COMPUTATIONAL TOOLS FOR AIRCRAFT SYSTEM ANALYSIS AND OPTIMIZATION

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Abstract

In this paper a model based QFD (MB-QFD) is introduced, and the different representations for design analysis of complex systems are discussed. Sensitivity analysis is a useful tool for trade studies and to achieve an overview of the importance of different aspects of the design. The introduction of an hierarchical aggregated design influence matrix greatly improves the ability to handle large systems. Also an influence matrix of uncertainties is used to trace the impact of uncertainties in the design and to identify hot spots where more effort is needed. Through the QFD "House of quality" it is possible to formulate design objective function in a formal way. Furthermore, functional correlation can be used to analyse coherent and conflicting requirements, and by studying the influence of design parameters on the design objective the most influential parameters can be selected for optimization. These tools are demonstrated on an aircraft design example..

1. Introduction

Design optimization for aircraft design has become a very active are of research. Early papers on the subject are exemplified by Wakayama S. Kroo I, 1995, where a wing planform for a subsonic aircraft was optimized. In Rohl, Peter J., Mavris, Dimitri N., Schrage, Daniel P, 1995 a supersonic aircraft was optimized both structurally and aerodynamically. In P Krus, A Jansson, P Berry, E Hansson, K Ovrebo 1996 the planform of a subsonic aircraft was optimized, and it was also demonstrated how simulation could be integrated to evaluate certain characteristics. In Krus In P Krus, J-O Palmberg, F Löhr, G Backlund 1995 simulation based optimisation was used for component selection and sizing of actuation systems. Although optimization as such is very useful, it is even more useful when combined with design analysis. The objective of design analysis is to obtain information about the nature of the design solution, and how it can be changed in order to fulfil the requirements, and how requirements can be negotiated to best fulfil the stakeholders requirements. Here different matrix methods are useful, since they can be used to display the mapping of relations between system parameters and system characteristics.

Figure 1 shows the concept development and system level design. This paper focus on the parts indicated as computational design methods. These involve design optimization (quantitative refinement), analysis and evaluation and sensitivity analysis and trade-off analysis. The tools described here have been implemented in a spread sheet program, which is an ideal form for displaying matrices and tables. Furthermore, it can be connected to models that resides in different tools using web-service technology.

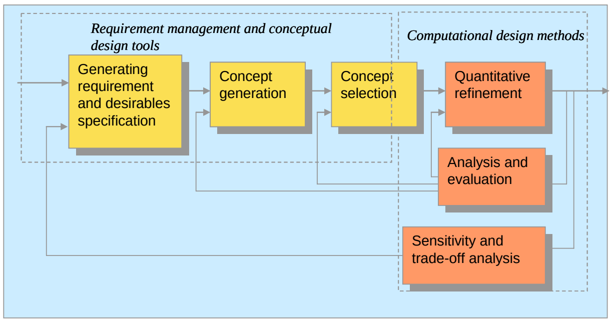

Figure 1. Conceptual design and system level design

1.1 Requirement Specification

Formal methods for establishing the requirements and the desirables of a design are needed in order to ensure the traceability between requirements, desirables and the design. This also involves initial analysis of the value of fulfilling certain customer requirements vs. the cost to do so. This also connects to the post-optimal analysis. Here the "house of quality", the central element of QFD, is useful

1.2 Concept Optimization and Selection

Based on computational models, optimisation of different configurations can be carried out. Optimisation has proven to be an extremely useful tool when connected to evaluation models. These can for instance be dynamic simulation models. The advantage with this approach is that it allows the designer to optimise complex non-linear systems in a convenient way, directing the designer's effort to the requirements and system objectives rather than to the actual computation of system parameters.

1.2 Computational Models

In order to evaluate different concepts, models at an adequate level of details have to be established. This is a very important task since the design becomes no more accurate than the underlying models. One of the most important shifts in paradigm occurring in engineering system design may well be the adoption of common system models as a foundation for system design. Efficient models for complete aircraft systems simulation can be established, where complete systems can be simulated more or less in real time. Furthermore, it is possible to connect different analysis tools such as aerodynamic codes, structural and simulation. This does, however, require proper management of model fidelity and accuracy through the design process.

