SUB-OPTIMAL MISSILE GUIDANCE WITH PULSE-MOTOR CONTROL LOGIC USING THE SINGULAR PERTURBATIONS METHOD

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Abstract

A planar non-linear missile guidance problem is studied and a guidance method is proposed. The solution includes finding the acceleration commands (guidance commands) and controlling ignition timing for a dual pulse rocket motor. A near optimal solution using the singular perturbations method for a switched system is used. This solution method results in a simple to implement analytic solution to the problem. In order to solve this problem, a new condition in the minimum principle was derived. The condition regards switched systems with system equations dependent on the switching time. During the solution, a modified rocket equation to include drag was developed.

1 Introduction

This study deals with the planar non-linear guidance problem for an interception missile. The model includes several non linearities; a) geometry (large deviations from the collision course);b) aerodynamic forces as function of the angle of attack and velocity, c) an axial two pulse rocket motor enabling ignition of the second pulse on demand; and d) a change in mass.

The forced singular perturbations technique (FSPT) **[2]**-**[4]** is used to simplify optimization problems by identifying fast and slow variables and separating the problem to two problems, the outer (free stream) and the inner (boundary layer). The technique has been used to simplify the solution of non-linear guidance problems for aircraft and missiles in planer and three dimensional engagements **[5]**-**[8]**.

The optimal control of switched systems with predetermined switching sequence was studied in **[9]** and **[10]**. Their solution looks at the problem as two separate problems. The first is to determine the switching times and the second to determine the optimal continuous control for the given switching times.

The optimization problem of a guided missile using a controllable pulse motor is studied in ref. **[11]**. The study shows the benefit of using a controllable pulse motor in medium range air-air applications. This study includes a three dimensional midcourse guidance scheme developed based on singular perturbations technique and additional approximations. The guidance commands and engine commands are weakly linked in order to achieve a simple solution to the problem.

The proposed solution method is based on the formulation of the problem as a switched system consisting of four stages with predetermined order. The stages are a) initial boost after launch; b) first coast stage after first pulse burnout; c) second pulse after controlled firing of the second pulse; and d) second coast phase after second pulse burnout. The duration of the first coast phase is controllable on tine and can be zero. The duration of the final cost stage is a result of previous decisions and can be zero as well. The optimal solution is based on the minimum principle as formulated for switched systems. As the optimal solution of the problem is complicated and not suitable for on-line real-time applications, A near optimal solution using the singular perturbation method is presented allowing a real time solution. Unlike **[9]**-**[11]**, the optimally conditions and the boundary conditions are used together to get a unified optimal solution for both the guidance commands (continuous control) and impulse ignition timing (switch time).

The optimal problem definition includes the hard constraint of ideal interception (achieving impact with the target) while optimizing a function that compromises minimum flight time and maximal terminal kinetic energy. Minimum flight time is critical in military missions (in anti-aircraft missions it means less chance of weapon release by the enemy, in other missions it gives a second intercept chance). The importance of maximal terminal kinetic energy is also significant, allowing higher end game maneuvers and lethality (in kinetic energy interceptors).

The problem is described in section 2. The formulation of the optimal solution is given in section 3. Section 4, 5 and 6 describe the near-optimal solution using the singular perturbations method. Section 4 presents the general formulation, section 5 the outer solution and section 6 the inner solution. In the appendix there is a detailed formulation of the minimum principle for switched systems with the system equations dependent on the switching time. Section 7 presents results from numeric examples. Conclusions from the work are presented in section 8.

2 Problem Formulation

The guidance scenario is described in Fig 1. A constant velocity non-maneuvering target (T) is intercepted by a missile(M). The forces acting on the missile, shown in Fig 2, are lift, L, (perpendicular to the velocity vector), drag, D, (apposing the velocity vector) and thrust, T (in the direction of the missile body). The missile's velocity magnitude is affected by thrust and drag forces. The maneuver is created by lift and thrust component that are controlled by the angle of attach (α).

The thrust profile consists of two predefined constant thrust pulses. The first pulse is fired at launch. The second pulse is fired on demand t_{c1} seconds after the burnout of the first pulse. The ignition time of the second pulse is a control parameter. During thrust operation the mass changes at a constant rate.

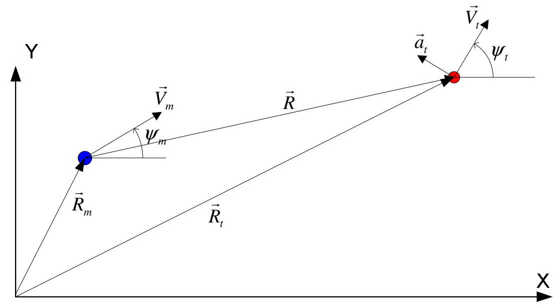

Fig. 1, Pursuit kinematics

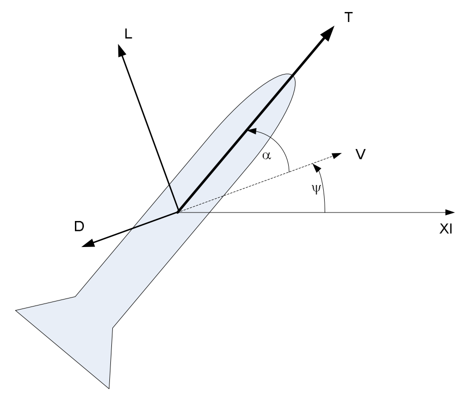

Fig. 2, Forces acting on the interceptor

The equations of motion describing the problem depend upon the flight phase. The general equations of motion are given by:

$$\dot{x} = V_T \cos\psi_T - V_M \cos\psi \tag{1}$$

$$\dot{y} = V_T \sin\psi_T - V_M \sin\psi \tag{2}$$

$$\dot{\psi} = T\sin\alpha/(m\cdot V_M) + L/(m\cdot V_M) \tag{3}$$

$$\dot{V} = T\cos\alpha/m - D/m \tag{4}$$

The aerodynamic model is given by a linear lift coefficient and a parabolic drag polar, with constant coefficients.

$$L = 0.5\rho V_M{}^2 SCl_\alpha \alpha = L' V_M{}^2 \alpha \tag{5}$$

$$D = 0.5\rho V_M{}^2 S\left(Cd_0 + kCl_\alpha{}^2\alpha^2\right) = \\ = D_0' V_M{}^2 + D_i' V_M{}^2 \alpha^2 \tag{6}$$

In the above equations (1)-(6), x and y are components of the relative position between the target and interceptor; V_T the target velocity (assumed constant); ψ_T the target heading; V_M the missile's velocity; ψ the missile's heading; m the missile mass; $L' = 0.5\rho SCl_\alpha$ the constant term in the lift force; $D_0' = 0.5\rho SC_{D0}$ the constant term in the zero-lift drag; $D_i' = 0.5\rho SC_{Di}$ the constant term in the induced drag force; α the angle of attack.

