TURBULENT SKIN FRICTION REDUCTION OWING TO ELECTROHYDRODYNAMIC INTERACTION IN UNIPOLAR CHARGED FLUID

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Abstract

A simplified model of turbulent skin friction, involving consideration of quasi-streamwise vortices in the cross-stream section, is used to study the effect of electrohydrodynamic interaction in viscous fluid containing volumetric charge of one sign. Through numerical solution of non-stationary electrohydrodynamic equations it is shown that a transfer of the volumetric charge due to cross-flow induced by the vortices causes an increase of the charge density between the vortices that, in turn, leads to a generation both horizontal and vertical components of the volumetric electrostatic force directed against the cross-flow. This retarding action of the electrostatic force results in a decrease of streamwise momentum transfer across the main flow and, as a consequence, the rate of time growth of the spanwise averaged skin friction reduces.

1 Introduction

The formation of streaky velocity structures in the near wall region of turbulent boundary layer was studied in [1] through a simplified two-dimensional computational model formulated in the plane normal to the main longitudinal shear flow. It was shown that the redistribution of the longitudinal velocity by streamwise vortices produces features very similar to those observed in the experiments, and that compact streamwise vortices form naturally from more general initial vorticity distributions. It was also shown that one effect of the formation of the streaks is an increase the average wall friction, and it was suggested that this effect is responsible for the higher skin friction in turbulent boundary layer.

Reduction of turbulent skin friction may be based on some action on mentioned quasi-streamwise vortices. In particular, the model [1] was developed in [2] to study the effect on the skin friction of oscillating the surface beneath the boundary layer in the spanwise direction. It was shown that the interaction between evolving streamwise vortices and a modified Stokes layer on the oscillating surface leads to reduction in the skin friction, the Reynolds stress and the rate of production of kinetic energy, consistent with predictions based on experiments and direct numerical simulations. Study of spanwise-wall oscillation in the turbulent boundary layer showed [2, 3] that the streamwise velocity is reduced near the wall as the vortex sheet in the Stokes layer is tilted into the spanwise direction during the wall oscillation to create a negative spanwise vorticity. The resultant reduction in the streamwise velocity in the near-wall region hampers the stretching of the quasi-streamwise vortices, thereby reducing the streamwise vorticity. Consequently, the near-wall events are weakened, leading to a reduction in the turbulent skin-friction drag.

Besides mechanical wall oscillation a number of methods available to achieve spanwise oscillations in the near wall region have been proposed, such as local oscillatory blowing [4], electromagnetic (using Lorentz force) oscillation [5], electrostatic (using plasma actuators) oscillation [6].

The present research is devoted to qualitative description of physical phenomenon of electrohydrodynamic interaction taking place in shear flow of a viscous fluid containing a volumetric charge of one sign near to a dielectric surface. This numerical research does not pretend to accurate quantitative description of the phenomenon under consideration because it is executed in the framework of the simplified computational model similar to [1] and extended for the case of charged fluid flow.

2 Physical pattern of flow

It is supposed that mentioned interaction may be realized in the near wall region of the ion jet generated by some source of ions or charged particles (direct current corona discharge in air [7], for example) and propagating along a dielectric wall (see Fig. 1).

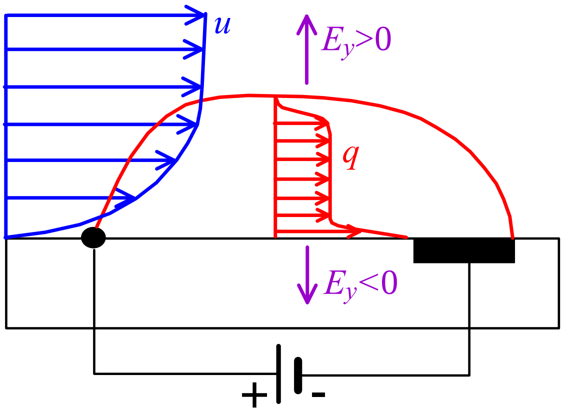

Fig. 1. The ion jet (red curve) generated by near wall corona discharge inside a boundary layer (blue curve).

