides a unique mechanism for remote actuation.

Investigations in the past decade have demonstrated that shape memory hybrid composites have many novel applications in the areas of shape control, fatigue resistance, vibration damping, acoustic radiation and transmission control and impact resistance [2]. Also, they discuss the effectiveness of SMA's in damping vibrations [3,4]. Active damping implies the activation of a mechanism by a control unit in order to reduce noise and vibrations. Shape memory alloys offer new possibilities for active damping devices.

2 SMA Mathematical Modeling

The three-dimensional constitutive model proposed by J. G. Boyd and D.C. Lagoudas [5,6] is used in this study. It has been identified as one of the most versatile SMA constitutive model. It can be described by using the phase transformation diagram shown in Fig. 1, where lines "*ab*" and "*cd*" denote the respective start and finish lines of the forward phase transformation (austenite to martensite), and lines "*ef*" and "*gh*" denote the start and finish lines of the reverse phase transformation (martensite to austenite) respectively. The critical stresses at the start and finish of the conversion of the martensite variants (twinned and detwinned) are denoted by σ_s and σ_f, respectively. Also, the slope of the forward and reverse phase transformation lines are denoted by C_M and C_A, respectively.

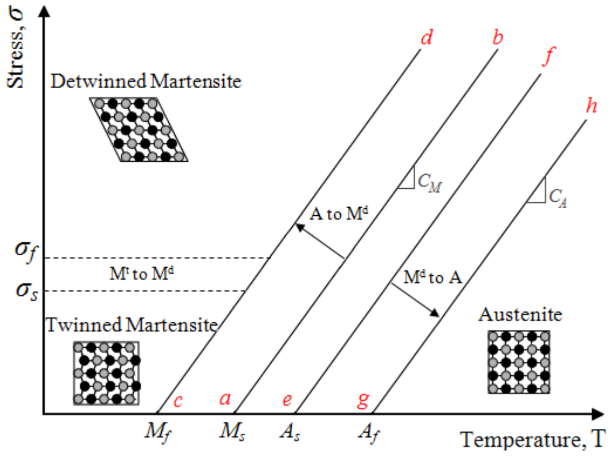

Fig. 1. Schematic of Stress-Temperature Phase Diagram for SMA [7]

The stress-strain relationship can be expressed as Eq. (1)

$$\Delta\varepsilon_{ij} = S_{ijkl}\,\Delta\sigma_{kl} + \alpha_{ij}\,\Delta T + Q_{ij}\,\Delta\xi \qquad (1)$$

where S_{ijkl}, σ_{kl} *and* ε_{ij} denote the elastic compliance, stress and strain tensor, respectively. α_{ij} and T are thermal expansion vector and temperature, respectively. The special characteristics of SMA's are attributed to microstructural changes affecting the quantitative relations between martensitic and austenitic phases. This phase transformation process is controlled by the evolution of an internal variable ξ that measures the martensitic volume fraction. ξ is governed by temperature and stress and varies from zero to one with unity representing 100% martensite. Tensor Q_{ij} is expressed by

$$Q_{ij} = \Lambda_{ij} + \Delta S_{ijkl}\,\sigma_{kl} + \Delta\alpha_{ij}\,(T - T_0) \qquad (2)$$

where Λ_{ij} is the transformation tensor and T_0 is the reference temperature.

Adopting the thermomechanical model of Lagoudas et al. [5], the transformation strain rate is given by

$$\dot{\varepsilon}^{I}_{ij} = \Lambda_{ij}\,\dot{\xi} \qquad (3)$$

Where

$$\Lambda_{ij} = \begin{cases} \dfrac{3H}{2\bar{\sigma}}\sigma'_{ij}, & \dot{\xi} > 0 \\ \dfrac{H}{\bar{\varepsilon}^{I}}\varepsilon^{I}_{ij}, & \dot{\xi} < 0 \end{cases} \qquad (4)$$

Here $\bar{\sigma} = \left[(3/2)\sigma'_{ij}\sigma'_{ij}\right]^{1/2}$ is the effective stress, $\bar{\varepsilon}^{I} = \left[(2/3)\varepsilon^{I}_{ij}\varepsilon^{I}_{ij}\right]^{1/2}$ is the effective transformation strain and $H = \varepsilon^{I}_{max}$ is the maximum axial transformation strain.