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1.4 Sensitivity Analysis for Traceability in Design

Sensitivity analysis is very closely tied to the requirement specification and it involves the estimation of sensitivities between design parameters and the functional characteristics. This can also be generalised to estimate the influence of aggregates of parameters, such as whole subsystem on requirements. This can be presented as an aggregated design impact matrix. Sensitivities can also be generalised to involve higher order functions, such as quadratic, which means that approximate analytical expression for the relations between design parameters and functional characteristics. This is a valuable tool for studying parameter variations around a design point when the actual underlying models are computer intensive. It is this activity that really ties the top-level

requirements and desirables to the low-level detail design.
useful

1.5 The House of Quality

The house of quality is a matrix method for mapping customer needs to system requirements See Hauser, and Clausing [8]. It was originally proposed by Taguchi as part of total quality deployment TQM. It can of course be used to map any domains against each other but traditionally it is directed towards customer needs. In this paper the house of quality is used to provide trace-ability in a quantitative way between individual design decisions/design parameters and customer requirements.

SC_1	■	SCI_{12}	SCI_{13}	SCI_{14}	
SC_2		■	SCI_{23}	SCI_{24}	
SC_3			■	SCI_{34}	
SC_4				■	
	System characteristics				
Customer needs	SC_1	SC_2	SC_3	SC_4	Customer priorities
CN_1	kc_{11}	kc_{12}	kc_{13}	kc_{14}	CP_1
CN_2	kc_{21}	kc_{22}	kc_{23}	kc_{24}	CP_2
CN_3	kc_{31}	kc_{32}	kc_{33}	kc_{34}	CP_3
CN_4	kc_{41}	kc_{42}	kc_{43}	kc_{44}	CP_4
System characteristics priorities	SCP_1	SCP_2	SCP_3	SCP_4	
Target values	v1	v2	v3	v4	

Table 1. The "House of quality" (or QFD-matrix). The roof is tilted in order to be able to be presented in a spread sheet.

Here the system requirements priorities can are calculated as:

$$SCP_i = \sum ks_{ij} CP_j \tag{1}$$

which can also be written as:

$$\mathbf{scp} = \mathbf{Ks}^T\mathbf{cp} \tag{2}$$

The roof of the house-of-quality displays the interaction between system characteristics. The quantitative evaluation of these from is showed later in this paper.

1.6 Design Matrices

In design, matrices can be used to describe the relationship between some design parameters that are to be determined and some aspects of the system behaviour. The notation *design parameters* are used by Nam P Suh, [9].Here the term *design parameters* are used for the parameters that can be manipulated by design; these are a subset of the *system parameters* that represents all parameters that describe the product. The behaviour of the system is called *functional requirements*, FR by Suh. Here the notation *system characteristics* is used. This is a little broader since it covers all aspects of behaviour and properties of the product. There might be cases where there might be some ambiguity to what is a design parameter and a system characteristics, but here it is simply a function of what is the input and what is output from a system analysis.

The relationship between two input variables and two output variables can be written as:

$$\mathbf{y} = \mathbf{Ax} \tag{3}$$

where A is a matrix and x is a vector that is mapped into y through A. This does of course assume linear relationships to be true. For non-linear systems such a relationship is still useful. Here sensitivity analysis can be used to obtain the sensitivity matrix for small variation around a nominal set of parameters. Sensitivity analysis can be used to quickly give an overview over what parts of the design that is important for the desired behaviour. Furthermore it can be used to study the influence of disturbances and uncertainties in parameters and constants. Sensitivity analysis is the primary tool for studying the degree of robustness in a system. Assuming the system:

$$\mathbf{y} = \mathbf{f(x)} \tag{4}$$

where **f** is a nonlinear function. However, using linearization around a nominal point, this can be written as

$$\mathbf{y_0} + \mathbf{\Delta y} = \mathbf{f(x_0)} + \mathbf{J\Delta x} \tag{5}$$

where J is the Jacobian, where

$$J_{ij} = \frac{\partial f_i(x)}{\partial y_j} \tag{6}$$

hence

$$\mathbf{\Delta y} = \mathbf{J\Delta x} \tag{7}$$

here the Jacobian **J** is also identical to the sensitivity matrix k The elements in the sensitivity matrix can also be expressed as:

$$k_{ij} = \frac{\partial y_i}{\partial x_j} \tag{8}$$

This can be displayed in a table form derived from the QFD-matrix where the system characteristics are used for y and system parameters are used for x .