A state vector is defined as $\bar{X} = \begin{bmatrix} x & y & \psi & V \end{bmatrix}^T$. Using this state vector, the equations of motion for each phase are written below.

Phase 1, initial boost. During this phase the first pulse is active ($T = T_1$) and the mass changes according to $m = m_0 - \dot{m}_1 t$. The equations of motion are:

$$\dot{\bar{X}} = g_1\left(\bar{X}, \alpha, t \in [0, t_{s1}]\right) = \left\{ \begin{array}{ll} V_T \cos\psi_T - V_M \cos\psi & (a) \\ V_T \sin\psi_T - V_M \sin\psi & (b) \\ \dfrac{T_1 \sin\alpha / V_M + L' V_M \alpha}{m_0 - \dot{m}_1 t} & (c) \\ \dfrac{T_1 \cos\alpha - V_M^{\,2}\left(D_0' + D_i' \cdot \alpha^2\right)}{m_0 - \dot{m}_1 t} & (d) \end{array} \right. \tag{7}$$

Phase 2, first coast phase. In this phase there is no thrust and the mass is constant ($m = m_0 - m_{P1}$). The equations of motion are:

$$\dot{\bar{X}} = g_2\left(\bar{X}, \alpha, t \in (t_{s1}, t_{s2}]\right) = \left\{ \begin{array}{ll} V_T \cos\psi_T - V_M \cos\psi & (a) \\ V_T \sin\psi_T - V_M \sin\psi & (b) \\ \dfrac{L'}{m_0 - m_{P1}} V_M \alpha & (c) \\ -\dfrac{V_M^{\,2}\left(D_0' + D_i' \cdot \alpha^2\right)}{m_0 - m_{P1}} & (d) \end{array} \right. \tag{8}$$

Phase 3, second boost. In this phase the second pulse is active ($T = T_2$) and the mass changes ($m = m_0 - m_{P1} - \dot{m}_2\left(t - t_{s2}\right)$). The equations of motion are:

$$\dot{\bar{X}} = g_3\left(\bar{X}, \alpha, t_{s2}, t \in (t_{s2}, t_{s3}]\right) = \left\{ \begin{array}{ll} V_T \cos\psi_T - V_M \cos\psi & (a) \\ V_T \sin\psi_T - V_M \sin\psi & (b) \\ \dfrac{T_2 \sin\alpha / V_M + L' V_M \alpha}{m_0 - m_{P1} - \dot{m}_2\left(t - t_{s2}\right)} & (c) \\ \dfrac{T_2 \cos\alpha - V_M^{\,2}\left(D_0' + D_i' \cdot \alpha^2\right)}{m_0 - m_{P1} - \dot{m}_2\left(t - t_{s2}\right)} & (d) \end{array} \right. \tag{9}$$

Phase 4, second coast phase. In this phase there is no thrust and the mass is constant ($m = m_0 - m_{P1} - m_{P2}$). The equations of motion are:

$$\dot{\bar{X}} = g_4\left(\bar{X}, \alpha, t \in (t_{s3}, t_f]\right) = \left\{ \begin{array}{ll} V_T \cos\psi_T - V_M \cos\psi & (a) \\ V_T \sin\psi_T - V_M \sin\psi & (b) \\ \dfrac{L'}{m_0 - m_{P1} - m_{P2}} V_M \alpha & (c) \\ -\dfrac{V_M^{\,2}\left(D_0' + D_i' \cdot \alpha^2\right)}{m_0 - m_{P1} - m_{P2}} & (d) \end{array} \right. \tag{10}$$

The general optimization problem is to minimize the final time and maximize the final velocity, thus

$$J = \kappa_t t_f - \kappa_V V_f \tag{11}$$

Where κ_t is a weight on the final time and κ_V the weight on the final velocity. Note that $\kappa_t = 0$ results in maximal final velocity and $\kappa_V = 0$ results in a minimum time problem.

The transition between the phases is defined using two time dependent manifolds. Transition from phase 1 to phase 2 is defined by η_1 (switch at the predefined first pulse burning time). The transition from phase 2 to 3 and from 3 to 4 are defined by η_2 (the ignition time is free but the ignition and burnout times are linked by the predefined second pulse burn time).

$$\eta_1\left(t_{s1}\right) = t_{s1} - \tau_{P1} = 0 \tag{12}$$

$$\eta_2\left(t_{s2},t_{s3}\right)=t_{s3}-t_{s2}-\tau_{P2}=0 \tag{13}$$

To complete the problem formulation, the following boundary conditions are used. These boundary conditions include the problem initial conditions and the required terminal conditions for hitting the target.

$$x(t_0)=x_0 \tag{14}$$

$$y(t_0)=y_0 \tag{15}$$

$$V_M(t_0)=V_0 \tag{16}$$

$$\psi(t_0)=\psi_0 \tag{17}$$

$$x(t_f)=0 \tag{18}$$

$$y(t_f)=0 \tag{19}$$

3 Optimal Solution

The optimal solution is based on Ponryagin's minimum principle adjusted to a switched system (see appendix B). The Hamiltonian is defined as

$$H=\begin{cases}\lambda^T g_1 & t\in\left[0,t_{s1}\right]\\ \lambda^T g_2 & t\in\left(t_{s1},t_{s2}\right]\\ \lambda^T g_3 & t\in\left(t_{s2},t_{s3}\right]\\ \lambda^T g_4 & t\in\left(t_{s3},t_f\right]\end{cases} \tag{20}$$

With $\lambda^T=\left[\lambda_x \quad \lambda_y \quad \lambda_\psi \quad \lambda_V\right]^T$ the vector of Lagrange's multipliers.

