PROBABILISTIC ANALYSIS ON CRACK GROWTH LIFE OF TURBINE DISK UNDER LCF-CREEP THROUGH EXPERIMENTAL DATA

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Abstract

Fatigue durability and inspection planning have long been important issues in the design and scheduled inspection of the aero-engines. Usually, the turbine disk acting as one of the most important structures of the aero-engine has to be designed for a finite life with an accepted probability of failure based on the S-N approach. Hence, cracks may propagate and become critical during the predicted "safe-life", unless discovered in time and repaired. The crack growth life of turbine disks at elevated temperatures is governed by the modes of degradation and failure including low cycle fatigue (LCF) failure and creep failure. Meanwhile, uncertainties related to the operating environment (speed, temperature, etc.) as well as in the structural properties (material properties, geometries, boundary conditions etc.), can result in considerable statistical scatter in the turbine disk life. The need for cost-effective designs has resulted in the development of probabilistic analysis to quantify the effects of these uncertainties to improve the components' reliability. Thus the purpose of this paper is to carry out a probabilistic analysis on crack growth life of turbine disk under LCF-creep based on experimental results.

First, an experimental system to achieve real-time fatigue-creep crack growth(FCCG) detection at high temperature is established by introducing a long-distance microscope with high magnification and resolution from distances of 15cm to 35cm. This setup consists of a dynamic testing machine, a machine controller, a temperature controlled box, a long-distance microscope and a high temperature furnace from room temperature to 1000℃.

Then the FCCG rate tests on 30 compact tension (CT) specimens made of GH4133B material at 600℃ are carried out. The tests are conducted on a 100KN capacity servo-hydraulic closed-loop machine employed trapezoidal load with hold time at upon peak load. Experiments on the fatigue crack growth have shown great dispersancy. The deterministic model for FCCG rate fitted by SINH model, which considers the parameters including temperature, hold time is established through experimental data. And the stochastic FCCG model for GH4133B is proposed and the probability of random time to reach a specified crack size can be obtained as well as the distribution function of crack size at the service time. Through comparison between the analytical and experimental results, it's found that the probabilistic FCCG model can fit the experimental data well. Once the stochastic FCCG model is established, it can be used for the probabilistic damage tolerance analysis and design of the turbine components made of GH4133B material.

At last, considering random characteristics of material parameters and load including rotational speed and temperature, a failure function is established based on the stochastic FCCG model. Then the centroidal Voronoi tessellation (CVT) sampling is used to improve the efficiency. The probabilistic methods of response simulation and Monte Carlo method are employed to carry out the probabilistic

analysis on crack growth life of turbine disk under fatigue-creep.

1 Introduction

The turbine disk is one of the most vital components of modern aero-engines. The typical loading spectrum experienced by an engine disk is characterized by low-frequency stress cycling resulting primarily from centrifugal forces, superposing dwell at peak load at high temperatures during aircraft's taking off and climbing. Crack growth can be significant problem in the design of turbine disk under LCF-creep. In turbine disks, nickel-base superalloys are widely used at high temperature in order to improve engine performances. For certain severe applications, superalloys are manufactured using powder metallurgy processes so as to obtain a superior mechanical property. However, considerable scatter in crack growth behavior usually is apparent even under carefully controlled experimental conditions due to material uncertainty and other unknown factors. As a result, probabilistic methods for the fatigue crack growth have received great attention in recent years. Many research projects [1-11] have investigated the stochastic fatigue crack growth models to depict the scattering of the crack growth process. Furthermore, there rarely is any degree of certainty regarding the load cycles and environment (e.g. corrosive environment) in field situations especially in the turbine disk. Uncertainties in these factors can amplify the scatter in crack growth life. Thus, it is necessary to perform probabilistic analysis on the crack growth life of turbine disk by considering the uncertainties related to material properties and loads so as to provide a quantitative description of the uncertainty in fatigue damage growth.

2 Experiments on Fatigue-Creep Crack Growth

To justify the applicability of the probabilistic models, fatigue crack growth data are needed. However, it is rather time-consuming to perform the experiments to obtain a set of statistical meaningful fatigue crack growth data. Up to now, there are only several data sets available for researchers including the ones released by Virkler et al.[12], Ghonem et al.[13], Yang et al.[14], Wu and Ni[15], Liao and Yang [16]. However, most of these studies were devoted to the material at room temperature. In contrast, the effort attached to the development of probabilistic crack growth model under fatigue-creep conditions remains more limited, due the difficulty in establishing crack growth rate function considering the creep effect and measuring crack size under fatigue-creep. Therefore, we carried out the crack growth experiments on 30 specimens made of nickel-base superalloy GH4133B at 600˚C under fatigue-creep in our laboratory in order to establish a probabilistic fatigue-crack growth model of GH4133B considering material uncertainty.

