A MULTI-POINT INVERSE DESIGN APPROACH OF NATURAL LAMINAR FLOW AIRFOILS

Deng L. *, Qiao Z. D. *
National Key Laboratory of Science and Technology on Aerodynamic Design and Research,
Northwestern Polytechnical University, China
Keywords: *multi-point; inverse; design; natural laminar; airfoil*

Abstract

A multi-point inverse design method of natural laminar flow (NLF) airfoils based on optimization is developed. XFOIL code is used to calculate pressure distributions and transition locations to reduce computational cost. The target pressure distributions need special care to design and N-factor design method is used to design target pressure distribution before recovery region with a substantial amount of natural laminar flow while maintaining aerodynamic constrains. The pressure in recovery region is designed according to the Stratford separation criteria. Optimization method based on Response Surface methodology (RSM) is used to calculate the target airfoil. The set of design points is selected to satisfy D-optimality and the reduced quadratic polynomial RS models without the second-order-cross items are employed to reduce the computational cost. The design cases indicate that RSM can be successfully applied to multi-objective inverse design and the design approach in this study is applicable to a wide range of airfoils.

NOMENCLATURE

A	[-]	Relax factor for *N*-factor design method
C_l	[-]	Lift coefficient
C_d	[-]	Drag coefficient
C_m	[-]	Pitching moment coefficient
C_p	[-]	Pressure coefficient
c_i, c_{ii}	[-]	Regression coefficient in RS model
Ma	[-]	Free Mach number
Re	[-]	Free stream Reynolds number based on chord
n_v	[-]	Number of variables
n_s	[-]	Number of experiments
n_{rc}	[-]	Number of regression coefficients
X	[m]	Coordinate on airfoil
X_{tru}	[m]	Transition location on upper surface
X_{trl}	[m]	Transition location on lower surface
I	[-]	Objective function
w	[-]	Weight factor in inverse design method
b	[-]	Base function
Special characters		
δ	[-]	=-1, 0 1
γ	[-]	Weight coefficient
Δ	[-]	Difference between the variables
Subscripts		
a		Analysis value, in $C_{p,a,j}$, $N_{a,j}$
T		Target value, in $C_{p,T,j}$, $N_{T,j}$
$crit$		Critical value for transition, in N_{crit}
j		Airfoil station
$1,2$		The number of objective in Optimization
d		Design value of airfoil
cp		Control point
k		Station of base function
Supscripts		
(p)		Predicted value

1 INTRODUCTION

Since the advent of powered flight, drag reduction has been a major issue in airplane design. For example, reductions in drag allow airplanes to operate more efficiently by less fuel and bigger payloads and longer ranges.

There have been many concepts to reduce drag. Since 1930's, there has been great interest in designing airfoils and wings for natural laminar flow to reduce viscous drag. Many NLF airfoils have been developed in the past decades. For example, in the early 1940's, NACA has designed several advanced NLF airfoils, such as NLF(1)-0215F[1] and NLF(1)-0414F[2,3], and HSNLF(1)-0213[3,4,5] for high speed flows.

The basic requirement in NLF airfoil design is to find an airfoil at design conditions (for example, at special C_l, Ma and Re) with reasonable amount of laminar flow, which means that the transition location requirement on both sides should to be met. Because of that, the key issue in designing NLF airfoils is to calculate reasonably accurate transition locations. In industrial design applications, the most widely used method for streamwise transition prediction is the e^N method. This is a method based on linear stability theory and experimental data. In the 1950's, Van Ingen[6] and Smith and Gamberoni[7] independently used the results from the linear stability theory and compared them with experimental data of viscous boundary layers. They found that transition from laminar to turbulent frequently happened when the amplification of disturbances is calculated from stability theory reached about 8100, which is e^N when N equals to 9. This is the well known criterion for Tollmien-Schlichting instability. It has been proven to provide reasonably accurate transition locations on airfoils[8].

Computation fluid dynamics has brought about a dramatic change in the aerodynamic design process. One of the early uses was based on the cut-and-try approach where a designer iteratively modifies and verifies a design [9]. The Numerical optimization technique is based on a rational, directed, design procedure. It can be used to generate an optimum geometry that has desirable characteristics while satisfying some design constraints [10-12]. This method has many attractive aspects: a combination of design parameters can be improved; multiple constraints can be imposed and multi-point design can be performed. One drawback is that numerical optimization would be expensive. A more desired design approach is to solve an inverse problem due to its well-established procedure and relatively small computational time[13-15]. Here a geometry is generated based on a desired pressure distribution that is the opposite of the traditional analysis mode[16]. However it is difficult to define the pressure distribution and there is no guarantee that there exits a geometry that will produce the specified pressure distributions.