The characteristic temperatures of SMAs are the stress dependent temperatures at which the forward and reverse martensite-austenite transformations start and finish. M_s, M_f, A_s and A_f are the martensitic start, martensitic finish, austenitic start and austenitic finish temperatures under stress-free state, respectively.

The state equation for the martensitic volume fraction is

$$\dot{\xi} = \frac{A\,\dot{\sigma}_{ij} + B\,\dot{T}}{\rho\left[b_7 + \dfrac{3}{2}H^2 b_4\right]} \qquad (5)$$

where

$$A = \Lambda_{ij} + \Delta S_{ijkl}\,\sigma_{kl} + \Delta\alpha_{ij}\,\Delta T \qquad (6)$$

$$B = \Delta\alpha_{ij}\,\sigma_{ij} + \rho\left[\Delta c\ln\left(\frac{T}{T_0}\right) + \Delta s_0\right] \qquad (7)$$

also ρ is the density of SMA and b_7 and b_4 are constants.

A computer code based on the above formulation is developed for 1-D analysis of SMA wires. Material properties of the Nickel-Titanium alloy are listed in Table 1.

Table 1. Material Properties of the Ni-Ti Alloy [5]

Mechanical Properties	Transformation Temperatures
$E_A = 30.0$ GPa	$M_f = 260.0$ K
$E_M = 13.0$ GPa	$M_s = 273.0$ K
$\alpha_A = 11.0\times10^{-6}$ 1/K	$A_s = 299.4$ K
$\alpha_M = 6.6\times10^{-6}$ 1/K	$A_f = 314.7$ K
$\rho = 6450$ kg/m³	

3 Smart Vibration Control System

In the proposed mechanism, SMA wires are embedded into the plate. Incorporating pre-strained shape memory alloy (SMA) wires into the plate allows the development of new 'smart' materials, which can change their vibration frequency in a reversible way with temperature changes. This smart composite plate is shown in Fig. 2. Boundary conditions of the plate can be free (F), simply-supported (S), clamped (C) or any combinations of them [8,9]. Embedded SMA wires are fixed at both ends. During heating the SMA wires by a heat power supply, the pre-strained martensitic wires cannot recover their original shape during the reverse transformation to the austenitic phase, so the recovery stress is increased in the wires and therefore the stiffness of the composite plate increases. Finally, the natural frequencies (f_n) increase by increasing the stiffness of the composite plate. As a result, transformation recovery stresses are generated in the composite, leading to a shift in the resonance vibration frequency.

The changes of tensile modulus and the internal recovery stress due to the thermal expansion and phase transformation (martensite to austenite) of the wires are considered in this study. Also the effect of direction of SMA wires on the response of the composite plate is studied.

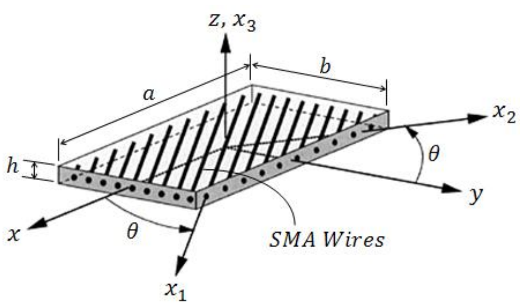

Fig. 2. Smart Composite Plate

It is assumed that the plate is made up of Aluminum. Its length, width and thickness are 0.3m, 0.3m and 0.001m, respectively. Radius of each SMA wire (r) is 0.333mm.