The material investigated in this study is GH4133B, a nickel-base superalloy for turbine disks. The chemical composition of the alloy is given as follows (in wt%):C 0.06, Cr 19-22, Al 0.75-1.15, Ti 2.3-3.0, Nb 1.3-1.7, Mg 0.001-0.01, Zr 0.01-0.1, and the balance nickel. The applied heat treatment consists of 1080˚C ± 10˚C/8h, air cool + 750˚C±10˚C/16h, air cool. Fatigue-creep crack growth test following the ASTM recommendations [17] were carried out on compact tension (CT) specimen cut from the turbine disk of a certain aero-engine. The dimensions of the specimen shown in Fig.1 are 40mm wide (counting from the loading line to the back face of the specimen) and 5mm thick.

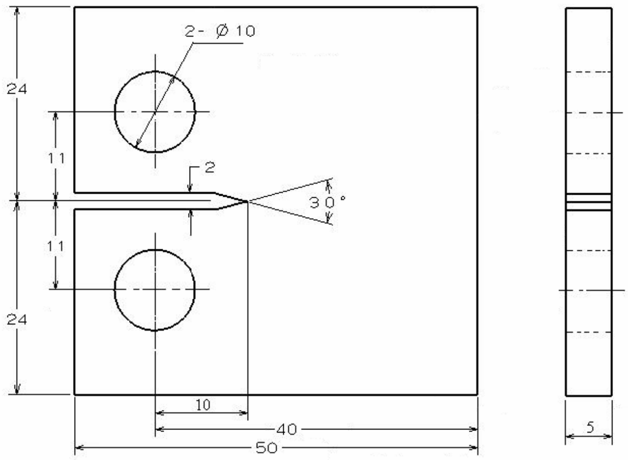

Fig. 1 CT specimen

The tests were conducted on a 100KN capacity servo-hydraulic closed-loop testing system equipped with a heating furnace in load control mode and interfaced with an MTS controller at 600°C. Trapezoidal loading waveforms were applied with hold time at maximum load. For all tests, load rise and fall times were 5s and hold time values of 10s were used to obtain the fatigue-creep crack growth (FCCG) data. All tests are run to fracture. The constant amplitude FCCG experiments on 30 CT specimens were performed.

2.1 Experimental results

The experimental FCCG curves under constant amplitude loading are shown in Fig.2. It should be noted that due to the uncertainty in material properties there is a considerable degree of scatter of the FCCG behavior. This means that both the crack size at a specified load cycle and the number of load cycles at a given crack length have great statistical dispersion. Therefore, it is necessary to perform the probabilistic analysis on the FCCG behavior of alloy GH4133B.

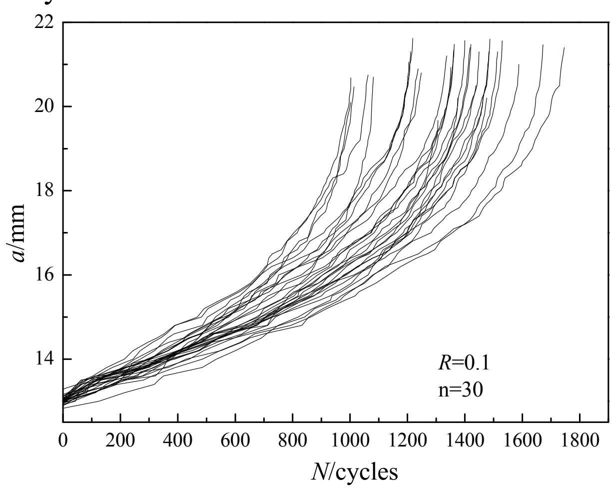

Fig.2 Experimental FCCG curves for GH4133B

The typical crack growth curve has a sigmoidal shape. The curve describing the relationship between crack growth rate, da/dN, and the stress intensity factor ΔK usually roughly divided into three regions. Since Paris law can only be used to model region Ⅱ, a hyperbolic sine model (SINH) [18] is employed in this study, which has the advantages to model the crack growth rate for a variety range of stress intensity factor. Besides this, the environmental parameters such as the effect of temperature, loading, creep can be incorporated in the crack growth model. The median FCCG model for GH4133B at 600°C is expressed as

$$\log\left(da/dN\right) = C_1 \cdot \sinh\left(C_2 \cdot \left(\log \Delta K + C_3\right)\right) + C_4 \qquad (1)$$

where C_1=0.5, C_2=8.788, C_3=-1.888, C_4= -2.486 with the R-squared value is 0.92298, as shown in Fig.3.