The objective of the present study is to build an inverse design tool for NLF airfoils that is capable of multi-objective design. N-factor method is used to design the target pressure distributions by an N-factor distribution of an initial airfoil and a target N-factor distribution that forces the flow undergoing transition at desired location while maintaining aerodynamic constraints. Optimization technique based on RSM is used in inverse design process with an objective function of difference between the analysis and target pressure distributions. One way of dealing with the multiple responses is to use the weighted sum with some drawbacks, which have been discussed by Das[17]. Optimization of multiple responses can be performed by building a composite response from individual responses. This composite response function is called as the desirability function[18]. In this study, the set of candidate design points is selected to satisfy D-optimality and quadratic polynomials without the second-order-cross items are employed to construct RS model to reduce computational cost.

The airfoil design method iteratively designs a new airfoil until the constraints are satisfied or the prescribed number of iterations is reached.

2. FLOW ANALYSIS

In present inverse design study, the computational cost is mainly used to construct RS model. In previous study, the RANS (Reynolds-average Navier-Stokes) equations coupled with a boundary layer solver with a transition prediction based on e^N method are used to flow analysis, but in order to reduce computational cost, XFOIL code[19] is used in present study. XFOIL has been proven to be well suited to rapid analysis of subcritical airfoils even in the presence significant transitional separation buddle. It was successfully used in inverse design [15] and well suited to making comparison of the trends in performance characteristics of airfoils with systematically varying design specifications.

3. TARGET PRESSURE DISTRIBUTION

In inverse airfoil design, it is difficult to define one or several pressure distributions that will produce the desired aerodynamic performance and meet all of the aerodynamic constraints, especially in multi-objective design. In this study, N-factor design method is employed to design the target pressure on upper surface before recovery region, however, when nature laminar flow is desired on both surfaces, it becomes more difficult to maintain some of the geometric constraints while trying to obtain laminar flow on both surfaces. As a result, a trade-off between geometric constrains and the amount of nature laminar flow is often necessary. Target pressure distribution on lower surface is designed by taking a weighted average of the linearly scaled upper target pressure distribution and the lower surface analysis pressure distribution of the initial airfoil.

The target pressure distribution is designed iteratively to meet the target N-factor distribution and aerodynamic constraints. In order to produce a reasonable geometry from the multiple target pressure distributions, it is necessary to modify the distributions, such as pressure peak or trailing edge pressure or even the total shapes.

3.1 Target *N*-factor Distribution

The target *N*-factor distribution is produced by interpolating 4 control points with a polynomial fit. In order to delay the transition, *N*-factor distribution should has a buffer zone above which the target *N*-factors are not allowed to grow rapidly so that the boundary layer will remain laminar prior to the desired transition location. After this zone, the *N*-factor grows rapidly to N_{crit} , that is the critical *N* at desired transition location.

The first control point ($X_{cp,1}$,$N_{cp,1}$) is located at the beginning of the buffer zone. Ahead of this point, the analysis *N*-factors are kept as the target *N*-factors. The second control point($X_{cp,2}$,$N_{cp,2}$) is located at the beginning of the steep *N*-factor distribution zone. The third control point($X_{cp,3}$,$N_{cp,3}$) is placed at the desired transition location, while the forth control point($X_{cp,4}$,$N_{cp,4}$) represents the end of the steep *N*-factor gradient. A detailed introduction can be found in [14].