	System components/parameters				
System characteristics	X_1	X_2	X_3	X_4	System characteristics priorities
SC_1	k_{11}	k_{12}	k_{13}	k_{14}	SCP_1
SC_2	k_{21}	k_{22}	k_{23}	k_{24}	SCP_2
SC_3	k_{31}	k_{32}	k_{33}	k_{34}	SCP_3
SC_4	k_{41}	k_{42}	k_{43}	k_{44}	$SRCP_4$
Component priorities	XP_1	XP_2	XP_3	XP_4	

Table 2. The sensitivity matrix, with calculation of component priorities.

Here component priorities can be calculated as:

$$\mathbf{xp} = \mathbf{K}^T\mathbf{scp} \tag{9}$$

Here K is really the sensitivity matrix (defined later in the next section). This representation can also be seen as another representation of the design matrix since

$$\mathbf{y_{sc}} = \mathbf{Kx_d} \tag{10}$$

if the system is linear. As an example the QFD-matrix of a tentative transport aircraft is shown.

	System characteristics												
Customer requirements	Range	Liftoff distance	Landing distance	Takeoff weight	Required weight quotient	Optimal cruise speed	Landing speed	Liftoff speed	Stall speed	Emissions	MTBF	Cost	Customer requirement priorities
Long range	9												1.25
High speed		9				9					9		0.25
Field performance		9	9				9						0.25
Safe								3	3		9		2.00
Economical						9				3	9	9	1.25
System characteristics priorities	11.25	4.50	2.25	0.00	0.00	13.50	2.25	6.00	6.00	3.75	31.50	11.25	
Normalized system characteristics priorities	1.46	0.59	0.29	0.00	0.00	1.76	0.29	0.78	0.78	0.49	4.10	1.46	
Sign	1	-1	-1	-1	-1	1	-1	-1	-1	-1	1	-1	
Demand or whish	1.00	1.00	1.00	0.00	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	
Units	km	m	m	N		m/s	m/s	m/s	m/s		hour	kEUR	

Table 3. QFD matrix for transport aircraft

To perform sensitivity analysis it is necessary to have some kind of models for how the system parameters influence the system characteristics. Model-based QFD as it is used here implies that there is a chain of tools that connects the “house of quality” through sensitivity analysis, with underlying mathematical models. The aim of the exercise is to produce input that can be used for design optimization of the system.

2. Design Optimization

Consider the following optimisation problem. The system characteristics y are computed from the system parameters x_s

$$\mathbf{y} = \mathbf{f}(\mathbf{x_s}) \tag{11}$$

The object function is in general a function of system characteristics and system parameters (It can also be defined as a separate system characteristic). The optimization process is indicated in the figure below:

Consider the following optimisation problem. The system characteristics y are computed from the system parameters x_s

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A system model is used to obtain the system characteristics of the system. The object function is a function of the system characteristics.

$$f_{obj} = g(\mathbf{y}) \tag{13}$$

It is most often practical to express the problem as a minimization problem where the total objective function is expressed as a sum of sub-objective functions that are related to each system characteristics. Using the information from the QFD-matrix, a useful formulation is

$$f_i = SCP_i\left(\frac{y_t}{y+\varepsilon}\right)^{\varphi\gamma} \tag{14}$$

where φ is equal to one, if the systems characteristics should be maximized and minus one otherwise. The quotient y_t/y represents the degree of “unfulfilment”. ε is a very small number that should prevent the singularity. SCP is the system characteristics priorities and γ is an additional exponent that normally is set to two.

There may also be a violation flag that indicates if implicit constraints are violated, that also is a function of system characteristics.

$$c_{viol} = c(\mathbf{y}) \tag{15}$$

Another way to deal with constraints is to use a penalty function that is included in the objective function instead.

2.1 Penalty Functions for Constraints

$$f_{objf} = \sum_{i=1}^{N} f_i + 1\sum_{i=1}^{N} f_{p,i} \tag{16}$$

The model is made from modules that can be of varying fidelity. There is a progression of work on models for the different modules. There is a trend to relay more on dynamic simulation for the performance evaluation.