The co-state equations are

$$\dot{\lambda}_x=0 \;\Rightarrow\; \lambda_x=k1=C\cos\gamma \tag{21}$$

$$\dot{\lambda}_y=0 \;\Rightarrow\; \lambda_y=k2=C\sin\gamma \tag{22}$$

$$\begin{aligned}\dot{\lambda}_\psi&=-\lambda_x V_M\sin\psi+\lambda_y V_M\cos\psi\\ &=CV_M\sin\left(\gamma-\psi\right)\end{aligned} \tag{23}$$

$$\dot{\lambda}_V=\begin{cases}\lambda_x\cos\psi+\lambda_y\sin\psi-\lambda_\psi\dfrac{-T_1\sin\alpha/V_M{}^2+L'\alpha}{m_0-\dot{m}_1 t}+\lambda_V\dfrac{2V_M\left(D_0'+D_i'\cdot\alpha^2\right)}{m_0-\dot{m}_1 t} & p1\\ \lambda_x\cos\psi+\lambda_y\sin\psi-\lambda_\psi\dfrac{L'\alpha}{m_0-m_{P1}}+\lambda_V\dfrac{2V_M\left(D_0'+D_i'\cdot\alpha^2\right)}{m_0-m_{P1}} & p2\\ \lambda_x\cos\psi+\lambda_y\sin\psi-\lambda_\psi\dfrac{-T_2\sin\alpha/V_M{}^2+L'\alpha}{m_0-m_{P1}-\dot{m}_2\left(t-t_{s2}\right)}+\lambda_V\dfrac{2V_M\left(D_0'+D_i'\cdot\alpha^2\right)}{m_0-\dot{m}_2\left(t-t_s\right)} & p3\\ \lambda_x\cos\psi+\lambda_y\sin\psi-\lambda_\psi\dfrac{L'\alpha}{m_0-m_{P1}-m_{P2}}+\lambda_V\dfrac{2V_M\left(D_0'+D_i'\cdot\alpha^2\right)}{m_0-m_{P1}-m_{P2}} & p4\end{cases} \tag{24}$$

The transversally conditions for a switched system lead to the following boundary conditions

$$\lambda_\psi\left(t_f\right)=0 \tag{25}$$

$$\lambda_V\left(t_f\right)=-\kappa_V \tag{26}$$

$$H\left(t_f\right)=-\kappa_t \tag{27}$$

$$H\big|_{t_{s2}^-}-H\big|_{t_{s2}^+}+\int_{t_{s2}}^{t_{s3}}\lambda^T\frac{\partial g_3}{\partial t_{s2}}=-H\big|_{t_{s3}^-}+H\big|_{t_{s3}^+} \tag{28}$$

$$\lambda_i\big|_{t_{sj}^-}=\lambda_i\big|_{t_{sj}^+} \quad i=x,y,\psi,V \quad j=1,2,3 \tag{29}$$

The optimal continuous control is found using the optimality condition

$$H_\alpha = 0 = \tag{30}$$

$$= \begin{cases} \dfrac{\lambda_\psi \dfrac{T_1 \cos\alpha / V_M + L' V_M}{m_0 - \dot m_1 t} - \lambda_V \dfrac{T_1 \sin\alpha + 2 V_M{}^2 D_i ' \cdot \alpha}{m_0 - \dot m_1 t}}{} & p1 \\ \dfrac{\lambda_\psi \dfrac{L' V_M}{m_0 - m_{P1}} - \lambda_V \dfrac{2 V_M{}^2 D_i ' \cdot \alpha}{m_0 - m_{P1}}}{} & p2 \\ \dfrac{\lambda_\psi \dfrac{T_2 \cos\alpha / V_M + L' V_M}{m_0 - m_{P1} - \dot m_2 (t - t_{s2})} - \lambda_V \dfrac{T_2 \sin\alpha + 2 V_M{}^2 D_i ' \cdot \alpha}{m_0 - m_{P1} - \dot m_2 (t - t_{s2})}}{} & p3 \\ \lambda_\psi \dfrac{L' V_M}{m_0 - m_{P1} - m_{P2}} - \lambda_V \dfrac{2 V_M{}^2 D_i ' \cdot \alpha}{m_0 - m_{P1} - m_{P2}} & p4 \end{cases}$$

4 Singular Perturbations Modeling

The above stated optimal solution, results in a two-point boundary-value problem (TPBVP). A suboptimal solution based on the singular perturbations method allows for a much simpler solution. This solution is based on the time scale separation between fast and slow states.

According to the forced singular perturbation method [2]-[4], slow and fast variables are identified. Following [8] we identify the fast states (ψ) and the slow states (x, y, V). We will force the equations of motion to exhibit the fast states by multiplying the left side of the equations for ψ and λ_ψ by a small parameter ε, resulting in the following differential equations for the states and co-states (boundary conditions, Hamiltonian and the Optimality conditions remain). Note the equations given are the general equations. Using the mass and thrust values the detailed equations for each phase can be derived.

$$\dot x = V_T \cos\psi_T - V_M \cos\psi \tag{31}$$

$$\dot y = V_T \sin\psi_T - V_M \sin\psi \tag{32}$$

$$\varepsilon \dot\psi = \frac{T \sin\alpha}{m V_M} + \frac{L' V_M \alpha}{m} \tag{33}$$

$$\dot V = \frac{T \cos\alpha}{m} - \frac{D_0 ' V_M{}^2 + D_i ' V_M{}^2 \alpha^2}{m} \tag{34}$$

$$\dot\lambda_x = 0 \tag{35}$$

$$\dot\lambda_y = 0 \tag{36}$$

$$\begin{aligned} \varepsilon \dot\lambda_\psi &= -\lambda_x V_M \sin\psi + \lambda_y V_M \cos\psi = \\ &= C V_M \sin(\gamma - \psi) \end{aligned} \tag{37}$$

$$\begin{aligned} \dot\lambda_V &= \lambda_x \cos\psi + \lambda_y \sin\psi \\ &- \lambda_\psi \left(-\frac{T}{m} \frac{\sin\alpha}{V_M{}^2} + \frac{L'}{m} \alpha \right) \\ &+ \lambda_V \frac{2 V_M}{m} \left(D_0 ' + D_i ' \cdot \alpha^2 \right) \end{aligned} \tag{38}$$

5 Outer Solution

Setting $\varepsilon = 0$ in the equation for ψ (33) yields $\alpha = 0$ in all the phases, i.e. flight at a constant heading. Setting $\varepsilon = 0$ in the equation for λ_ψ (37), implies $\gamma = \psi^{out}$, i.e. the position co-states are directly related to the heading. The continuous controller in the outer solution changes from the angle of attack α to the heading angle ψ^{out}. Since the heading is a controller, it is not guarantied to satisfy its own initial condition.