3.2 Extrapolation of Analysis *N*-factor

In the stability analysis code, N-factors can be calculated when the laminar boundary layer is attached. If boundary layer separates before the 4th control point, it is necessary to extend the analysis *N*-factors from the laminar separation point to the 4th control point. It is determined that the change in *N*-factor is proportional to the change in pressure over the surface of the airfoil, thus, the analysis *N*-factors are artificially extended to the 4th control point using linear extrapolation based on the pressures:

$$N_j = N_{j-1} + \frac{N_{j-1} - N_{j-2}}{C_{p,a,j-1} - C_{p,a,j-2}} \left(C_{p,a,j} - C_{p,a,j-1} \right) \quad (1)$$

3.3 *N*-factor Design method

When the target *N*-factor and the extrapolated analysis *N*-factor have been calculated, the target pressure distribution can be calculated by *N*-factor method before recovery region. In this study, the change in pressure coefficient required at airfoil station *j* is defined by the change between the analysis and target *N*-factors at *j*:

$$\Delta C_{p,j} = A \Delta N_j \quad (2)$$

$$\Delta C_{p,j} = C_{p,T,j} - C_{p,a,j} \quad (3)$$

$$\Delta N_j = N_{T,j} - N_{a,j} \quad (4)$$

Where, A is 0.012 typically. Once $\Delta c_{p,j}$ is calculated, this change is applied to all of the station downstream of *j* as well, and at the same time, the analysis *N*-factor downstream is modified by ΔN_j to maintain a smooth and continuous target pressure distribution.

3.4 Pressure Distribution in Recovery region

In ref.[14], the pressure in recovery region is designed by taking a weighed average of the pressure distribution of the initial airfoil and the linearly scaled target pressure distribution in recovery region. When the laminar separation is considered in recovery region, linear recovery or Stratford separation criteria may be used to prevent the laminar separating [20]. This rule states that separation will occur when:

$$C_p' \sqrt{x' \frac{dC_p'}{dx}} = \left(\frac{\mathrm{Re}}{10^6} \right)^{0.1} s \quad (5)$$

Where, the constant *s* is 0.35 when $d^2C_p/dx^2 < 0$ (concave recovery) and 0.39 when $d^2C_p/dx^2 > 0$ (convex recovery). The Reynolds number in the Stratford formula is based on the local effective length of the boundary layer, *x*', and the maximum velocity, U_m. C'_p is the canonical pressure coefficient. The detailed introduction can be found in Ref.[21].

4. RESPONSE SURFACE METHOD (RSM)

RSM is a set of techniques in which approximate relations between the input variables and the responses of a system are found. In using RSM, the designer performed a limited number of computational analyses using experimental design theory to prescribe values for the independent variables. Number of the experiments is governed by the order of the surfaces constructed and the type of the experiments design. There are usually three design point selection techniques, which are full factorial design, central composite design (CCD) and D- optimality. In this study, D- optimality is selected to reduce computational cost.

The model used to describe the relationship between the response and predictor variables is known as the response model and may be written in general as follows,

$$y = F(x_1, x_2, \ldots x_{n_v}) + \varepsilon \qquad (6)$$

Where, ε represents the total error, and is considered as a statistical error. The function F is normally chosen to be a low order polynomial, typically the 2nd order polynomial functions. In this study, quadratic polynomial RS models without the second-order-cross items (reduced model) are formed. It is written as follow,

$$y^{(p)} = c_0 + \sum_{i=1}^{n_v} c_i x_i + \sum_{i=1}^{n_v} c_{ii} x_i^2 + \varepsilon$$
$$p = 1, \ldots, n_s \qquad (7)$$

In Eq.7, the number of regression coefficients n_{rc} is $2(n_v+1)$. n_s, the number of experiments is selected as 1.5~3 times of the regression coefficients[22,23]. For an optimization problem with 26 design variables, n_s is about 80~150, that is much less than that of the full model. There are no redundant terms included in reduced model which decreases the error of the model and increases the prediction capabilities. No matter for reduced or full 2nd order polynomial RS, the regression coefficients can be solved using least square method. Previous study shows that, the results of reduced model with more design variables are better than the full model[24].

5. DESIGN VARIABLES

The design process begins with an initial airfoil. The new airfoil geometry is modified by adding smooth perturbations defined as a linear combination of the base functions, b_k,

$$\Delta y(x) = \sum_{k=1}^{n_v} \delta_k \gamma_k b_k(x) \qquad (8)$$

Where, δ_k is (-1, 0, 1); γ_k is weight coefficient; b_k is base function and $\gamma_k b_k$ is design variable at x_k; x_k is the location of the maximum height of the base function. The base functions are defined as follows[25],

$$\begin{aligned}
&b_k(x) = \sin(\pi(1-x)^{e(k)}) && x < 0.2 \\
&b_k(x) = \sin^3(\pi x^{e(k)}) && 0.2 \le x < 0.9 \\
&b_k(x) = \sin(\pi x^{e(k)}) && 0.9 \le x \qquad (9)\\
&e(k) = \ln(0.5) \big/ \ln(1-x_k) && x < 0.2 \\
&e(k) = \ln(0.5) \big/ \ln(x_k) && 0.2 \le x
\end{aligned}$$