Positive volumetric charge of the ion jet generates the component of the electric field strength E_y directed from the wall above the jet and to the wall at the bottom of the jet. The volumetric charge density q increases significantly in relatively thin near wall region of the jet because the drift electric current induced by the electric field strength and directed to the wall must be compensated by the diffusive current created by a gradient of the charge density and directed from the wall to satisfy the condition of absence of total current on dielectric surface [8]. One can expect that if the thickness of this diffusive charged layer will be comparable with the viscous sublayer of the turbulent boundary layer, then the assumed electrohydrodynamic interaction can influence on the evolution of streamwise vortices and on subsequent variation in the skin friction.

According to asymptotic approach used in [8], the near wall ion jet consists of the main part, where the process of volumetric charge diffusion is negligible, and thin diffusive layer in the vicinity of the wall. The charge distribution in the diffusive layer is governed by proper electrohydrodynamic equations at zero charge density and given vertical component of the electric field strength on the outer boundary of the layer. In turn, the electric field strength is defined from simultaneous solution of the external and internal problems [8].

2 Statement of the boundary value problem

A flow of viscous incompressible fluid near to a flat dielectric wall is considered. It is supposed that the fluid contains volumetric electric charge of single sign (positive, for distinctness) and the external electric field is applied perpendicular to the wall. The Cartesian coordinate system *Oxyz* is introduced with the origin placed on the wall, the axis *Ox* directed along the main shear flow, the axis *Oy* directed perpendicular to the wall, and the axis *Oz* directed along a span (see Fig. 2).

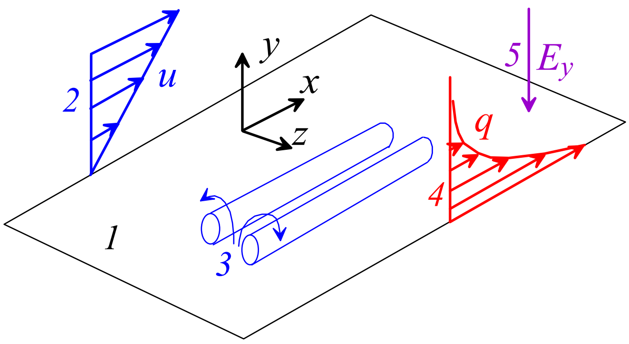

Fig. 2. The sketch of the considered flow: *1* – dielectric wall, *2* – main shear flow, *3* – streamwise vortices, *4* – volumetric charge, *5* – perpendicular electric field.

At the initial moment of time a shear flow along the axis *Ox* presents. It is supposed that variation of flow parameters in the streamwise direction is negligible, i. e. $\partial / \partial x = 0$. In this case the equations of motion of the transverse flow (in *y*-*z* cross-section) decouple from those of the streamwise component. The streamwise component of the fluid velocity does not influence on transverse flow but depends on it. The considered flow is governed by the system of the non-stationary equations of electrohydrodynamics

$$\nabla^* \cdot \mathbf{V}^* = 0 \qquad (1)$$

$$\frac{\partial \mathbf{V}^*}{\partial t^*} + \left(\mathbf{V}^* \cdot \nabla^*\right)\mathbf{V}^* + \frac{\nabla^* p^*}{\rho^*} - \frac{q^* \mathbf{E}^*}{\rho^*} = \\ = \nu^* \nabla^{*2} \mathbf{V}^*$$

$$\varepsilon_0^* \nabla^* \cdot \mathbf{E}^* = q^*, \quad \nabla^* \times \mathbf{E}^*$$

$$\frac{\partial q^*}{\partial t^*} + (\mathbf{V}^* + b^* \mathbf{E}^*) \nabla^* q^* + \frac{b^* q^{*2}}{\varepsilon_0^*} = \\ = D^* \nabla^{*2} q^*$$