The ensuing equations of motion, the co-state equations, the Hamiltonian and the optimally condition reduce to

$$\dot x^{out} = V_T \cos\psi_T - V_M{}^{out} \cos\psi^{out} \tag{39}$$

$$\dot y^{out} = V_T \sin\psi_T - V_M{}^{out} \sin\psi^{out} \tag{40}$$

$$\dot{V}^{out} = \begin{cases} \dfrac{T_1 - V_M^{out2} D_0'}{m_0 - \dot{m}_1 t} & p1 \\ -\dfrac{V_M^{out2} D_0'}{m_0 - m_{P1}} & p2 \\ \dfrac{T_2 - V_M^{out2} D_0'}{m_0 - m_{P1} - \dot{m}_2 \left(t - t_{s2}\right)} & p3 \\ -\dfrac{V_M^{out2} D_0'}{m_0 - m_{P1} - m_{P2}} & p4 \end{cases} \tag{41}$$

$$\lambda_x^{out} = k1 = C\cos\gamma \tag{42}$$

$$\lambda_y^{out} = k2 = C\sin\gamma \tag{43}$$

$$\dot{\lambda}_V^{out} = \begin{cases} C + \dfrac{2\lambda_V^{out} V_M^{out} D_0'}{m_0 - \dot{m}_1 t} & p1 \\ C + \dfrac{2\lambda_V^{out} V_M^{out} D_0'}{m_0 - m_{P1}} & p2 \\ C + \dfrac{2\lambda_V^{out} V_M^{out} D_0'}{m_0 - m_{P1} - \dot{m}_2 \left(t - t_{s2}\right)} & p3 \\ C + \dfrac{2\lambda_V^{out} V_M^{out} D_0'}{m_0 - m_{P1} - m_{P2}} & p4 \end{cases} \tag{44}$$

$$H^{out} = \begin{cases} \lambda_x^{out}\left(V_T\cos\psi_T - V_M^{out}\cos\psi^{out}\right) & \\ +\lambda_y^{out}\left(V_T\sin\psi_T - V_M^{out}\sin\psi^{out}\right) & p1 \\ +\lambda_V^{out}\dfrac{T_1 - V_M^{out2} D_0'}{m_0 - \dot{m}_1 t} & \\ \hline \lambda_x^{out}\left(V_T\cos\psi_T - V_M^{out}\cos\psi^{out}\right) & \\ +\lambda_y^{out}\left(V_T\sin\psi_T - V_M^{out}\sin\psi^{out}\right) & p2 \\ -\lambda_V^{out}\dfrac{V_M^{out2}}{m_0 - m_{P1}} D_0' & \\ \hline \lambda_x^{out}\left(V_T\cos\psi_T - V_M^{out}\cos\psi^{out}\right) & \\ +\lambda_y^{out}\left(V_T\sin\psi_T - V_M^{out}\sin\psi^{out}\right) & p3 \\ +\lambda_V^{out}\dfrac{T_2 - V_M^{out2} D_0'}{m_0 - m_{P1} - \dot{m}_2\left(t - t_{s2}\right)} & \\ \hline \lambda_x^{out}\left(V_T\cos\psi_T - V_M^{out}\cos\psi^{out}\right) & \\ +\lambda_y^{out}\left(V_T\sin\psi_T - V_M^{out}\sin\psi^{out}\right) & p4 \\ -\lambda_V^{out}\dfrac{V_M^{out2}}{m_0 - m_{P1} - m_{P2}} D_0' & \end{cases} \tag{45}$$

$$H_\alpha^{out} = 0 = \begin{cases} \lambda_\psi^{out}\dfrac{T_1 / V_M^{out} + L' V_M^{out}}{m_0 - \dot{m}_1 t} & p1 \\ \lambda_\psi^{out}\dfrac{L'}{m_0 - m_{P1}} V_M^{out} & p2 \\ \lambda_\psi^{out}\dfrac{T_2 / V_M^{out} + L' V_M^{out}}{m_0 - m_{P1} - \dot{m}_2\left(t - t_{s2}\right)} & p3 \\ \lambda_\psi^{out}\dfrac{L'}{m_0 - m_{P1} - m_{P2}} V_M^{out} & p4 \end{cases} \tag{46}$$

The optimally condition (46) of the outer solution leads to the value of λ_ψ^{out}. It is found to be $\lambda_\psi^{out} = 0$ which satisfied the boundary condition.

From the Hamiltonian at the terminal time and the boundary condition for λ_V the value of C is calculated to be

$$C = \frac{-\kappa_t - \kappa_V \dfrac{V_M^{out}\left(t_f\right)^2}{m_0 - m_{P1} - m_{P2}} D_0'}{V_T\cos\left(\psi_T - \psi\right) - V_M^{out}\left(t_f\right)} \tag{47}$$

Solution of the equation for the velocity results in an analytic expression for the velocity in the outer solution in general and at the switching points and final time in particular. Note the solution in the first and third phases includes both thrust and drag. This equation is formulated in detail at appendix B as the modified rocket equation.

$$V^{out}(t)=\begin{cases}\sqrt{\frac{T_1}{D_0'}}\tanh\left[\begin{array}{l}\frac{\sqrt{T_1 D_0'}}{\dot m_1}\ln\left(\frac{m_0}{m_0-\dot m_1 t}\right)\\ +a\tanh\left(V_0\sqrt{\frac{D_0'}{T_1}}\right)\end{array}\right] & p1\\ \frac{V^{out}(t_{s1})}{\frac{D_o'}{m_0-m_{P1}}\cdot V^{out}(t_{s1})\cdot(t-t_{s1})+1} & p2\\ \sqrt{\frac{T_2}{D_0'}}\tanh\left[\begin{array}{l}\frac{\sqrt{T_2 D_0'}}{\dot m_2}\cdot\\ \ln\left(\frac{m_0-m_{P1}}{m_0-m_{P1}-\dot m_2(t-t_{s2})}\right)\\ +a\tanh\left(V^{out}(t_{s2})\sqrt{\frac{D_0'}{T_2}}\right)\end{array}\right] & p3\\ \frac{V^{out}(t_{s3})}{\frac{D_o'}{m_0-m_{P1}-m_{P2}}\cdot V^{out}(t_{s3})\cdot(t-t_{s3})+1} & p4\end{cases} \tag{48}$$

$$V^{out}(t_{s1})= =\sqrt{\frac{T_1}{D_0'}}\tanh\left[\begin{array}{l}\frac{\sqrt{T_1 D_0'}}{\dot m_1}\ln\left(\frac{m_0}{m_0-m_{P1}}\right)\\ +a\tanh\left(V_0\sqrt{\frac{D_0'}{T_1}}\right)\end{array}\right] \tag{49}$$

$$V_M{}^{out}(t_{s2})= =\frac{V^{out}(t_{s1})}{\frac{D_o'}{m_0-m_{P1}}\cdot V^{out}(t_{s1})\cdot(t_{s2}-t_{s1})+1} \tag{50}$$

$$V^{out}(t_{s3})=\sqrt{\frac{T_2}{D_0'}}\cdot \tanh\left[\begin{array}{l}\frac{\sqrt{T_2 D_0'}}{\dot m_2}\ln\left(\frac{m_0-m_{P1}}{m_0-m_{P1}-m_{P2}}\right)\\ +a\tanh\left(V^{out}(t_{s2})\sqrt{\frac{D_0'}{T_2}}\right)\end{array}\right] \tag{51}$$

$$V_M{}^{out}(t_f)= =\frac{V^{out}(t_{s3})}{\frac{D_o'}{m_0-m_{P1}-m_{P2}}V^{out}(t_{s3})\cdot(t_f-t_{s3})+1} \tag{52}$$