In this study, 26 total base functions were used, 13 on both the upper and lower sides, with x_k = 0.03, 0.06, 0.13, 0.20, 0.27, 0.40, 0.50, 0.60, 0.73, 0.80, 0.87, 0.94, 0.97, which are shown in Fig. 1.

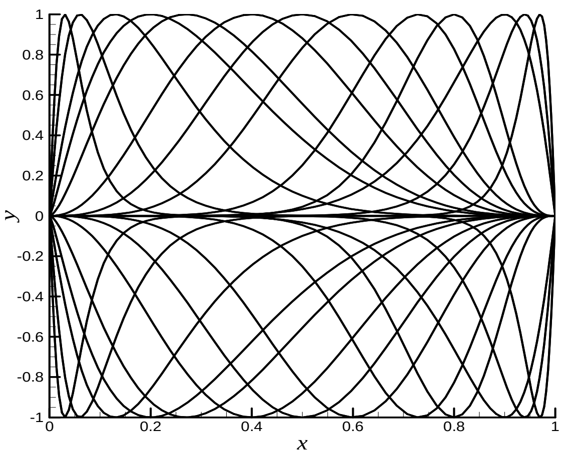

Figure 1 Base Functions Used to Modify the Geometry

6. INVERSE DESIGN METHOD

When the target pressure distributions have been designed, RSM is employed to design an airfoil that will produce the target pressure distributions. The objective is to minimize the pressure distribution differences with an initial airfoil-NACA0012 in present study:

$$I = \int_{BW} (C_p - C_{p,T})^2 ds \qquad (11)$$

Where, *BW* is wall.

In present 2-point inverse design, a composite objective function is built using the weighted sum:

$$I = w_1 \times I_1 + (1 - w_1) \times I_2 \qquad (11)$$

Where, I_i and w_i are the individual objective function and importance factor for the i^{th} response respectively.

The inverse design procedure is shown in Fig.1 and it can be divided into 2 parts: the design of the target pressure distributions and the design of new airfoil by RSM method.

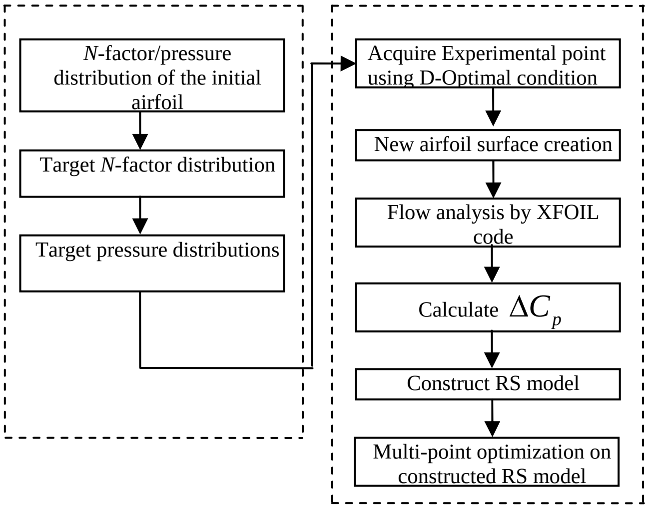

Figure 2 the Design Procedure

7. RESULTS AND DISCUSSION

7.1 Airfoil for Wind Turbine Application

WA21ak5 airfoil is designed in Northwestern Polytechnical University for large wind turbine applications. In order to check the multi-objective optimization approach, the pressure distributions at design conditions of WA21ak5 are employed as the target pressure distributions. The design conditions are shown in Table 1:

Table 1 Design conditions for WA21ak5

	Condition I	Condition II
$C_{l,d}$	1.2	0.6
Ma	0.30	0.15
Re	6.0×10^6	1.5×10^6

Condition I is the primary design condition and the weight factor is 0.6. At condition II, there is about 50% NLF on both surfaces. NACA0012 airfoil is selected as the initial airfoil. Fig.3 is the convergence history of objective function and it is clear that objective function converge to 0 in about 100 iterations.