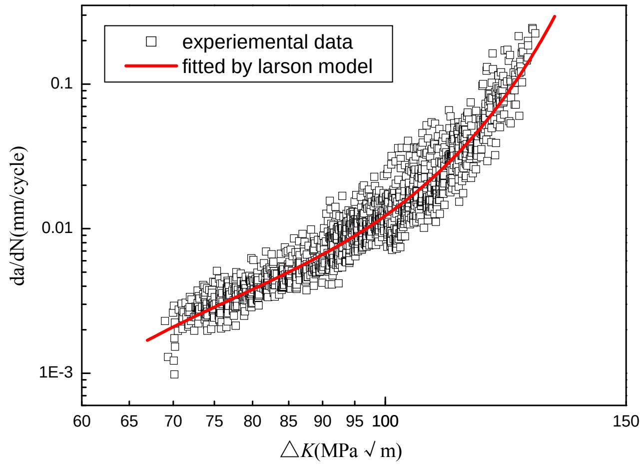

Fig.3 Median FCCG curve

2.2 Stochastic FCCG model

The time-dependent uncertainty in FCCG under constant amplitude load cycling is modeled by a non-negative stationary random process, $X_T^{\Delta t}$ reflecting the uncertainty of material at fatigue-creep failure referred to Yang's model [11, 13],

$$da/dt = X_T^{\Delta t}(t) L(a) \qquad (2)$$

where a is the crack size and $L(a)$ is the function describing the deterministic FCCG (see Equation(1)). Since crack growth increments must be non-negative, the noise $X(t)$ generally is non-Gaussian, with mean value μ_X and standard deviation σ_X. To facilitate the analysis of this stochastic differential equation, an auxiliary zero-mean stationary Gaussian process $Z_T^{\Delta t}$, is introduced by the transformation,

$$Z_T^{\Delta t}(t) = \log X_T^{\Delta t}(t) \qquad (3)$$

where the standard deviation can be obtained from

$$\sigma_Z = \sqrt{\ln\left(1 + V_X^2\right)} \Big/ \ln 10 \qquad (4)$$

where V_X is the coefficient of variation of $X(t)$. Then by the transformation, it satisfies,

$$\mu_X = \exp\left[\frac{\left(\sigma_Z \ln 10\right)^2}{2}\right] \qquad (5)$$

$$\sigma_X = \exp\left[\frac{(\sigma_Z \ln 10)^2}{2}\right]\sqrt{\exp(\ln 10\sigma_Z)^2 - 1} \quad (6)$$

A general auto-covariance function of the following form is assumed for the random process $X(t)$ based on the correlation stripe from the microcosmic observations [19]

$$\operatorname{cov}[X(t_1), X(t_2)] = \exp(-\varsigma|t_2 - t_1|)\sigma_X^2 \quad (7)$$

where ζ^{-1} indicates the correlation time for $X(t)$. When the correlation time ζ^{-1} approaches to zero, it indicates that $X(t)$ is a lognormal white noise random process. When ζ^{-1} approaches to infinite, random process $X(t)$ is simplified as a lognormal random variable X.

From Equation (2), it can be gotten that

$$\int_{a_0}^{a} \frac{da}{L(a)} = \int_0^t X_T^{\Delta t}(\tau)\, d\tau = W(t) \quad (8)$$

where random process $W(t)$ is the integration of $X(t)$. Thus, the distribution function of crack size $F(a)$ at a specified service time t may be derived and expressed as

$$F_{a(t)}(x) = F_{W(t)}[y(x)] = F_{W(t)}[\bar{t}(x)] \quad (9)$$

where $y(x) = \int_{a_0}^{x} \frac{da}{L(a)} = \bar{t}(x)$, $\bar{t}(x)$ is the median service time for a crack to growth from initial crack size a_0 to x. To calculate Equation (9), the mean value and standard deviation of $W(t)$ should be obtained. From Equation (8), we can know

$$\mu_W = E[W(t)] = \int_0^t E\left[X_T^{\Delta t}(\tau)\right] d\tau = \mu_X t \quad (10)$$

$$\sigma_W^2 = E\left[(W(t) - \mu_W)^2\right] = \int_0^t \int_0^t \operatorname{cov}[X(t_1), X(t_2)]\, dt_1 dt_2 \quad (11)$$