As an illustrative example a simple model of a transport aircraft is optimized and analysed. The model is made in Excel, but can also use modules located on other computers through web-service technology, see Johansson, Jouannet and Krus 2003. The relevant system parameters (parameters that are of interest to vary) are organized hierarchically on one worksheet.

System group	System group	System group	System parameter	Value	sdev	Unit	Type	Lower limit	Upper limit
Aircraft	Structure	Wing	B	2,00E+01	0,1	m	DP	1,00E+01	2,00E+01
Aircraft	Structure	Wing	Cr	6,032316911	0,1	m	DP	1,00E+00	2,00E+01
Aircraft	Structure	Wing	Ct	2,126565664	0,1	m	DP	1,00E-01	5,00E+00
Aircraft	Structure	Wing	tc	0,014448987	0,01		DP	5,00E-02	2,00E-01
Aircraft	Structure	Wing	lambda	0,001282934	0,05	rad	DP	0,00E+00	3,00E-01
Aircraft	Structure	Wing	ClmaxC	1,5	0,3		UP	8,00E-01	3,20E+00
Aircraft	Structure	Wing	CLmaxL	3,4	0,1		UP	1,75E+00	7,00E+00
Aircraft	Structure	Wing	CLmaxLo	2,4	0,5		UP	1,25E+00	5,00E+00
Aircraft	Structure	Wing	e0	0,75	0,1		UP	3,75E-01	1,50E+00
Aircraft	Structure	Wing	emax	0,9	0,1		UP	4,50E-01	1,80E+00
Aircraft	Structure	Wing	Snom	20	2		UP	1,00E+01	4,00E+01
Aircraft	Structure	Wng	ARnom	10	2		UP	5,00E+00	2,00E+01
Aircraft	Structure	Wng	tcnom	0,1	0,02		UP	5,00E-02	2,00E-01
Aircraft	Structure	Wng	lambdanom	0	0,1	rad	UP	0,00E+00	0,00E+00
Aircraft	Structure	Wing							
Aircraft	*Structure*								
Aircraft	*Airframe*								
Aircraft	*Propulsion system*								
Aircraft	*Systems*								
Aircraft	*Load*	*Load*							
Aircraft									
Mission									

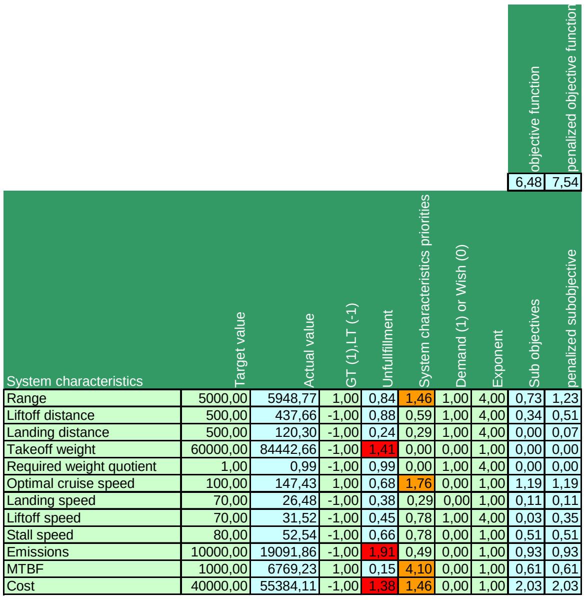
Table 4. Worksheet with relevant system parameters. Only Wing is expanded.

The system characteristics are displayed on another worksheet where the results from the different model modules are linked.

System Characteristics	Unit	Value	Relative model uncertainty
Range	km	5,95E+03	0,1
Liftoff distance	m	4,38E+02	0,1
Landing distance	m	1,20E+02	0,1
Takeoff weight	N	8,44E+04	0,1
Required weight quotient		9,92E-01	0,1
Optimal cruise speed	m/s	1,47E+02	0,1
Landing speed	m/s	2,65E+01	0,1
Liftoff speed	m/s	3,15E+01	0,1
Stall speed	m/s	5,25E+01	0,1
Emissions		1,91E+04	0,1
MTBF	hour	6,77E+03	0,10
Cost	kEUR	5,54E+04	0,10

Table 5. Table of system characteristics with links to models through "Value".