Here $\mathbf{V}^* = \left(u^*, v^*, w^*\right)$ is the vector of the fluid velocity, $\mathbf{E}^* = \left(0, E_y^*, E_z^*,\right)$ is the vector of the electric field strength, ρ^* is the fluid density, ν^* is the kinematic viscosity, p^* is the static pressure, q^* is the volumetric charge density, $(q^* > 0)$, b^* and D^* are the coefficients of mobility and diffusion of ions, $\varepsilon_0^* = 8.85 \cdot 10^{-12}$ F/m is the dielectric permeability of vacuum. The upper index asterix denotes dimensional values. For numerical solution the flow variables are non-dimensionalized with respect to the wall units according to the expressions $\mathbf{x}^* = (\nu^*/u_\tau^*)\mathbf{x}$, $t^* = (\nu^*/u_\tau^{*2})t$, $\mathbf{V}^* = u_\tau^* \mathbf{V}$, $\mathbf{E}^* = \left(u_\tau^*/b^*\right)\mathbf{E}$, $p^* = \rho^* u_\tau^{*2} p$, $q^* = \left(\varepsilon_0^* u_\tau^{*2}/b^* \nu^*\right) q$, $D^* = \nu^* D$, where $u_\tau^* = (\tau_w^*/\rho^*)^{1/2}$ is the friction velocity, τ_w^* is the shear stress at the wall. For calculation of the flow in the $y - z$ cross-section the vortex method is used thereby excluding the pressure from a consideration. The streamfunction ψ and the vorticity ω are introduced into consideration according to the relations $v = -\partial\psi/\partial z$, $w = \partial\psi/\partial y$, $\omega = \partial w/\partial y - \partial v/\partial z$.

In the framework of the vortex method under the above assumption $\partial/\partial x = 0$ the equation system (1) takes the following non-dimensional form:

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nabla^2 u \qquad (2)$$

$$\nabla^2 \psi = \omega \qquad (3)$$

$$\frac{\partial \omega}{\partial t} + v\frac{\partial \omega}{\partial y} + w\frac{\partial \omega}{\partial z} + \\ + NE_y \frac{\partial q}{\partial z} - NE_z \frac{\partial q}{\partial y} = \nabla^2 \omega \qquad (4)$$

$$\nabla^2 \varphi = -q, \quad E_y = -\frac{\partial \varphi}{\partial y}, \quad E_z = -\frac{\partial \varphi}{\partial z} \qquad (5)$$

$$\frac{\partial q}{\partial t} + \left(v + E_y\right)\frac{\partial q}{\partial y} + \left(w + E_z\right)\frac{\partial q}{\partial z} + \\ + q^2 = D\nabla^2 q \qquad (6)$$

Here the electric potential φ and the parameter of electrohydrodynamic (EHD) interaction $N = \varepsilon_0^*/\rho^* b^{*2}$ are introduced.

The charged fluid flow is simulated in the rectangular domain $0 \le y \le Y$, $0 \le z \le Z$, where $Y >> 1, Z >> 1$. On the left and the right boundaries $z = 0$ and $z = Z$ the periodic conditions are applied, the no-slip conditions are applied on the wall $y = 0$, and the conditions of decaying of main flow disturbances is applied far away at $y = Y$. Thus, the initial and boundary conditions for equation (2) governing the main shear flow are taken to be:

$$u\left(y, z, 0\right) = y, \quad u(0, z, t) = 0 \qquad (7)$$

$$u(Y, z, t) = Y, \quad u(y, 0, t) = u(y, Z, t)$$

The boundary conditions for the equations (3)-(4) describing the cross-flow take the form

$$\psi(0, z, t) = \psi(Y, z, t) = \omega(Y, z, t) = 0 \qquad (8)$$

$$\psi(y, 0, t) = \psi(y, Z, t)$$

$$\omega(y, 0, t) = \omega(y, Z, t)$$

The first three conditions (8) mean an absence of flow through the lower and upper boundaries of the calculation domain. For solution of the equation (4) it is needed to determine any boundary condition for the function ω at $y = 0$. The values of this function on the wall $\omega\left(0, z, t\right)$ during calculations are defined in iterations so as to satisfy the no-slip condition $\partial\psi(0,z,t)/\partial y = 0$.