Substituting Equation (7) into Equation (11), we have

$$\sigma_W = \frac{\sigma_X}{\varsigma}\sqrt{2\left(e^{-\varsigma t} + \varsigma t - 1\right)} \quad (12)$$

$$V_W = \frac{\sqrt{2\left(e^{-\varsigma t} + \varsigma t - 1\right)}}{\varsigma t}\sqrt{e^{(\ln 10\sigma_Z)^2} - 1} \quad (13)$$

where σ_W is the standard deviation, V_W is the coefficient of variation of $W(t)$. If we obtain the distribution function of $W(t)$, we can solve Equation (9) based on the experimental results. Thus, various distribution functions which are defined in the positive domain, such as Weibull, lognormal, gamma, etc., will be investigation for approximating that of $W(t)$. Here we assume $W(t)$ follows a two-parameters Weibull distribution, then the cumulative distribution function is expressed

$$F_{W(t)}(x) = 1 - \exp\left\{-(x/\beta)^{\alpha}\right\} \quad (14)$$

where the shape parameter $\alpha(t)$ and the scale parameter $\beta(t)$, function of time, satisfy the following expression

$$V_W = \left[\Gamma\left(1 + 2/\alpha\right) - \Gamma^2\left(1 + 2/\alpha\right)\right]^{1/2} \Big/ \Gamma\left(1 + 1/\alpha\right) \quad (15)$$

$$\mu_W = \beta\Gamma\left(1 + 1/\alpha\right) \quad (16)$$

where $\Gamma(\cdot)$ is the Gamma function. Then we can acquire

$$F_{a(t)}(x) = 1 - \exp\left\{-\left(\bar{t}(x)/\beta(t)\right)^{\alpha(t)}\right\} \quad (17)$$

$$F_{T(a_1)}(t) = \exp\left\{-\left(\bar{t}(a_1)/\beta(t)\right)^{\alpha(t)}\right\} \quad (18)$$

$F_{a(t)}(x)$ and $F_{T(a)}(t)$ relate to the correlation time ζ^{-1}. In the condition of lognormal white noise random process, there is the smallest statistical dispersion for crack growth accumulation, leading to the most unconservative life prediction, whereas the life prediction using the lognormal random variable always achieves the conservative result.

2.3 Verification of Gaussian process Z(t)

The above mentioned universal probabilistic fatigue crack growth model employs the zero-mean Gauss process Z(t), which should be verified based on the experimental data.

Equation (2) can be written as

$$\log(da/dN) = Z_T^{\Delta t}(t) + \log L(a) \quad (19)$$

Rearranging Equation (19) yields

$$Z_T^{\Delta t}(t) = \log X_T^{\Delta t}(t) = \log(da/dN) - \log L(a) \quad (20)$$

where the crack growth rate $(da/dN)_i$ (i=1,2,…n. n is the total number of the data points) was calculated by the seven-point incremental polynomial method. Thus the data set $Z(t)_i$ can be obtained. The goodness-of-fit for $Z(t)$ was confirmed with a Kolmogorov-Smirnov (K-S) test statistic, as shown in Table1.

Table1 One-Sample K-S test for Z(t)

		Zt
N		1608
Normal Parameters	Mean	.0000
	Std. Deviation	.11447
Most Extreme Differences	Absolute	.032
	Positive	.032
	Negative	-.020
Kolmogorov-Smirnov Z		1.274
Asymp. Sig. (2-tailed)		.078

From Table1, it is known that μ_Z=0, σ_Z=0.11447, the K-S value is 1.274. Asymptotic significance of 2-tailed 0.078 is greater than the significance level $\alpha = 0.05$, which demonstrates the zero-mean normal distribution for $Z(t)$ is acceptable at least a 5% level of significance.