Using these system characteristics along with information regarding target values and system characteristics priorities from the QFD-matrix, a scalar objective function is set up.



objective function	penalized objective function
6,48	7,54

System characteristics	Target value	Actual value	GT (1),LT (-1)	Unfullfillment	System characteristics priorities	Demand (1) or Wish (0)	Exponent	Sub objectives	penalized subobjective
Range	5000,00	5948,77	1,00	0,84	1,46	1,00	4,00	0,73	1,23
Liftoff distance	500,00	437,66	-1,00	0,88	0,59	1,00	4,00	0,34	0,51
Landing distance	500,00	120,30	-1,00	0,24	0,29	1,00	4,00	0,00	0,07
Takeoff weight	60000,00	84442,66	-1,00	1,41	0,00	0,00	1,00	0,00	0,00
Required weight quotient	1,00	0,99	-1,00	0,99	0,00	1,00	4,00	0,00	0,00
Optimal cruise speed	100,00	147,43	1,00	0,68	1,76	0,00	1,00	1,19	1,19
Landing speed	70,00	26,48	-1,00	0,38	0,29	0,00	1,00	0,11	0,11
Liftoff speed	70,00	31,52	-1,00	0,45	0,78	1,00	4,00	0,03	0,35
Stall speed	80,00	52,54	-1,00	0,66	0,78	0,00	1,00	0,51	0,51
Emissions	10000,00	19091,86	-1,00	1,91	0,49	0,00	1,00	0,93	0,93
MTBF	1000,00	6769,23	1,00	0,15	4,10	0,00	1,00	0,61	0,61
Cost	40000,00	55384,11	-1,00	1,38	1,46	0,00	1,00	2,03	2,03

Figure 2. Table for formulation and evaluation of system objectives

For design optimization a wide range of methods can be used. However, in a complex design a non-gradient method is preferable, since gradients usually are not available. Therefore genetic algorithms (GA) or the COMPLEX-RF method is preferable, see Krus and Gunnarsson 1993. The COMPLEX-RF is a method is a modified version of the constraint SIMPLEX (COMPLEX) by Box 1965. When an optimal design point has been found it is analysed using sensitivity analysis. In this way the influence of all design parameters on the system characteristics is obtained.

3. Normalized Sensitivities

If the system is complex and the sensitivity matrix large, it may be difficult to get an overview of the system since the different parameters may have values of different orders of magnitude. The system characteristics are normally also of different orders of magnitude. In order to make it easier to get an overview of the sensitivities some kind of normalised dimensionless sensitivities are needed. The first approach to normalise the sensitivities is to employ the following definition

$$k_{ij}^{0} = \frac{x_{s,j}}{y_{s,i}} \frac{\partial y_{s,i}}{\partial x_{s,j}} \qquad (17)$$

In this way a non-dimensional value is obtained, that indicates how many percent a certain system characteristic is changed when a system parameter is changed one percent. In this way it is much easier to assess the relative importance of the different system parameters. Using this approach the following table is obtained.

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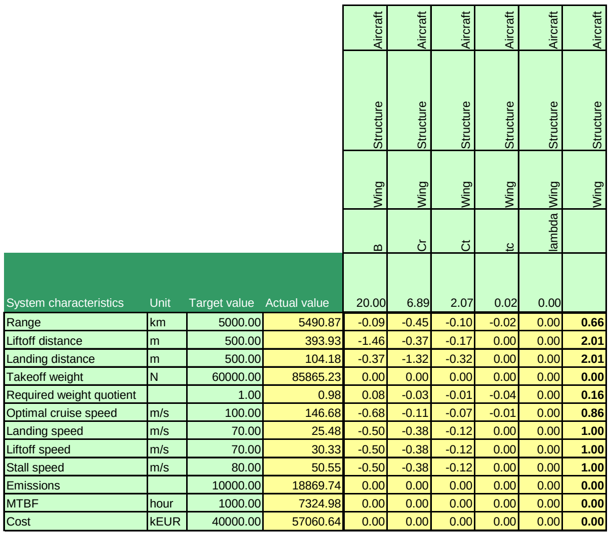
3.1 Aggregated Normailized Sensitivities

However, since normalised sensitivities are used, it is possible to aggregate all parameters associated with one component by simply adding them together. This has, however the drawback that different sign in sensitivities may cancel each other out. Therefore it is better to add the absolute values of all sensitivities instead. This will at least give a value to the relative importance and hence the magnitude of impact from design changes in a component or subsystem to the different system characteristics.