The finite element method is used for obtaining natural frequencies and dynamic response of system to an external excitation with a specific frequency. In this situation by using ANSYS as a FEM software, a static analysis followed by a modal and harmonic analysis is performed at each temperature. In fact, the static analysis is done in order to apply the stress produced by the phase transformation. These pre-stresses are a function of material characteristic in the corresponding temperature. Therefore, after activating the SMA wires, it is possible to perform a pre-stressed modal analysis to find out the effects of activated wires on the natural frequencies of the system. Similarly, the harmonic analysis can be carried out and as a result of this analysis, the resonance frequencies and amplitude of the plate vibrations are established.

3.1 SMA Wires Actuation

The pre-strain of SMA wires before mounting on the system is about 5 percent. In the initial condition, the plate and the wires are at ambient temperature. By increasing the temperature of the SMA wires to a higher value than the austenite start temperature, A_s, and starting the phase transformation, wires are activated. As wires are constrained to the plate, the strain recovery induced by shape memory effect is not possible; hence, the stress goes up in the wires. Simultaneously, the elastic modulus of wires is increased as well.

The elastic modulus and stress of the SMA wires at each temperature above the austenite start temperature are indicated by a specific value. By using a developed code which is written in Maple software, these values are calculated up to the temperature of 400 Kelvin.

Table 2. Young's Modulus and Stress of SMA Wires at 8 Different Temperatures

T (K)	E (GPa)	σ (MPa)
298.1	13.00	0
299.4	13.00	-0.11
300.0	13.09	3.69
320.0	16.03	126.73
340.0	18.33	242.71
360.0	20.18	352.28
380.0	21.71	456.16
400.0	22.99	555.01

According to Table 2, the ambient temperature is considered as the start point which is 298.1K. This increase in temperature up to austenite start temperature (299.4K) will not cause any increase in the wires stress and the elastic modulus because no phase transformation happens. Between these two temperatures, SMA wires are at martensite phase. By increasing the temperature above the austenite start temperature and starting the phase transformation from martensite to austenite, the elastic modulus and stress of the wires will be increased. This increase in the elastic modulus and stress of the wires will lead to structural stiffening and hence, enhancing the natural frequencies of the system.

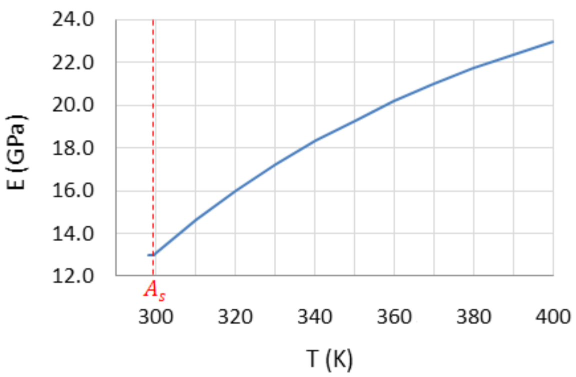

Fig. 3. Young's Modulus Changing by Temperature in SMA Wires

Fig. 3 illustrates the trend of elastic modulus changes. From T_0 to austenite start temperature, the phase transformation is not occurred, so the elastic modulus is constant.

4 FEM Modeling of SMA Wires

Nonlinear behavior of the material cannot be considered in the modal analysis. Therefore, in order to model the nonlinear behavior of SMAs in ANSYS software and calculation of natural frequencies, corresponding mechanical properties of SMA wires are defined. These mechanical properties that are Young's modulus and thermal expansion coefficient are constant at each temperature.

Uniaxial stress in the wires can be produced thermally instead of applying a mechanical load. Therefore, equivalent thermal expansion coefficient will be defined at each temperature. In fact, these equivalent thermal expansion factors are defined in a way that the corresponding stress in that temperature will be induced in the wire.

As the SMA wires are assumed to be isotropic, their properties in different directions are similar. Thus, according to the Hook's law and by comparing mechanical strain due to the stress and thermal strain induced by temperature, the equivalent thermal expansion coefficient, α, will be obtained in each temperature as in Eq. (8)

$$\alpha = \frac{\sigma}{E\left(T - T_0\right)} \tag{8}$$

where σ, E and T are axial stress, Young's modulus and temperature, respectively. T_0 is the reference (ambient) temperature and equals to 298.1K. The results for different temperature values are shown in Table 3.