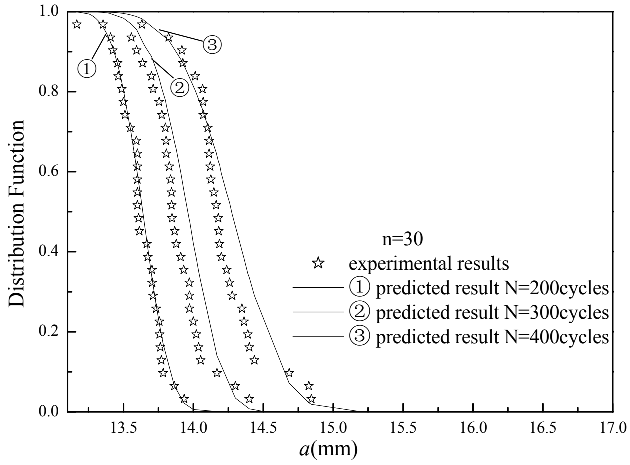

Fig.5 Crack size distribution at a given load cycle

2.4 Stochastic FCCG distribution

With the procedure described above, theoretical prediction for the distribution of service time to reach a given crack size, $F_{T(a)}(t)$, and the distribution of crack size at a specified load cycle, $F_{a(t)}(x)$, using the Weibull distribution are plotted in Fig.4 and Fig.5, respectively. Three different crack size (i.e. a=13.5mm, 14mm and 15mm) were considered for $F_{T(a)}(t)$, and three different service times (i.e. t=200, 300 and 400cycles) were considered for $F_{a(t)}(x)$.

It is demonstrated that the correlations between the Weibull approximation and the experimental test data are very satisfactory. As a conclusion, the Weibull approximation model can be used to predict the FCCG under constant amplitude cyclic loadings.

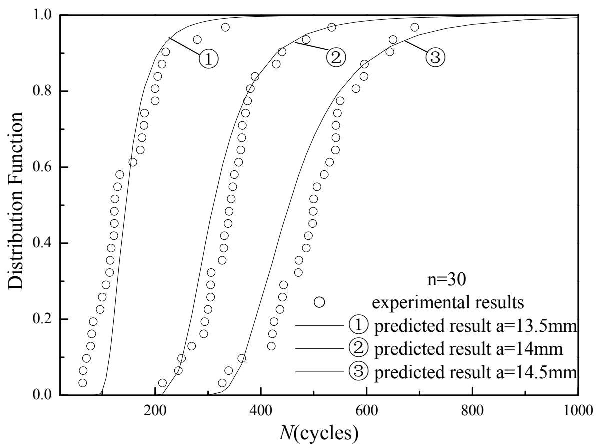

Fig.4 Number of load cycles distribution to reach the specified crack sizes

3 Probabilistic Analysis on Crack Growth Life of Turbine Disk

3.1 Crack growth life prediction of the disk

A high-pressure (HP) turbine disk of a certain kind of engine made of GH4133B, installed in a certain type of aero-engine was chosen in this work. The two-dimensional FE analysis with plane strain conditions was conducted using the commercial code ANSYS. The FE model is shown in Fig.6. The J-integral technique was employed to obtain the stress intensity factor (SIF) range, where the initial crack size a0 is set as 0.8mm in this study due to the capacity of the flaw detection device.

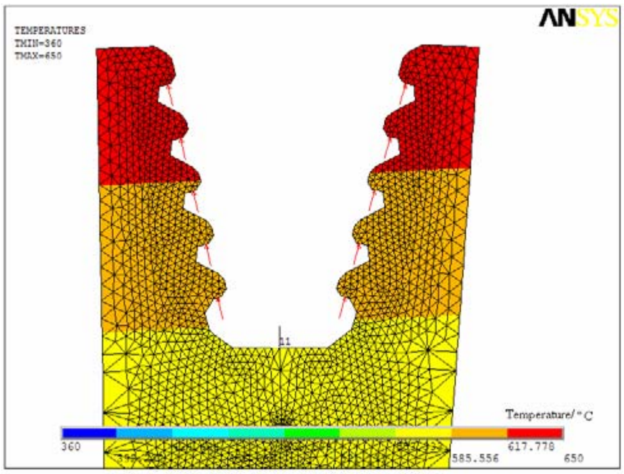

Fig.6 FE mesh and boundary constraint conditions

With varying the craze size, the corresponding SIF range can be obtained by the use of J-integral method and the SIF range is fitted by a least square method, as plotted in

Fig.7. Then the critical crack size a_{cr} of 8.4mm can be obtained based on the fracture toughness of GH4133B.