				Aircraft	Aircraft	Aircraft	Aircraft	Aircraft	Aircraft
				Structure	Structure	Structure	Structure	Structure	Structure
				Wing	Wing	Wing	Wing	Wing	Wing
				B	Cr	Ct	tc	lambda	
System characteristics	Unit	Target value	Actual value	20.00	6.89	2.07	0.02	0.00	
Range	km	5000.00	5490.87	-0.09	-0.45	-0.10	-0.02	0.00	**0.66**
Liftoff distance	m	500.00	393.93	-1.46	-0.37	-0.17	0.00	0.00	**2.01**
Landing distance	m	500.00	104.18	-0.37	-1.32	-0.32	0.00	0.00	**2.01**
Takeoff weight	N	60000.00	85865.23	0.00	0.00	0.00	0.00	0.00	**0.00**
Required weight quotient		1.00	0.98	0.08	-0.03	-0.01	-0.04	0.00	**0.16**
Optimal cruise speed	m/s	100.00	146.68	-0.68	-0.11	-0.07	-0.01	0.00	**0.86**
Landing speed	m/s	70.00	25.48	-0.50	-0.38	-0.12	0.00	0.00	**1.00**
Liftoff speed	m/s	70.00	30.33	-0.50	-0.38	-0.12	0.00	0.00	**1.00**
Stall speed	m/s	80.00	50.55	-0.50	-0.38	-0.12	0.00	0.00	**1.00**
Emissions		10000.00	18869.74	0.00	0.00	0.00	0.00	0.00	**0.00**
MTBF	hour	1000.00	7324.98	0.00	0.00	0.00	0.00	0.00	**0.00**
Cost	kEUR	40000.00	57060.64	0.00	0.00	0.00	0.00	0.00	**0.00**

Figure 3. Aggregated normalized sensitivity matrix. The matrix is expanded for the wing parameters.

This suggests that the aggregated sensitivities of components and subsystems is a good measurement of the design impact of that component/subsystem and is rather independent of the degrees of details, and degrees of freedoms in the model. The aggregated normalized sensitivity matrix therefore seems to be an excellent tool to ensure that the design efforts are properly balanced in the ensuing design steps, by identifying critical areas at an early stage of the design. The matrix can also be updated and maintained throughout the design process as the degrees of details in the design are increasing and the underlying models get more detailed. Furthermore, the same simple structure can be used at the top level even for the finished products, while more detailed information can be found at lower levels.

4 Modelling and the Influence of Uncertainties

Model based QFD requires models of all the different areas of interest, in order to evaluate influence of various design decisions. Furthermore, different domains have different tools and also different views of what an “accurate” model is. Very often tradition more than anything else dictates what kinds of models are used. This can lead to situation where very accurate models may be used for some areas, such as the detailed distribution of heat in a component using finite element software, combined with very poor models for boundary conditions, or detailed structural models together with very uncertain load cases. This is a very typical in design analysis that areas that "can" be analysed very accurately is allowed to draw resources from areas where it is difficult to reduce uncertainty. By managing the propagation if uncertainty in a model, from parameters, and models, to the system characteristics of interest, it is possible to avoid excessive computational costs. Instead the effort can be placed where it is most useful. In general, that means that very simple models can be used in many areas because, of the effect of irreducible uncertainties, or because the system is robust to a lot of certain uncertainties anyway. When uncertainty is present in the system characteristics, sufficient margin can instead be introduced to clear constraints.

When necessary, a physical prototype can be used to assess the influence of the uncertainties and to calibrate models and parameters for further detailed design.