The initial conditions for the equations (3)-(4) are defined by the initial distribution of the vorticity ω in the cross-section *y-z*. This distribution is given in the form of two vortex sheets which are elongated in the spanwise direction and take opposite signs but the same absolute value of the total circulation. This distribution for single sheet takes the form [1, 2]

$$\omega(y,z) = \frac{\Gamma\pi^{1/2}}{2hS}\exp\left(-\frac{n^2}{h^2}\right)\sin\left(\frac{\pi s}{S}\right) \qquad (9)$$

$$s = (\mathbf{r}-\mathbf{r}_1)\cdot\mathbf{s},\; n = (\mathbf{r}-\mathbf{r}_1)\cdot(\mathbf{i}\times\mathbf{s})$$

$$\mathbf{s} = (\mathbf{r}_2-\mathbf{r}_1)/S,\; s\in(0,S),\; S = |\mathbf{r}_2-\mathbf{r}_1|$$

Here Γ is the total circulation around a vortex or vortex strength, $\mathbf{r}$ is the radius-vector of the considered point in the cross-section *y-z*, $\mathbf{r}_1$ and $\mathbf{r}_2$ are the radius-vectors defining a spanwise size of the sheet, h is the parameter defining the sheet thickness, $\mathbf{i} = (1, 0, 0)$ is the unit vector along the axis *Ox*.

In addition to distributions of the vorticity ω_1, ω_2 in the form (9) the initial vorticity distribution near to the wall is set in the form

$$\omega_3(y,z,0) = \omega_w(z)\exp\left(-y^2/\delta^2\right) \qquad (10)$$

The unknown function $\omega_w(z)$ is defined by iterations through a solution of the equation (3) with the boundary conditions (8) and given distribution $\omega = \omega_1 + \omega_2 + \omega_3$ so as to satisfy the no-slip condition $\partial\psi(0,z,t)/\partial y = 0$.

The initial distribution of the volumetric charge and the boundary conditions for electric potential are determined by the solution of the equations (5)-(6) in stationary ($\partial/\partial t = 0$), one-dimensional ($\partial/\partial z = 0$) so-called “diffusion” approximation, that is without taking into account convective charge transfer. This approximation takes the form

$$E' = q,\quad DE''' - EE'' - E'^2 = 0 \qquad (11)$$

$$y = 0: DE'' - EE' = 0,\; E = E_0$$

$$y\to\infty: E\to E_e$$

The analytical solution of the problem (11) takes the form [8]

$$E(y) = E_e\frac{1+\beta}{1-\beta},\; q_0(y) = \frac{2E_e^2\beta}{D(1-\beta)^2} \qquad (12)$$

$$\beta(y) = \alpha\exp\left(\frac{E_e}{D}y\right),\; \alpha = \frac{E_0-E_e}{E_0+E_e}$$

Values of $E_0 < 0$ and $E_e < 0$ $\left(|E_0| > |E_e|\right)$ define a magnitude of the volumetric charge in diffusion layer and are the parameters of the formulated problem. In particular, the value of the volumetric charge on the wall is evaluated as $q(0) = \left(E_0^2 - E_e^2\right)/2D$.

The ideal dielectric surface which does not adsorb electric charge is considered. In this case the condition $\varepsilon_1 E_0 = \varepsilon_2 E_2$ satisfies on the wall, where ε_1 and ε_2 are the relative dielectric constants of the fluid and the wall, respectively, E_2 is the electric field strength in the wall. The initial distributions of the electric potential in the fluid and in the wall according to (12) take the form

$$\varphi_1(y,z,0) = -DE_e y - 2D\ln\frac{1-\alpha}{1-\beta} \qquad (13)$$

$$\varphi_2(y,z,0) = -E_2 y$$

According to this formulation the zero value of the electric potential is defined on the wall. The Laplace equation for electric potential $\nabla^2\varphi_2 = 0$ is solved inside the dielectric wall in the domain $Y_2 \le y \le 0$, $0 \le z \le Z$, where $Y_2 << -1$, with the condition of periodicity $\varphi_2(0,y,t) = \varphi_2(Z,y,t)$ and the condition on the lower boundary $\varphi_2(Y_2,z,t) = -E_0 Y_2/\varepsilon_2$. The Poisson equation (5) is solved in the flow region with the same condition of periodicity on the vertical boundaries and the condition on the upper boundary followed from (13)

$$\varphi_1(Y,z,t) = -DE_e Y - 2D\ln(1-\alpha)$$

On the dielectric wall the following conditions are applied:

$$\varphi_1(0,z,t) = \varphi_2(0,z,t)$$

$$\varepsilon_1 \partial\varphi_1(0,z,t)/\partial y = \varepsilon_2 \partial\varphi_2(0,z,t)/\partial y$$