Fig.4 and 5 show the comparisons of the pressure distributions of the WA21ak5 airfoil and the new designed airfoil at two design conditions. We can see that the distributions agree very well except the trailing edge. This is because that the geometry change in trailing edge is not sensitive to the design variables. Fig. 6 is the comparison of the airfoils. The results show that the multi-objective optimization approach can design the target airfoil from the multiple target pressure distributions.

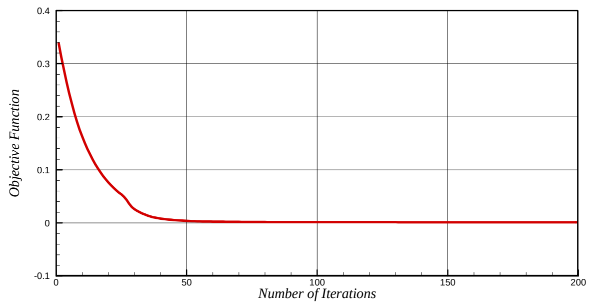

Figure 3 Convergence History of Objective Function

7.2 Airfoil for a General Aviation Application

NLF(1)-0414F airfoil is developed in NASA for general aviation application. The aerodynamic design goal includes 70% chord NLF on both surfaces at a Ma of 0.4, Re of 1×10^7 and $C_{l,d}$ of 0.4 with 14% chord thickness.

In this study, a NLF airfoil with 14% chord thickness is designed with the primary design condition kept as the aerodynamic design goals as NLF(1)-0414F. Except the primary design

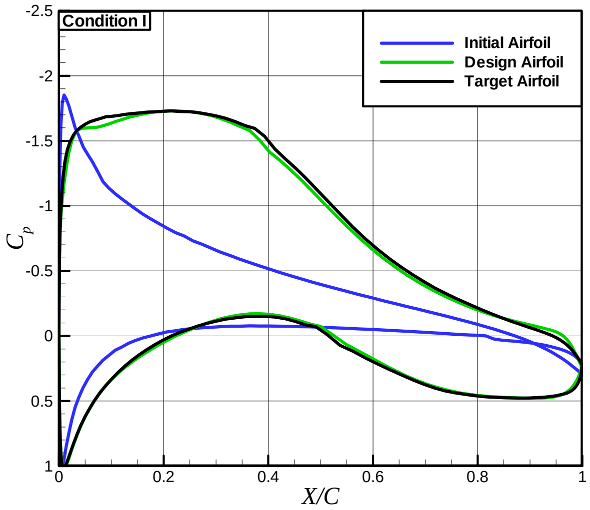

Figure 4 Comparison of Surface Pressure at Condition I

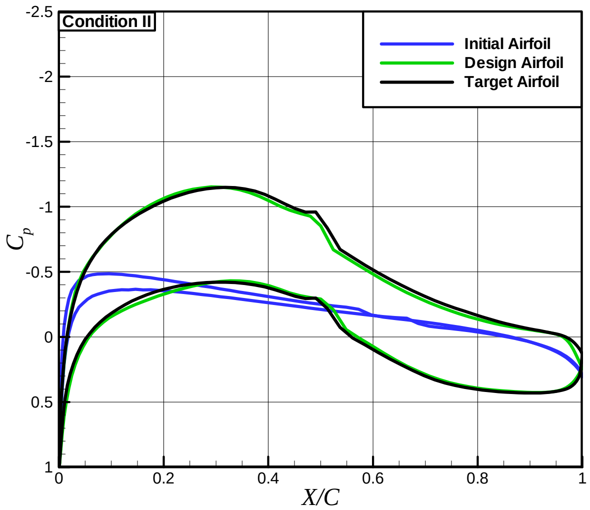

Figure 5 Comparison of Surface Pressure at Condition II

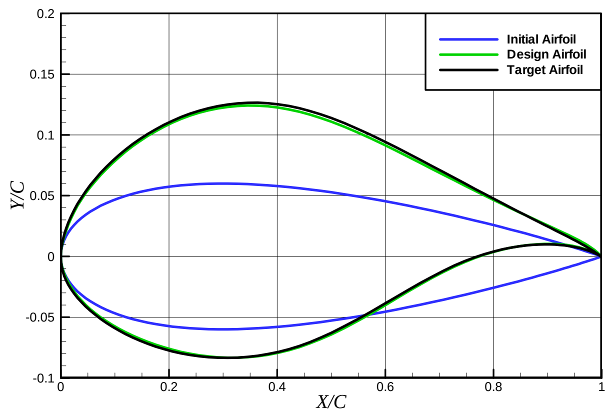

Figure 6 Comparison of Airfoil Geometry

condition, condition II is selected and shown in Table 2.