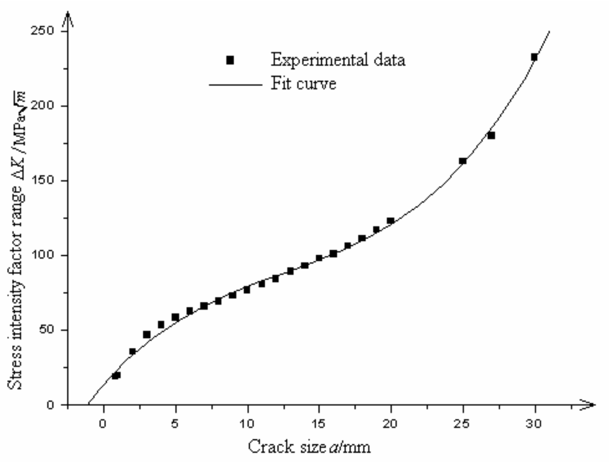

Fig.7 The curve of $\Delta K - a$

Fight time of the turbine disk is predicted by Larson model obtained through experimental data, as shown in Fig.8. The crack growth life for the turbine disk is 10181 cycles (7635hours), in which a_0=0.8mm, a_{cr}=8.4mm.

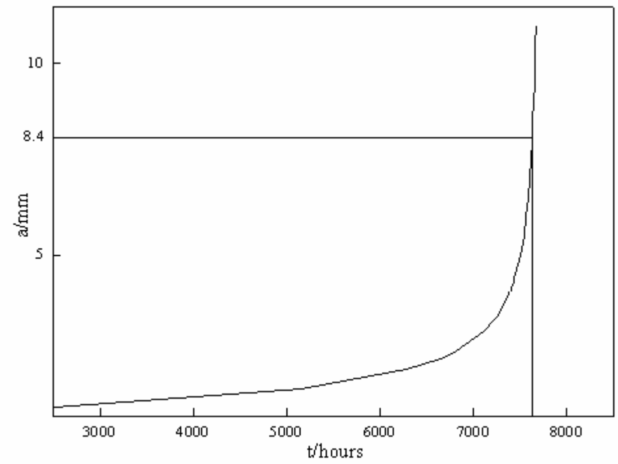

Fig.8 The relationship between flight time to crack size

3.2 Probabilistic analysis on crack growth life of the turbine disk

The uncertainties related to the operating environment as well as in the structural properties will result in the scatter of crack growth life. We aim to perform the probabilistic analysis on the crack growth life of the turbine disk mainly considering material properties through experimental data and operational load as random variables in this study. The uncertainty in FCCG of the turbine disk can be modeled as follows:

$$da/dt = X_T^{\Delta t} f(\Delta K) = X_T^{\Delta t} f(\omega, \Delta T, a) \qquad (21)$$

where $X_T^{\Delta t}$ reflects the uncertainty of material at fatigue-creep failure, ω is rotational speed, ΔT is the temperature difference of the disk-rim to disk-center, a is the crack size. Then crack growth life is made integrating equation (21) over the crack length

$$N = \int_{a_0}^{a_{cr}} \frac{1}{X_T^{\Delta t} \cdot f(\omega, \Delta T, a)} da \qquad (22)$$

The limit state function of the turbine disk under LCF-Creep can be defined as:

$$g(\boldsymbol{Y}) = N - N_0 = \int_{a_0}^{a_{cr}} \frac{1}{X_T^{\Delta t} \cdot f(\omega, \Delta T, a)} da - N_0 = F\left(X_T^{\Delta t}, \omega, \Delta T, N_0\right) \qquad (23)$$

where, $\boldsymbol{Y}$ is a vector of random variables, N_0 is specified crack growth life of the turbine disk. The expression that $g(\boldsymbol{Y})$ is less than zero denotes the failure state, and the probability of the turbine disk at failure is:

$$p_f = P\left(F\left(X_T^{\Delta t}, \omega, \Delta T, N_0\right) < 0\right) \qquad (24)$$

Structural probability of failure or reliability can be derived by using analytical or simulation methods to solve equation (24).The basic Monte Carlo method becomes prohibitively expensive and time-consuming to achieve high accuracy in estimating a low failure probability, especially for complicated systems that may need a large amount of computational effort for each deterministic analysis. Thus, efficient methods CVT (centroidal Voronoi tessellation) sampling (See [20]) is used to improve the efficiency, where the sample set is shown in Table2. Then we applied the combination technique of response surface (RS) analysis and Monte Carlo simulation to perform stochastic analysis on the crack growth life, with central composite design (CCD) sampling used in RS analysis method and CVT sampling method chosen in Monte Carlo simulation. The distribution function of crack growth life is plotted in Fig.9. Hence, the failure probability of crack growth rate at a specified value can be quantified, as shown in Table3.