The initial condition for the charge conservation equation (6) is defined by the expression (12), that is $q(x,y,0) = q_0(y)$, and the following conditions on the lower and the upper boundaries are used:

$$y = 0: \quad q\frac{\partial\varphi_1}{\partial y} + D\frac{\partial q}{\partial y} = 0 \qquad (14)$$

$$y = Y: \quad q = q_0(Y)$$

The first condition (14) signifies an absence of the electric current on dielectric wall. The periodicity conditions for the electric potential and the volumetric charge density are applied on the vertical boundaries

$$\varphi(y,0,t) = \varphi(y,Z,t),\ q(y,0,t) = q(y,Z,t)$$

Note that the problem parameters N and D depend on only physical properties of the fluid and the charge carriers. But some variation of the parameters E_0 and E_e will correspond to a change of the volumetric charge density in the diffusive layer. It may be achieved, for example, by a control of ion source intensity.

3 Results of calculations

Numerical solution of the formulated problem was obtained by finite-element method using Lagrange elements of second order. Check of the mesh influence on accuracy of calculations resulted in nonuniform triangular mesh consisting from 22315 elements in fluid region and 15955 elements in dielectric. Minimal and maximal sizes of elements equal 0.1 (on dielectric surface) and 1.5 (on upper and lower boundaries), respectively, for considered task geometry.

Calculations were executed at fixed values of the problem parameters $\Gamma = 300$, $E_e = -0.01$, D=1, $\varepsilon_1 = 1, \varepsilon_2 = 3$, Y=100, Y_2=-50, Z=100 and for constant geometrical parameters of the initial vorticity distributions in the vortex sheets (9), namely, $h = 0.75$ for both sheets, $\mathbf{r}_1 = (0{,}15{,}10)$ and $\mathbf{r}_2 = (0{,}10{,}50)$ for the left sheet ω_1 (negative values of vorticity and counter-clockwise induced cross-flow), $\mathbf{r}_1 = (0{,}10{,}50)$ and $\mathbf{r}_2 = (0{,}5{,}60)$ for the right sheet ω_2 (positive values of vorticity, clockwise induced flow), $\delta = 2$ in (10) for the near wall vorticity distribution ω_3. The calculated initial distribution of the streamwise vorticity $\omega_0=\omega_1+\omega_2+\omega_3$ is presented on Fig. 3. The values of the EHD-interaction parameter N and the electric field strength on the wall E_0 in calculations were varied.

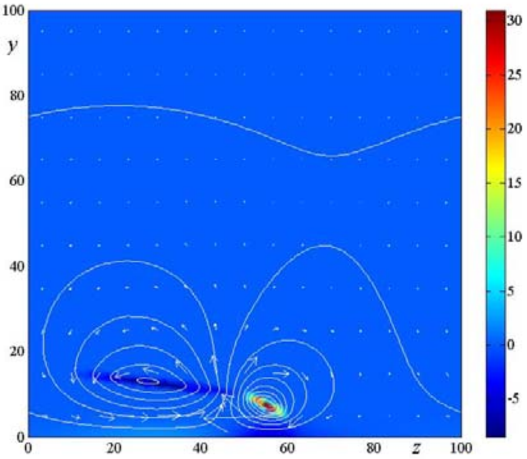

Fig. 3. The initial distribution of vorticity, streamlines and velocity vectors of the transversal flow.

Below the results of numerical simulation of the considered electrohydrodynamic flow obtained for task parameters N=10 and E_0=-1 at the time moment t=10 are discussed in detail.

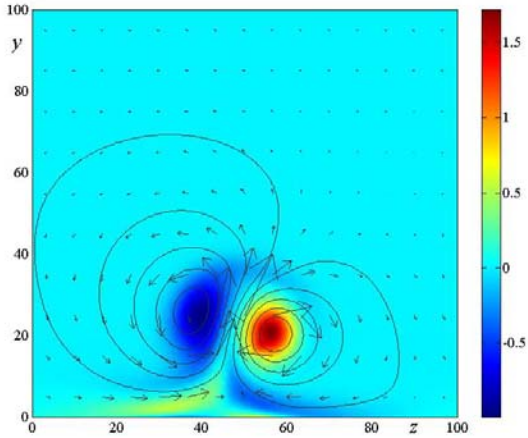

Fig. 4. The vorticity distribution, streamlines and velocity vectors of the transversal flow at t=10.