At condition I, there is 70% chord NLF on both surfaces； *N*-factor method is used to design the target pressure distributions on both surfaces and pressure distribution in recovery region is designed according to Stratford

Table 2 Design conditions for 14% thickness airfoil

	Condition I	Condition II
$C_{l,d}$	0.4	0.60
Ma	0.4	0.15
Re	1.0×10^7	6.0×10^6

separation criteria; while at condition II, there is 30% chord NLF on upper surface and *N*-factor method is used to design the target pressure distribution on upper surface while that of lower surface is kept as NLF(1)-0414F.

The initial pressure/N-factor distribution of NACA64$_1$-212 airfoil is used as initial pressure/ N-factor distribution to design the target pressure distributions. NACA0012 airfoil is selected as initial airfoil for optimization design. Converged results can be achieved after about 50 iterations. The thickness of the new designed airfoil is 13.7% chord, which is a little thinner than design goals.

Fig.7 shows the *N*-factor distributions of NACA64$_1$-212 and the new designed airfoil at condition I. That indicates that there is approximately 72% chord NLF on upper surface with a transition *N*-factor of 9. The target pressure distribution on lower surface is designed by linearly scaled of the upper surface target pressure distribution and 72% chord NLF is achieved on lower surface. Fig.8 and 9 are comparisons of the airfoils at condition I and II respectively. It indicates that the adverse pressure gradient in recovery region is less than NLF(1)-0414F to prevent the separation locations changing too much when separation happens.

Fig.10 shows the comparison of geometries. It indicates that, in general, the lower surface geometries of the new airfoil and NLF(1)-0414F appear to be very similar, but the upper surface is thinner than NLF(1)-0414F. Table 3 shows the aerodynamic performances between the designed airfoil and NLF(1)-0414F.

Table 3 Comparison of Aerodynamic Performances

	NLF(1)-0414F	New Airfoil
Condition I - C_d	0.00317	0.00317
- C_m	-0.0796	-0.0856
-Xtru	0.74	0.72
-Xtrl	0.72	0.72
Condition II: - C_d	0.00606	0.00516
- C_m	-0.0712	-0.0751
-Xtru	0.31	0.42
-Xtrl	0.72	0.74