Table 3. Equivalent Thermal Expansion Coefficient of SMA Wires at 8 Different Temperatures

T (K)	α (1/K)
298.1	0
299.4	6.889×10^{-6}
300.0	-1.525×10^{-4}
320.0	-3.619×10^{-4}
340.0	-3.165×10^{-4}
360.0	-2.823×10^{-4}
380.0	-2.567×10^{-4}
400.0	-2.370×10^{-4}

Now it is possible to model the reinforced structure by defining variable properties with temperature in ANSYS. Then, the natural frequencies at each temperature will be calculated by a modal analysis.

5 Results

In this research work, it is observed that the natural and resonance frequencies of the composite plate increase with continuously increasing the temperature of SMA wires above austenite start temperature. According to Fig. 4, which shows the effect of temperature on natural frequencies of the plat, with all simply supported edges, the x-axis represents the ratio of natural frequency of the structure at each temperature to that at reference temperature, T_0. The y-axis indicates the wire temperature.

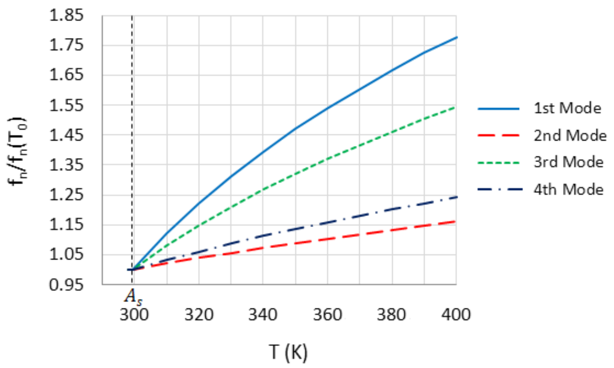

Fig. 4. Effect of Temperature on Natural Frequency (Simply-Supported Edges)

For analyzing the dynamic response of the plate, a uniform unit pressure as an excitation load is applied to its top surface. Fig. 5 illustrates the resonance frequency shift by increasing the temperature. In this diagram the x-axis is the ratio of excitation frequency to the first natural frequency at reference temperature, $f_{n1}(T_0)$, and the y-axis is non-dimensional amplitude. For example, at 380K the resonance frequency enhances about 65%.

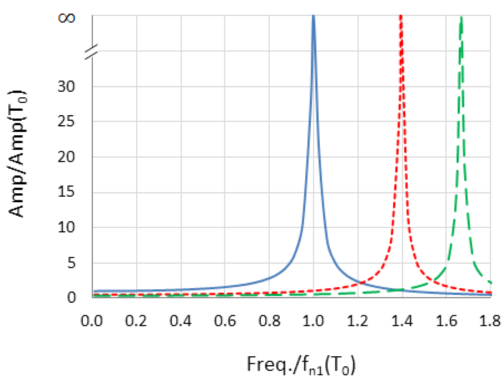

Fig. 5. Effect of Temperature on Resonance Frequency (Simply-Supported Edges)

The effect of temperature on the dynamic amplitude of plate can be considered by showing the amplitude changes from zero excitation frequency to $0.5f_{n1}(T_0)$. Fig. 6 illustrates these alterations. The amplitude is decreased by increasing the activation temperature of SMA wires. According to the Fig. 6, when the activation temperature is 380K and the excitation frequency is half of the first natural frequency at T_0, the dynamic response amplitude decreases to about 75 percent of the amplitude at reference temperature.