Evolution of the streamwise vortex sheets is reflected on Fig. 4. It is evident that the interaction of the sheets results in their moving away from the wall. The more concentrated right sheet (positive vorticity) becomes more similar to axially symmetrical vortex and deforms drastically the left sheet. The extremal values of vorticity in both sheets reduce significantly due to viscous dissipation.

The transversal flow induced by streamwise vorticity distribution results in essential alteration of main shear flow, as Fig. 5 demonstrates. The ascending transversal flow arising between the vortex sheets lifts up low-speed fluid from the wall. On the contrary, the descending transversal flow arising near the vertical boundaries of the considered domain presses high-speed fluid to the wall. In consequence the streamwise skin friction $\tau_w = \partial u(x,0)/\partial y$ decreases in some region between the vortex sheets and increases in fringe regions. The average value of the skin friction on the site $0 \le z \le 100$ increases (in accordance with [1]).

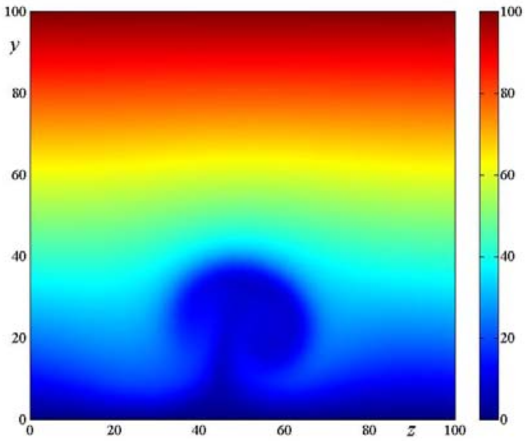

Fig. 5. The distribution of main flow velocity at t=10.

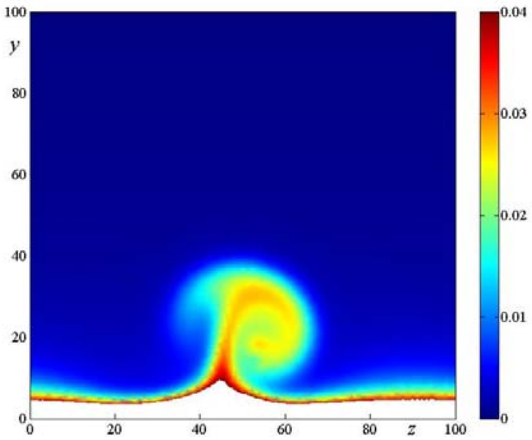

Fig. 6. The distribution of charge density at t=10.

The ascending transversal flow lifts the volumetric charge from the diffusive layer in main region of the shear flow, as Fig. 6 shows. The white area on Fig. 6 corresponds to values of the volumetric charge density exceeding 0.04. Distribution of maximal values of the charge density reaching on the wall is shown on Fig. 7. Note that repulsive action of own electric field results in appearing two maximums in the volumetric charge density distribution.

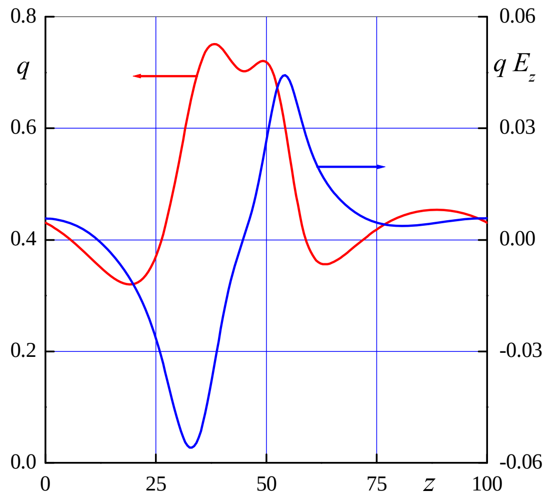

Fig. 7. The distributions of volumetric charge density and horizontal electrostatic force on the wall at t=10.