The relative position and the velocity are found by forward integration (remembering that the target velocity and heading are constant and the missile heading is also constant in the outer solution).

$$x^{out}(t)= =x_0+V_T\cos\psi_T\cdot t_f-R(t)\cos\psi^{out} \tag{53}$$

$$y^{out}(t)= y_0+V_T\sin\psi_T\cdot t_f-R(t)\sin\psi^{out} \tag{54}$$

with R the distance traveled by the interceptor during the outer solution, given by

$$R(t)=\int_0^t V_M{}^{out}dt \tag{55}$$

Using the expressions for the velocity, $R(t_f)$ can be calculated. The expression is partially analytic

$$\begin{aligned}R(t_f)=&\int_0^{t_{s1}}\sqrt{\frac{T_1}{D_0'}}\tanh\left[\begin{array}{l}\frac{\sqrt{T_1 D_0'}}{\dot m_1}\ln\left(\frac{m_0}{m_0-\dot m_1 t}\right)\\ +a\tanh\left(V_0\sqrt{\frac{D_0'}{T_1}}\right)\end{array}\right]dt+\\ &+\ln\left(\frac{D_o'}{m_0-m_{P1}}\cdot V^{out}(t_{s1})\cdot(t_{s2}-t_{s1})+1\right)\frac{m_0-m_{P1}}{D_o'}+\\ &+\int_{t_{s2}}^{t_{s3}}\sqrt{\frac{T_2}{D_0'}}\tanh\left[\begin{array}{l}\frac{\sqrt{T_2 D_0'}}{\dot m_2}\ln\left(\frac{m_0-m_{P1}}{m_0-m_{P1}-\dot m_2(t-t_{s2})}\right)\\ +a\tanh\left(V(t_{s2})\sqrt{\frac{D_0'}{T_2}}\right)\end{array}\right]dt+\\ &+\ln\left(\frac{D_o'}{m_0-m_{P1}-m_{P2}}\cdot V^{out}(t_{s3})\cdot(t_f-t_{s3})+1\right)\frac{m_0-m_{P1}-m_{P2}}{D_o'}\end{aligned} \tag{56}$$

Since λ_V is known at the final time, $\lambda_V(t_f)=-\kappa_V$, integration will be performed backwards in time from this point. Like in the expression for the distance traveled, a partial analytic expression is obtained.

$$\lambda_V^{out}(t_{s3}) = -\frac{C\cdot(t_f - t_{s3})}{\left[\frac{D_0'}{m_0 - m_{P1} - m_{P2}} V^{out}(t_{s3})\cdot(t_f - t_{s3}) + 1\right]} - \frac{\kappa_V}{\left[\frac{D_0'}{m_0 - m_{P1} - m_{P2}} V^{out}(t_{s3})\cdot(t_f - t_{s3}) + 1\right]^2} \tag{57}$$

$$\lambda_V^{out}(t_{s2}) = \lambda_V^{out}(t_{s3})\cdot\mu(t_{s3}) - C\cdot\int_{t_{s2}}^{t_{s3}}\mu(s)\,ds \qquad \mu(t) = \exp\int_{t_{s2}}^{t}\frac{-2V_M(\tau)}{m_0 - m_{P1} - \dot{m}_2(\tau - t_s)}D_0'\,d\tau \tag{58}$$

$$\lambda_V^{out}(t_{s1}) = -\frac{C\cdot(t_{s2} - t_{s1})}{\left[\frac{D_0'V^{out}(t_{s1})}{m_0 - m_{P1}}(t_{s2} - t_{s1}) + 1\right]} + \frac{\lambda_V(t_{s2})}{\left[\frac{D_0'V^{out}(t_{s1})}{m_0 - m_{P1}}(t_{s2} - t_{s1}) + 1\right]^2} \tag{59}$$

$$\lambda_V^{out}(t_0) = \lambda_V^{out}(t_{s1})\cdot\mu(t_{s1}) - C\cdot\int_{t_0}^{t_{s1}}\mu(s)\,ds \qquad \mu(t) = \exp\int_{t_0}^{t}\frac{-2V_M(\tau)}{m_0 - \dot{m}_2(\tau - t_0)}D_0'\,d\tau \tag{60}$$

The Hamiltonian "jump" condition (28) results in

$$0 = -\lambda_V^{out}(t_{s2})\frac{T_2}{m_0 - m_{P1}} + \lambda_V^{out}(t_{s3})\frac{T_2}{m_0 - m_{P1} - m_{P2}} - \dot{m}_2\int_{t_{s2}}^{t_{s3}}\lambda_V^{out}(t)\frac{T_2 - V_M^{out}(t)^2 D_0'}{m_0 - m_{P1} - \dot{m}_2(t - t_{s2})}dt \tag{61}$$

Assuming the problem parameters ψ^{out}, t_{s2}, t_f are known, the above set of equations is the solution to the outer solution. To find these three unknowns, three conditions should be satisfied, hitting the target ($x_f = y_f = 0$) and satisfying the Hamiltonian jump condition. That is, the whole optimization in the outer solution degenerates from a TPBVP to a set of three equations with three unknowns.

$$\bar{f}(t_s, t_f, \psi) = \bar{0} \qquad \bar{f} = \begin{bmatrix} f_x & f_y & f_H \end{bmatrix}^T \tag{62}$$

$$f_x = x_f = x_0 + V_T\cos\psi_T\cdot t_f - R(t_f)\cos\psi^{out} \tag{63}$$

$$f_y = y_f = y_0 + V_T\sin\psi_T\cdot t_f - R(t_f)\sin\psi^{out} \tag{64}$$

$$f_H = -\lambda_V^{out}(t_{s2})\frac{T_2}{m_0 - m_{P1}} + \lambda_V^{out}(t_{s3})\frac{T_2}{m_0 - m_{P1} - m_{P2}} - \dot{m}_2\int_{t_{s2}}^{t_{s3}}\lambda_V^{out}(t)\frac{T_2 - V_M^{out}(t)^2 D_0'}{m_0 - m_{P1} - \dot{m}_2(t - t_s)}dt \tag{65}$$

6. Inner (boundary layer) solution

In the boundary layer, a stretched time $\tau = t/\varepsilon$ is used. In this stretched time, the equations of motion for the first phase become (with $(\dot{x})$ being a derivative with respect to the stretched time τ):

$$\dot{x}^{in} = \varepsilon\left(V_T\cos\psi_T - V_M^{in}\cos\psi^{in}\right) \tag{66}$$

$$\dot{y}^{in} = \varepsilon\left(V_T\sin\psi_T - V_M^{in}\sin\psi^{in}\right) \tag{67}$$

$$\dot{\psi}^{in} = \frac{T}{m_0 - \dot{m}_1\varepsilon\tau}\frac{\sin\alpha}{V_M^{in}} + \frac{L'}{m_0 - \dot{m}_1\varepsilon\tau}V_M^{in}\alpha \tag{68}$$

$$\dot{V}^{in} = \varepsilon\left(\frac{T}{m_0 - \dot{m}_1\varepsilon\tau}\cos\alpha - \frac{V_M^{in2}}{m_0 - \dot{m}_1\varepsilon\tau}\left(D_0' + D_i'\cdot\alpha^2\right)\right) \tag{69}$$