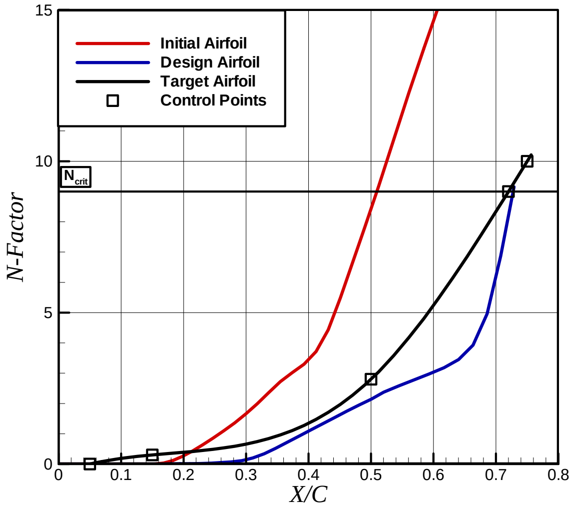

Figure 7 Comparisons of the *N*-Factors on Upper Surface at Condition I

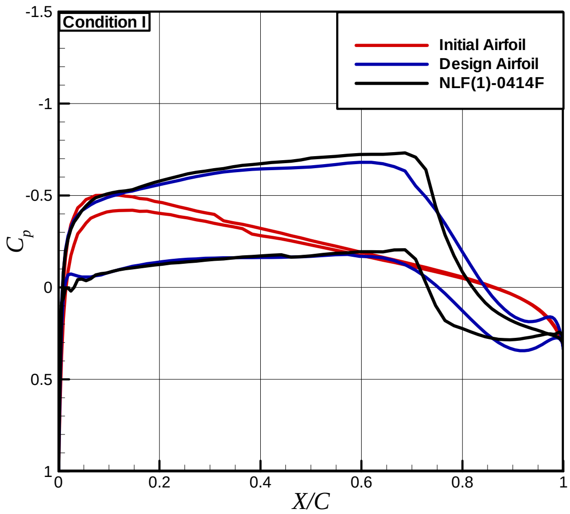

Figure 8 Comparison Surface Pressures at Condition I

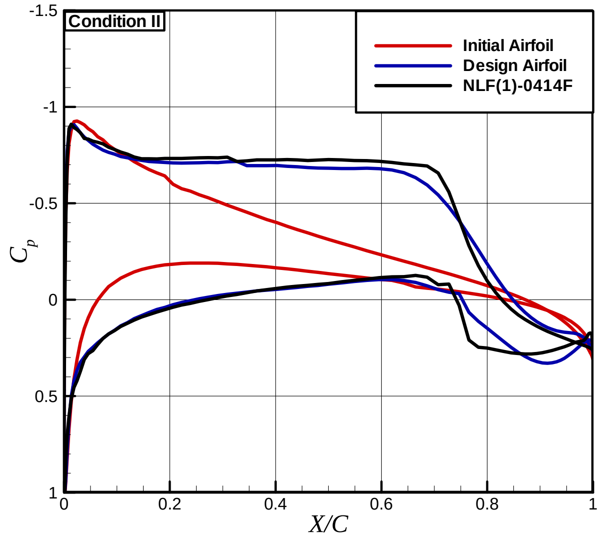

Figure 9 Comparison Surface Pressures at Condition II

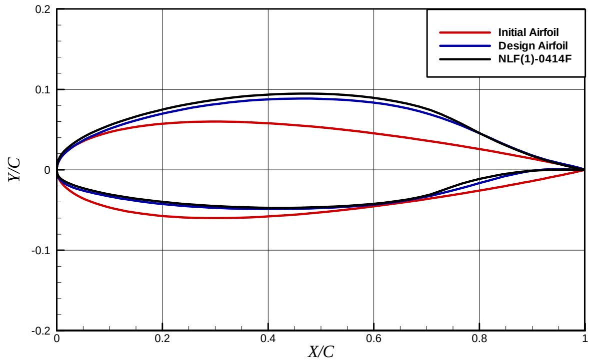

Figure 10 Comparisons of Airfoil Geometries

8 CONCLUSIONS

A multi-objective inverse approach based on RSM method for NLF airfoil design is performed. From the results, it shows that XFOIL can be used in airfoil design process and has been proven to be able to provide reasonably accurate aerodynamic performances and transition locations. The target pressure distributions designed by *N*-factor method can meet the aerodynamic constraints but the geometry constraints are difficult to meet in the target pressure distributions design process. The set of design points is selected to satisfy D-optimality, and in this study, 26 variables are employed. The reduced quadratic polynomial RS models without the second-order-cross items are constructed as RS model to reduce computational cost. The results show that the optimization approach based on RSM can be used in inverse design process. The pressure distributions of new airfoil agree well with the target pressure distributions except the trailing edge because the change in trailing edge in optimization process is not so sensitive to variables.

Although two design cases are performed in present study, the design approach and the design philosophies are applicable to a wide range of airfoils and other applications.

References

[1]Somers, Dan M., Subsonic Natural-Laminar-Flow Airfoils. *Natural Laminar Flow and laminar Flow Control*, R. W. Barnwell and M, Y. Hussaini, eds., Springer-Verlag New York, Inc., 1992, pp. 143-176

[2]McGhee, Robert J., Viken, Jeffry K., Et Al., Experimental Results for a Flapped NLF Airfoil with High Lift/Drag Ratio. *NASA TM-85788*, May 1984

[3]Viken, J. K., Viken, S. A., Prenninger, W., and Morgan, H. L., Campbell, R. L., Design of the Low Speed NLF(1)-0414F and the High Speed HSNLF(1)-0213 Airfoils with High-Lift Systems, *NASA CP-2487*, pp.637-671, May 1987

[4]Waggoner, Edgar G., Campbell, Richard L., Phillips, Pamela S., Viken, J.K., Computational Design of Natural Laminar Flow Wings for Transonic Transport Application, *NASA CP-2398*, Vol. II, April 1985 .