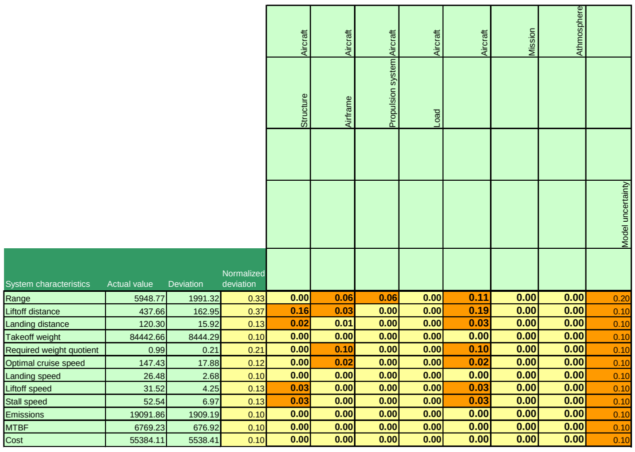
The influence of disturbances can be studied in a couple of ways. First, it is possible to study the sensitivities in the same way as from design variables. The other way is to study the influence of removing the uncertainty of one uncertainty variable. The uncertainty of a system characteristic can be calculated as:

$$s_{y,i} = \sqrt{k_{u,i,1}^2 s_{x,u,1}^2 + k_{u,i,2}^2 s_{x,u,2}^2 \dots + k_{u,i,3}^2 s_{x,u,3}^2} \qquad (18)$$

where s is the standard deviation. The normalized influence of the uncertainties is calculated

$$\Delta s_{0,y,ij} = \frac{s_{y,u,i}}{y_i}\left(1-\sqrt{\frac{s_{y,u,i}^2 - k_{u,i,1}^2 s_{x,u,ij}^2}{s_{y,i}^2}}\right) \qquad (19)$$

This matrix indicates the effect of totally removing the uncertainty of an uncertainty variable on specific system characteristics. This is extremely useful to balance the fidelity of models for different areas of a design. Due to the nature of these expressions, small uncertainties are quickly shadowed by larger uncertainties. This means that it is meaningful only to reduce the dominating uncertainties. In the example shown in Table 6, the influence on range from the properties of the wing is totally shadowed by the uncertainties in airframe (weight calculations), the propulsion, and the model uncertainty. This provides a valuable tool for allocating resource in the right areas, in order to reduce uncertainty in the system characteristics. Without this kind of tool it is a high risk that the department with the highest status and prestige will be able to argue most successfully for resources.



				Aircraft	Aircraft	Aircraft	Aircraft	Aircraft	Mission	Athmosphere	
				Structure	Airframe	Propulsion system	Load				
											Model uncertainty
System characteristics	Actual value	Deviation	Normalized deviation								
Range	5948.77	1991.32	0.33	0.00	0.06	0.06	0.00	0.11	0.00	0.00	0.20
Liftoff distance	437.66	162.95	0.37	0.16	0.03	0.00	0.00	0.19	0.00	0.00	0.10
Landing distance	120.30	15.92	0.13	0.02	0.01	0.00	0.00	0.03	0.00	0.00	0.10
Takeoff weight	84442.66	8444.29	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
Required weight quotient	0.99	0.21	0.21	0.00	0.10	0.00	0.00	0.10	0.00	0.00	0.10
Optimal cruise speed	147.43	17.88	0.12	0.00	0.02	0.00	0.00	0.02	0.00	0.00	0.10
Landing speed	26.48	2.68	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
Liftoff speed	31.52	4.25	0.13	0.03	0.00	0.00	0.00	0.03	0.00	0.00	0.10
Stall speed	52.54	6.97	0.13	0.03	0.00	0.00	0.00	0.03	0.00	0.00	0.10
Emissions	19091.86	1909.19	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
MTBF	6769.23	676.92	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10
Cost	55384.11	5538.41	0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10

Table 6. Hierarchical robustness matrix indicating the the influence of uncertainties on system characteristics from uncertainties in system parameters and models.