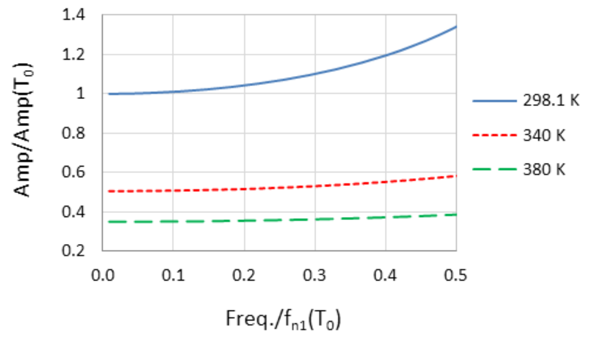

Fig. 6. Effect of Temperature on Dynamic Amplitude (Simply-Supported Edges)

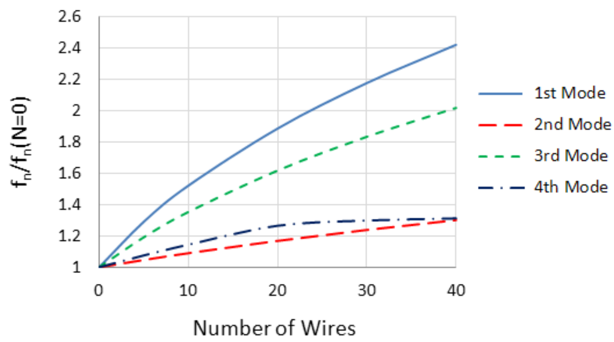

Fig. 7. Effect of SMA Wires Number on Natural Frequency (Simply-Supported Edges)

It is also found that the stress generated in the 'activated' composite results in a frequency shift, which increases by increasing number and the volume fraction of the embedded SMA wires. Figs. 7 and 8 show alteration trend of natural frequencies as a function of number and volume fraction of SMA wires, respectively.

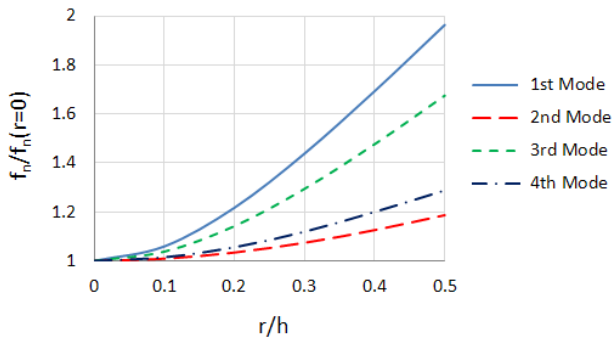

Fig. 8. Effect of SMA Wires Volume on Natural Frequency (Simply-Supported Edges)

The changes in 1st natural frequency by increasing the temperature for 7 different B.C.'s configurations of the plate are shown in Fig. 9.

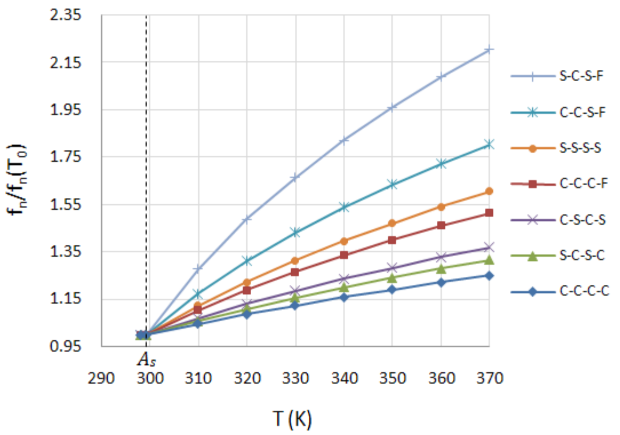

Fig. 9. Effect of B.C.'s on 1st Natural Frequency Changing by Temperature

6 Conclusion

According to the significant changes of natural and resonance frequencies and dynamic response amplitude of the plate using shape memory alloys in the form of wires, SMAs could develop intelligent systems for vibration control applications in many industries such as mechanics, aerospace and civil engineering.

Raising the activation temperature above the austenite start temperature and therefore provoking phase transformation from martensite to austenite or parent phase, the stiffness of the plate is increased and hence improves the performance of vibration control system.

Alteration of natural frequencies of the plate enhances by increasing the SMA wires number and volume fraction.

By reinforcing the constraints of the plate, the change in natural frequency is decreased. In this situation, the minimum alteration in first natural frequency corresponds to the condition in which all edges of the plate are clamped.

7 References

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