Setting $\varepsilon = 0$ in the equations results in having the slow states constant at their initial values with the heading being the only changing state in the boundary layer. In the boundary layer the heading changes from its initial value to the outer solution value.

$$\dot{x}^{in} = 0 \Rightarrow x^{in} = x_0 \tag{70}$$

$$\dot{y}^{in} = 0 \Rightarrow y^{in} = y_0 \tag{71}$$

$$\dot{\psi}^{in} = \frac{T_q}{m_0} \frac{\sin\alpha}{V_M{}^{in}} + \frac{L'}{m_0} V_M{}^{in} \alpha \tag{72}$$

$$\dot{V}^{in} = 0 \Rightarrow V^{in} = V_0 \tag{73}$$

The co-state equations, the Hamiltonian and the optimally condition degenerate to:

$$\dot{\lambda}_x{}^{in} = 0 \Rightarrow \lambda_x{}^{in} = k1 = C\cos\gamma \tag{74}$$

$$\dot{\lambda}_y{}^{in} = 0 \Rightarrow \lambda_y{}^{in} = k2 = C\sin\gamma \tag{75}$$

$$\begin{aligned} \dot{\lambda}_\psi{}^{in} &= -\lambda_x{}^{in} V_0 \sin\psi^{in} + \lambda_y{}^{in} V_0 \cos\psi^{in} = \\ &= CV_0 \sin\left(\gamma - \psi^{in}\right) \end{aligned} \tag{76}$$

$$\dot{\lambda}_V{}^{in} = 0 \Rightarrow \lambda_V{}^{in} = \lambda_V{}^{out}\left(t=0\right) \tag{77}$$

$$\begin{aligned} H^{in} &= \lambda_x{}^{in}\left(V_T \cos\psi_T - V_0 \cos\psi^{in}\right) \\ &+ \lambda_y{}^{in}\left(V_T \sin\psi_T - V_0 \sin\psi^{in}\right) + \\ &+ \lambda_\psi{}^{in}\left(\frac{T_1}{m_0}\frac{\sin\alpha}{V_0} + \frac{L'}{m_0} V_0 \alpha\right) \\ &+ \lambda_V{}^{in}\left(\frac{T_1}{m_0}\cos\alpha - \frac{V_0{}^2}{m_0}\left(D_0{}' + D_i{}' \cdot \alpha^2\right)\right) \end{aligned} \tag{78}$$

$$\begin{aligned} H_\alpha{}^{in} &= \lambda_\psi{}^{in}\left(\frac{T_1}{m_0}\frac{\cos\alpha}{V_0} + \frac{L'}{m_0} V_0\right) \\ &- \lambda_V{}^{in}\left(\frac{T_1}{m_0}\sin\alpha + \frac{2V_0{}^2}{m_0} D_i{}' \cdot \alpha\right) = 0 \end{aligned} \tag{79}$$

Note: Although the general expression for the Hamiltonian is time dependent, the expression in the boundary layer is time independent, thus the Hamiltonian is constant in it. Matching the value from the two solutions results in

$$H^{in}\left(\tau\right) = const = H^{out}\left(t=0\right) \tag{80}$$

From the optimally condition the heading adjoint can be extracted. By substituting this expression in the Hamiltonian we obtain an equation with one unknown, namely the optimal control.

$$\lambda_\psi{}^{in} = \lambda_V{}^{out}\left(t=0\right) \frac{\left(\frac{T_1}{m_0}\sin\alpha + \frac{2V_0{}^2}{m_0} D_i{}' \cdot \alpha\right)}{\left(\frac{T_1}{m_0}\frac{\cos\alpha}{V_M} + \frac{L'}{m_0} V_0\right)} \tag{81}$$

$$\begin{aligned} H^{in} &= \lambda_x{}^{in}\left(V_T \cos\psi_T - V_0 \cos\psi^{in}\right) \\ &+ \lambda_y\left(V_T \sin\psi_T - V_0 \sin\psi^{in}\right) \\ &+ \lambda_V{}^{out}\left(0\right)\left(\frac{T_1}{m_0}\sin\alpha + \frac{2V_0{}^2}{m_0} D_i{}' \cdot \alpha\right) \\ &\cdot \frac{\left(\frac{T_1}{m_0}\frac{\sin\alpha}{V_0} + \frac{L'}{m_0} V_0 \alpha\right)}{\left(\frac{T_1}{m_0}\frac{\cos\alpha}{V_0} + \frac{L'}{m_0} V_0\right)} + \\ &+ \lambda_V{}^{out}\left(0\right)\left(\frac{T_1}{m_0}\cos\alpha - \frac{V_0{}^2}{m_0}\left(D_0{}' + D_i{}' \cdot \alpha^2\right)\right) \\ &= H^{out}\left(t=0\right) \end{aligned} \tag{82}$$

An analytic solution is not obtained, only a numeric one. In order to get an analytic solution, 2nd order Taylor series approximations of the trigonometric functions are used; $\sin\alpha = \alpha - \alpha^3/3! + \alpha^5/5! - \ldots \simeq \alpha$ and $\cos\alpha = 1 - \alpha^2/2! + \alpha^4/4! - \ldots \simeq 1 - \alpha^2/2$. Some algebra and neglecting terms of α^3 and higher results in

$$\alpha = \pm\sqrt{\frac{\Delta H_{in0}^{out}\left(\frac{T_1}{m_0}\frac{1}{V_0} + \frac{L'}{m_0} V_0\right)}{\Delta H_{in0}^{out}\frac{T_1}{2m_0}\frac{1}{V_0} + \lambda_V{}^{out}\left(0\right)\left(\frac{T_1}{2m_0} + \frac{V_0{}^2}{m_0} D_i{}'\right)\left(\frac{T_1}{m_0}\frac{1}{V_0} + \frac{L'}{m_0} V_0\right)}} \tag{83}$$

$$\Delta H_{in0}^{out} = -CV_0 + CV_0 \cos\left(\psi - \psi^{out}\right) \tag{84}$$

7. Close Loop solution

The closed loop solution is necessary for on-line applications. In the closed loop solution, the instantaneous states are used each calculation step as initial values. Using these values the outer solution is solved followed by the inner solution to calculate the control. It should be noted that special treatment is given to problem parameters such as pulse burning time, pulse ignition time and current flight phase