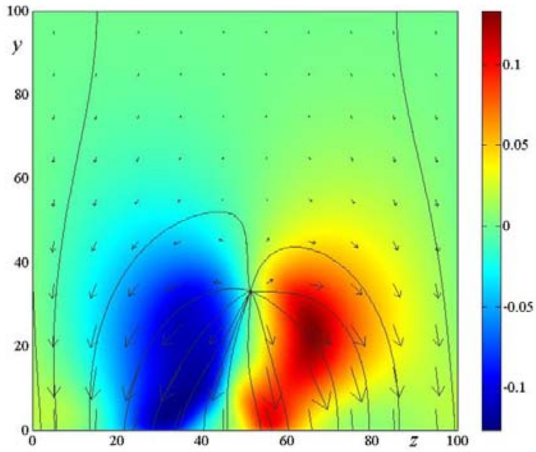

Fig. 8. The distributions of horizontal component of electric field strength E_z, field lines and vectors at t=10.

Positive volumetric charge in the arising burst induces the horizontal component of the electric field strength E_z opposite directed on different sides of the burst, as it is shown on Fig. 8. Positive values of E_z on Fig. 8 mean that the horizontal force is directed to the right, negative values – to the left. Note that carrying out of positive charge leads to an origin of electric potential maximum at some distance from the wall. Therefore the point of this maximum is the source of the field lines.

The induced horizontal component of the electric field strength generates horizontal volumetric electrostatic force $F_z=qE_z$, as it is shown on Fig. 9. The sign (i. e. the direction) of this force coincides with the sign of the field component E_z, but the spatial distribution of F_z is determined in the main by volumetric charge density. Therefore extremal values of the horizontal force are reached on the wall and are presented on Fig. 7. The principal feature of the horizontal force distribution shown on Fig. 9 is that this force is positive (rightwards directed) below the right vortex (inducing clockwise cross-flow) and is negative below the left vortex (generating counter-clockwise cross-flow). That is the horizontal force decelerates the transversal flow beneath the both vortices.

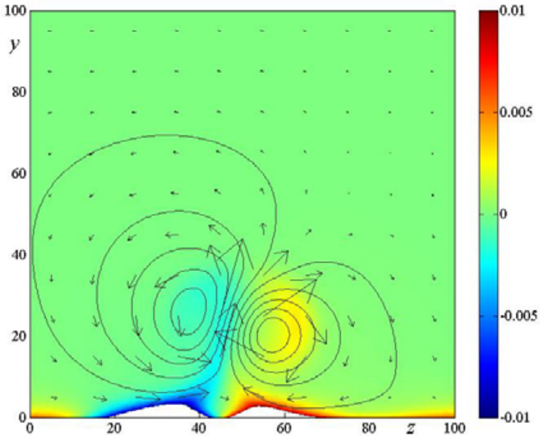

Fig. 9. The distributions of horizontal electrostatic force F_z, streamlines and velocity vectors of the transversal flow at t=10.

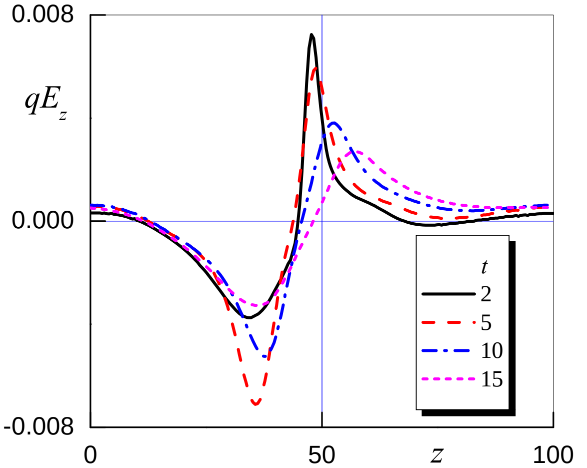

Fig. 10. The evolution of horizontal electrostatic force F_z at the distance y=5 from the wall.