Table2 CVT sample set and response of crack growth life of turbine disk

	ω (rad/s)	ΔT(˚C)	X	N(cycles)
1	1103.45	281.36	1.6791086898	6088.6
2	1283.16	301.89	1.6640871789	6065.8
3	1194.16	282.44	0.6451916867	15772
4	1317.53	281.54	0.5245821803	19175
5	1289.59	299.01	0.5351864867	18852
6	1073.01	281.28	0.5446554268	18793
7	1099.46	298.82	1.6366715455	6245.9
8	1285.37	278.01	1.6564734850	60965
9	1105.95	299.06	0.5502448813	18573
10	1272.59	290.23	1.6467298803	6138.9
11	1070.34	279.67	0.5037736321	18316
12	1317.16	279.76	1.1740189586	11537
13	1107.63	302.37	1.6833562284	7822.7
14	1223.89	292.92	1.0922995464	11930
15	1287.51	302.22	1.6468711334	6161.3

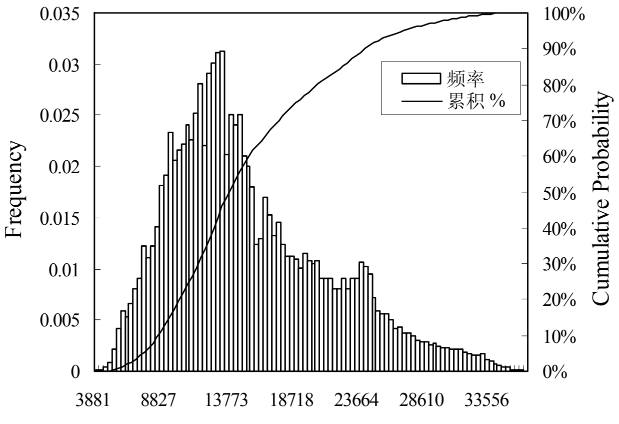

Fig.9 The cumulative density function of crack growth life for turbine disk

Table3 Probabilistic results of crack growth life for turbine disk

	50% reliability	99% reliability	99.87% reliability	99.9% reliability
life/cycles	14000.1	5639.4	4765.7	4654.6

4 Conclusion

The FCCG experiments on GH4133B were carried out on 30 CT specimens at 600˚C, where the crack size was measured with an optical crack tracking device. The prediction for the median FCCG curve using Larson model is very good in this study, and various stochastic models for FCCG under constant amplitude loadings have been investigated. These models are based on the general assumption that the FCCG rate is a lognormal random process with a median value of unity.

The second moment approximation such as Weibull approximation has been used to demonstrate the validity of stochastic crack growth model of GH4133B. Theoretical predictions for the distribution of service time to reach a given crack size and the distribution of crack size at a specified service time correlate well using the Weibull approximation. The stochastic model for FCCG presented in this paper is based on the median crack growth curve (see Equation (1)). Therefore, it is important to accurately establish the crack growth model under the experimental conditions.

In order to predict the turbine disk remaining lifetime, the determination of the fracture parameters such as SIF and J-integral value is a crucial point. Thanks to the development of FE techniques, it is now accessible by a fracture analysis based on FEM. Furthermore, a probabilistic fracture mechanics (PFM) model for the stochastic crack growth life of the turbine disk is established. Then the reliability analysis on the crack growth rate is carried out with a combined approach of response surface and Monte Carlo simulation method. As a result, the distribution function of turbine disk crack growth life under LCF-creep is determined considering the uncertainties including material properties through experimental data and operational loads including rotational speed and temperature. Thus the damage tolerance risk of the turbine disk under LCF-creep can be quantified.

The results are of importance for the assessment of life extension of disks in the respect that an inspection schedule can be derived from the calculated failure probabilities. The proposed method will give results on the impact of a proposed schedule on the reliability. However, it is noted that the initial crack size is treated as a deterministic value in this study. Although the careful pre-cracking experiment is conducted, there is statistical scatter of the initial crack size. Therefore, the random distribution of the initial crack size should be studied.

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