8. Results

A sample run is shown based on the following parameters:

Parameter	Value	Units
x_0	10000	m
y_0	0-10000	m
V_0	500	m/s
ψ_0	0	
m_0	100	kg
m_{P1}	25	kg
m_{P2}	25	kg
τ_{P1}	3	sec
τ_{P2}	3	sec
Isp_1	240	sec
Isp_2	240	sec
$\dot{m}_1$	8.33	kg/s
$\dot{m}_2$	8.33	kg/s
T_1	19,620	N
T_2	19,620	N
V_t	200	m/s
ψ_t	0	
L'	0.1	kg/m·rad
D_0'	0.005	kg/m
D_i'	0.01	$kg/m \cdot rad^2$
k	0.85	
κ_t	$k \cdot V_0 / R_0$	sec^{-1}
κ_V	$(1-k)/V_0$	sec/m

Two sample runs based on this data are shown. The first is the near optimal solution calculated using the closed-loop method described in this paper. The second run is a numeric reference optimal solution calculated by the collocation method. Fig. 3 shows the trajectories of the target (red, doted), the missile's optimal trajectory (dashed green) and near optimal solution (blue). As expected by the time scale separation, the trajectories show an initial jurn followed by and almost straight path to intercept. Fig 4, shows the time histories of the velocity and heading. The velocities of both solutions are almost the same. The heading shows the expected behaviour of an initial turn followed by an almost constant heading flight. Fig 5 describes the main control parameters. The angle of attack going to zero towards the end with the exception that the near optimal solution control value grows towards the end. This is due to the fact that near the end the time scale separation is no longer valid. The outer solution calculation of the expected final time and heading are almost constant also hinting that the time scale separation is valid for this case.

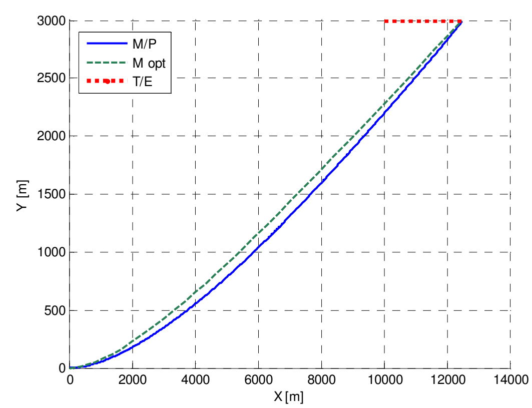

Fig. 3, Trajectories in the X_Y plane

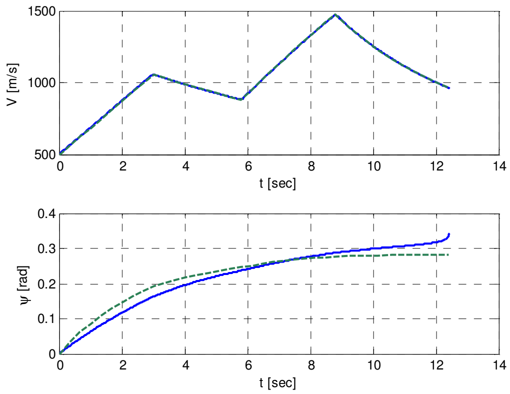

Fig. 4, Velocity and Heading time histories

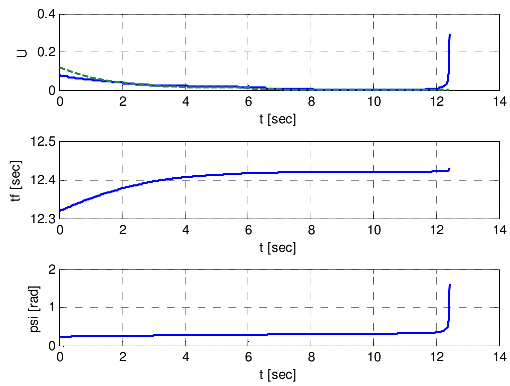

Fig. 5, Main control parameters

9. Conclusions

A near-optimal guidance logic including pulse motor control has been derived. The use of singular perturbations technique combined with switched systems modeling enabled an almost full analytic solution of the problem. The results were tested against a reference optimal solution and have been shown to provide a very good approximation. The use of a near-optimal solution has two main benefits, a) a closed loop solution and b) a simple almost analytic solution. These two benefits allow for a simple on-line implementation of this guidance scheme. This solution method is based on the separation between fast and slow variables and is suitable for midcourse guidance. The solution holds up to small ranges from the target where a more conventional (PN type) terminal guidance is recommended. Additional contributions of this work are a) the development of the minimum principle for switched systems with system equations dependent on the switching time, and b) the development of a modified rocket equation including the effect of drag..

10. References

[1] Bryson E., Ho Y.C., Applied Optimal Control, optimization, estimation and control, Hallsted Press, 1975

[2] A. Merari and J. Shinar, “Applicability of Forced Singular Perturbation Technique in Flight Mechanics”, TAE No. 391, Technion – Israel Institute of Technology, 1979

[3] H.G. Visser, “An Introduction to the Technique of Singular Perturbations Applied to Performance Optimization Problems in Atmospheric Flight Mechanics”, LR-374, Delft University of Technology, 1982

[4] Anthony J. Calise and Daniel D. Moerder, “Singular Perturbation Techniques for Real Time Aircraft Trajectory Optimization and Control”, NASA Contractor Report 3597, 1982

[5] B. Sridhar and N.K. Gupta, “Missile Guidance Laws Based on Singular Perturbation Methodology”, *J. Guidance and Control*, Vol. 3, No. 2, 1980, pp. 158-165

[6] N. Rajan and M.D. Ardema, “Interception in Tree Dimentions:An Energy Formulation”, *J. Guidance*, Vol. 8, No. 1, 1985, pp 23-30

[7] V.H.L. Cheng and N.K. Gupta, “Advanced Midcourse Guidance for Air-to-Air Missiles”, *J. Guidance*, Vol. 9, No. 2, 1986, pp 135-142

[8] Josef Shinar, Hendrikus G. Visser, “Synthesis and Validation of Feedback Guidance Laws for Air-to-Air Interceptions”, LR-615, Delft University of Technology, Technion – Israel Institute of Technology, 1990

[9] X. Xu, P.J.Antsaklis, “Optimal Control of Switched Systems Based on Parameterization of the Switching Instants”, *IEEE transactions on automatic control*, Vol. 49, N0. 1, Jan 2004

[10] X. Xu, P.J. Antsaklis, “Optimal control of switched systems via non-linear optimization based on direct differentiations of value function”, *Int. J. Control*, Vol. 75, Nos 16/17, 2002, pp. 1406-1426.