[5]Sewall, W. G., McGhee, R.J., Hahne, D.E., Jordan, Frank L. Jr., Wind Tunnel results of the High-Speed NLF(1)-0213 Airfoil, *NASA CP-2487*, pp. 697-726, Mar 1987

[6]Van, Ingen J.L., A Suggested Semi-Empirical method for the Calculation of the Boundary Layer transition Region, *Inst. of Tech.*, Delft, 1956

[7]Smith, A.M.O., and Gamberoni, N., Transition, Pressure Gradient, and Stability Theory, *Douglas Aircraft, ES-26388*, 1956

[8]Krumbein, A., and Stock, H.W., Laminar-Turbulent Transition Modeling in Navier-Stokes Solver Using Engineering Methods, *ECCOMAS 2000, September, Barcelona, ISBN: 84-89925-70-4, Deposito Legal: B-37139-2000*

[9]Rubbert, P.E., Design Applications of CFD at Boeing, *NASA Ames Research Center, Moffett Field*, CA 1987

[10]Harold, Y., Multi-Point Design and Optimization of a Natural Laminar Flow Airfoil for a Mission Adaptive Compliant Wing, AIAA Paper 2008-293

[11]Jen-Der, L., and Antony, J., Natural-Laminar-Flow Airfoil and Wing Design by Adjoint Method and Automatic Transition Prediction. *AIAA2009-897*, 2009

[12]Driver, J., Zingg, D. W., Optimized Natural-Laminar Flow Airfoil. AIAA Paper 2006-247, 2006

[13]Srilatha, K.R., Dwarakanath, G.S., Design of a Natural Laminar Flow Airfoil for a Light Aircraft. *Journal of Aircraft*, Vol.27, No.11, pp. 966-968, 1990

[14]Green, B.E., an Approach to the Constrained Design of Natural Laminar Flow Airfoils, *NASA CR-201686*, Feb. 1997

[15]Ashok, G., and Michael, S.S., Low Speed NLF Airfoils: Case Study in Inverse Airfoil Design. *Journal of Aircraft*, Vol. 38, No. 2, pp. 57-63, 2001

[16]Giles, M.B., and Drela, M., Two-dimensional Transonic Aerodynamic Design Method. *AIAA Journal*, Vol. 25, No. 9, pp.1199-1206, 1987

[17]Das, I., and Dennis, J.E., A Closer Look at Drawbacks of Minimizing Weighted Sums of Objective for Pareto Set Generation in Multicriteria optimization Problems, *Structural Optimization*, Vol. 14, pp. 63-69, 1997

[18]Myers, R.H., Montgomery, D.C., *Response surface methodology: process and product optimization using designed experiments*. New York: Wiley. 2002

[19]Drela, M., XFOIL: An Analysis and Design System for Low Reynolds Number Airfoils, *Low Reynolds Number Aerodynamics, edited by T.J. Mueller*, Vol. 54 of lecture Notes in Engineering, Springer-Verlag, New York, pp. 1-12, 1989

[20]Stratford, B.S., The Prediction of Separation of The Turbulent Boundary Layer, *Journal of Fluid Mechanics*, Vol. 5, pp. 1-16, 1959

[21]Stratford, B.S., Applied Aerodynamics: A Digital Textbook, *http://www.desktopaero.com/appliedaero/preface/welcome.html*, Version 5.0, January 2007

[22]Ahn, J., Kim, H.-J., Lee, D.-H., Rho, O.-H., Response Surface Method for Airfoil Design in Transonic Flow, *Journal of Aircraft*, Vol. 38, No. 2, March-April 2001

[23]Ahn, J., Yee, k., Lee, D.-H., Tow-Point Design Optimization of Transonic Airfoil Using Response Surface Methodology, AIAA Paper 99-0403, Jan. 1999.

[24]Deng, L., Qiao, Z. D., Xiong, J. T., Multi-Objective of Low-Speed Airfoils Using Optimization based on Response Surface Methodology[C]//*14th National Conference of CFD. Guiyang: China Aerodynamics R&D Center*, 2009:810-815

[25] Hager, J. O., Eyi, S., and Lee, K. D., Multi-point Design of Transonic Airfoils Using Optimization. AIAA Paper 92-4225, 1992

Copyright Statement