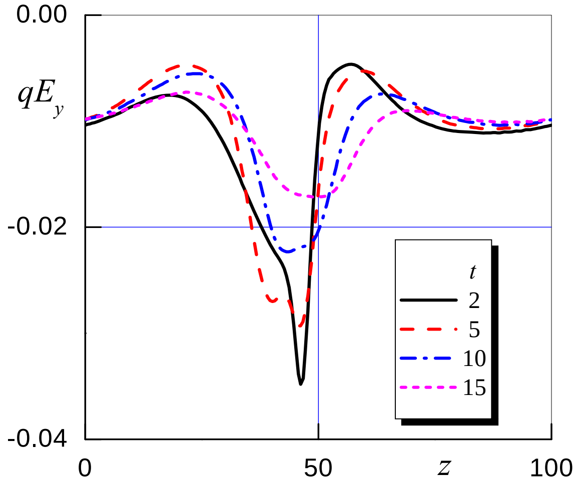

Fig. 11. The evolution of vertical electrostatic force F_y at the distance y=5 from the wall.

Qualitative behavior of the horizontal force in time is reflected on Fig. 10. It is seen that both positive and negative forces reach their extremal value (but in different time) and thereupon decays. Besides the horizontal force, a concentration of the volumetric charge between the vortices results in an appearance of additional negative vertical component of the electrostatic force in this flow region (see Fig. 11). This force component is directed opposite to the ascending transversal flow and therefore makes a contribution in its attenuation too.

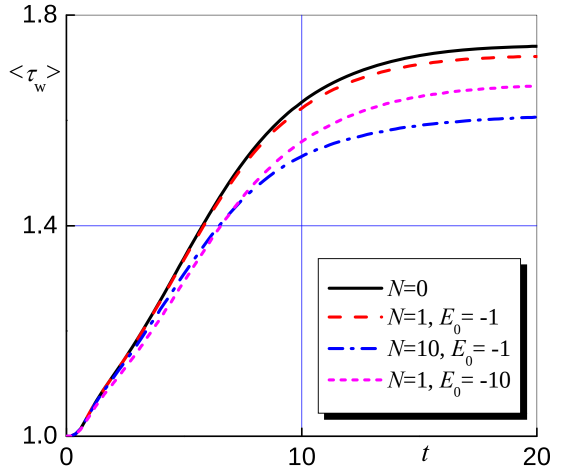

Fig. 12. The influence of EHD-interaction on evolution of average skin friction.

Mentioned decelerating influence of volumetric electrostatic force on transversal flow leads to a weakening of the streamwise momentum transfer across the shear flow. It, in turn, results in a delay of growth in time of the skin friction averaged along a span, as it is shown on Fig. 12. Here the time dependences of the average skin friction for different values of the parameter of EHD-interaction and vertical component of the electric field strength on the wall are presented. It is evident that remarkable decrease of the skin friction may be received only at high enough values of the parameter of EHD-interaction or the volumetric charge density (i. e. large absolute value of the parameter E_0).

As for a possibility of practical realization of the considered method of turbulent drag reduction, it must be noted the following. If the carriers of electric charge are the ions of air, then the parameter N is very small (of the order of 10^{-4} at atmospheric pressure) due to high mobility of ions. Therefore possible effect of the skin friction reduction will be negligible in this case. Significant amplification of this effect may be possible if to use charged micro particles because their mobility may be greater up to several orders as compared to ions. Disadvantage of usage of charged particles may consist in very small coefficient of diffusion of particles in comparison with ions. Because of it the charged diffusive layer may be too thin and carrying out the volumetric charge from this layer by the transversal flow may be difficult. In any case this method demands further study.

4 Conclusion

Modeling of evolution of streaky velocity structures in near-wall region of turbulent boundary layer in unipolar charged fluid flow over dielectric wall is carried out through numerical solution of simplified two-dimensional non-stationary boundary value problem of electrohydrodynamics. It is shown that volumetric charge transfer by transversal fluid flow results in concentration of volumetric charge between streamwise vortices and generation of electrostatic forces retarding this transversal flow. As a consequence of this decelerating force action, the intensity of bursts of low-speed fluid from the wall attenuates and the rate of time growing of the average skin friction reduces.

5 Acknowledgments

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