[11] V.H.L. Cheng, P.K.A. Menon, N.K. Gupta, M.M. Briggs, “Reduced-Order Pulse-Motor Ignition Control Logic”, *J. Guidance*, Vol. 10, No. 4, 1987, pp. 343-350

[12] George P. Sutton, Oscar Biblarz, *Rocket Propulsion Elements*, Seventh Edition, John Wiley & Sons, Inc., New York, 2001, pp. 102-160.

[13] George B. Thomas, Jr., Roll L. Finney, *Calculus and Analytic Geometry,* Sixth Edition, Addison-Wesley

Publishing Company., Massachusetts, 1984, pp. 231-236.

Appendix A The Modified Rocket Equation

The classic Rocket Motor Equation is a well known simple and important tool in evaluating rocket motor performance. It calculates the increase in velocity during the operation of a rocket motor based the basic rocket motor parameters assuming no losses due to drag and gravity. The classic rocket motor equations is given by [12]

$$\Delta V = \frac{T}{\dot{m}} \ln \frac{m_0}{m_f} = Isp \cdot g \ln \frac{m_0}{m_f} \tag{85}$$

With ΔV the gained velocity during the rocket motor operation, m_0 initial mass, m_f the final mass (after motor burnout), Isp the specific impulse, assumed constant ($Isp = T/\dot{m}g$), T the thrust (not assumed constant), $\dot{m}$ propellant consumption rate and g_0 the standard gravity coefficient.

Using the assumptions of constant thrust, constant mass flow rate, constant altitude, and constant drag coefficient, the velocity equation is given by (quoting equation (41) during the pulse phases):

$$\dot{V} = \frac{T - D'V^2}{m_0 - \dot{m}t} \tag{86}$$

This equation, with the initial condition $V(0) = V_0$ has an analytic solution

$$V(t) = \sqrt{\frac{T}{D'}} \tanh\left[\frac{\sqrt{TD'}}{\dot{m}} \ln\left(\frac{m_0}{m_0 - \dot{m}t} \right) + \tanh^{-1}\left(V_0 \sqrt{\frac{D'}{T}} \right) \right] \tag{87}$$

This equation is a modified rocket equation that includes the effect of drag under the assumptions: a) constant thrust force, b) constant propellant consumption rate, c) constant altitude, and d) constant drag coefficient

This equation (87) looks different from the classic rocket motor equation. In order to validate the relation between this equation and the classic rocket motor equation, the limit of V(t) in the new equation will be evaluated for $D' \to 0$ using l'Hôpital's rule [13].

$$\lim_{D' \to 0} V(t) = \lim_{D' \to 0} \frac{\sqrt{T} \tanh\left[\frac{\sqrt{TD'}}{\dot{m}} \ln\left(\frac{m_0}{m_0 - \dot{m}t} \right) + \tanh^{-1}\left(V_0 \sqrt{\frac{D'}{T}} \right) \right]}{\sqrt{D'}} \tag{88}$$

Some algebra results in

$$\lim_{D' \to 0} V(t) = \frac{T}{\dot{m}} \ln\left(\frac{m_0}{m_0 - \dot{m}t} \right) + V_0 \tag{89}$$

Or

$$\Delta V = \frac{T}{\dot{m}} \ln \frac{m_0}{m_f} \tag{90}$$

Which is identical to the classic rocket equation.

Appendix B The Minimum Principle for Switching System

The development given in this appendix is based on the method shown in [1]. In [1] a development of the minimum principle for switching systems without the dependence on the switching time.

A switched dynamic system is described in eq. (91) below. The initial system equations g_1 are dependent on the state, control and time. The second system equations g_2 are dependent on the switching time as well. Note that since this is a physical (causal) system, g_1 can not depend on the future switching instance.

$$\dot{x}=\begin{cases} g_1\left(x(t),u(t),t\right) & t\in\left[t_0,t_s\right] \\ g_2\left(x(t),u(t),t,t_s\right) & t\in\left(t_s,t_f\right] \end{cases} \tag{91}$$

Where $x(t)\in R^n$, $u(t)\in R^m$

The switch is determined by the general interior manifold

$$\psi\left(x_s,t_s\right)=0 \tag{92}$$

The control $u(t)$ is PWC, g_1 and g_2 are continuous and have all partial derivatives required, but, in general, $g_1\left(t_s\right)\neq g_2\left(t_s\right)$ i.e. no continuity of the state derivatives at the switching instance.

We want to minimize the general cost function

$$J=\varphi\left(x_f,t_f\right)+\int_{t_0}^{t_f}L\left(x,u,t\right)dt \tag{93}$$

for some specified initial conditions, but where the switching time and the final time are free.

The Hamiltonian in this case is defined as

$$H\left(x,u,t,\lambda\right)=\begin{cases} L+\lambda^T g_1 & t\in\left[t_0,t_s\right] \\ L+\lambda^T g_2 & t\in\left(t_s,t_f\right] \end{cases} \tag{94}$$

With $\lambda(t)\in R^n$ the vector of continuous Lagrange multipliers. The equations for the Lagrange multipliers are

$$\dot{\lambda}^T=-\frac{\partial H}{\partial x}=\begin{cases} -\frac{\partial L}{\partial x}-\lambda^T\frac{\partial g_1}{\partial x} & t\in\left[t_0,t_s\right] \\ -\frac{\partial L}{\partial x}-\lambda^T\frac{\partial g_2}{\partial x} & t\in\left(t_s,t_f\right] \end{cases} \tag{95}$$

The optimal solution should fulfill the system equations, the differential equations for the Lagrange multipliers and the following conditions

1) Optimality condition for the unconstrained continuous control

$$\frac{\partial H}{\partial u}=0 \tag{96}$$

2) The known transversality conditions used for boundary values on the co-states

$$\lambda^T\delta x\Big|_{t_0}=0 \tag{97}$$

$$\lambda^T\delta x\Big|_{t_0}=0 \tag{98}$$

$$\lambda^T\delta x\Big|_{t_0}=0 \tag{99}$$

3) A new transtersality condition at the switching instance

$$\left(\nu^T\frac{\partial\psi}{\partial x_s}-\lambda^T\Big|_{t_s^-}+\lambda^T\Big|_{t_s^+}\right)dx_s=0 \tag{100}$$

4) A new condition for optimality of the switching instance

$$\left(\nu^T\frac{\partial\psi}{\partial t_s}+H\Big|_{t_s^-}-H\Big|_{t_s^+}+\int_{t_s}^{t_f}\lambda^T\frac{\partial g_2}{\partial t_s}dt\right)\delta t_s=0 \tag{101}$$

Note in equations (100) and (101) ν is a vector of constant Lagrange multipliers

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