5. Robust Optimization

With uncertainties involved the optimal solution can not be guaranteed. Therefore, a robust optimum is wanted, that is a point which not only gives a good nominal optimal value, but one that also can tolerate variations in the uncertain values. For this robust optimization can be used, see [15]. Here, a simple approach is used in order to minimize cost. For those system characteristics subject to constraints, these constraints are simply moved in such a way that proper margins are introduced. The uncertainty (deviation) of a system characteristic can be calculated as in (20). The associated constraint is then calculated as:

$$y'_{ci} = y_{ci} + \lambda \varphi_i s_{yi} \qquad (20)$$

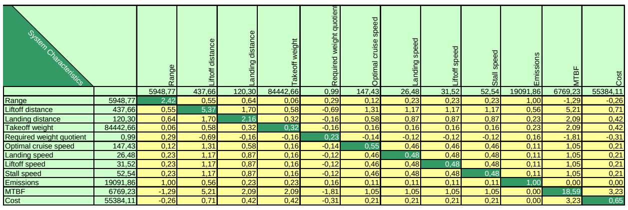
Here $\varphi_i = 1$ if it is a lower constraint and $\varphi_i = -1$ if it is an upper constraint. λ is a factor indicating how large the margin should be compared to the estimated deviation. If a robustness (sensitivity) analysis has been performed, there is no extra computational cost involved in doing the robust optimization.

6. Dependencies Between System Characteristics

In a design the different system characteristics may be conflicting or more or less pulling in the same direction. Information about this is very useful when setting up the requirements for a design since it can show what areas that can be improved without scarifying to much in other areas, or to see what areas that might be worth sacrificing in order to improve others. A simple measure of this is the systems characteristics dependency matrix, SCD. The elements are here defined as:

$$SCD_{ik} = \sum_{j=1}^{m} k_{ij}^{0} k_{kj}^{0} \tag{21}$$

SCD is always a symmetric matrix. The diagonal element represents a measure of how sensitive, or "controllable", a system characteristic is with respect to the design parameters.



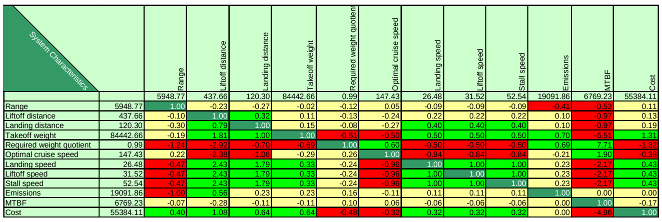
System Characteristics		Range	Liftoff distance	Landing distance	Takeoff weight	Required weight quotient	Optimal cruise speed	Landing speed	Liftoff speed	Stall speed	Emissions	MTBF	Cost
		5948,77	437,66	120,30	84442,66	0,99	147,43	26,48	31,52	52,54	19091,86	6769,23	55384,11
Range	5948,77	2,42	0,55	0,64	0,06	0,29	0,12	0,23	0,23	0,23	1,00	-1,29	-0,26
Liftoff distance	437,66	0,55	5,37	1,70	0,58	-0,69	1,31	1,17	1,17	1,17	0,56	5,21	0,71
Landing distance	120,30	0,64	1,70	2,16	0,32	-0,16	0,58	0,87	0,87	0,87	0,23	2,09	0,42
Takeoff weight	84442,66	0,06	0,58	0,32	0,32	-0,16	0,16	0,16	0,16	0,16	0,23	2,09	0,42
Required weight quotient	0,99	0,29	-0,69	-0,16	-0,16	0,23	-0,14	-0,12	-0,12	-0,12	0,16	-1,81	-0,31
Optimal cruise speed	147,43	0,12	1,31	0,58	0,16	-0,14	0,55	0,46	0,46	0,46	0,11	1,05	0,21
Landing speed	26,48	0,23	1,17	0,87	0,16	-0,12	0,46	0,48	0,48	0,48	0,11	1,05	0,21
Liftoff speed	31,52	0,23	1,17	0,87	0,16	-0,12	0,46	0,48	0,48	0,48	0,11	1,05	0,21
Stall speed	52,54	0,23	1,17	0,87	0,16	-0,12	0,46	0,48	0,48	0,48	0,11	1,05	0,21
Emissions	19091,86	1,00	0,56	0,23	0,23	0,16	0,11	0,11	0,11	0,11	1,00	0,00	0,00
MTBF	6769,23	-1,29	5,21	2,09	2,09	-1,81	1,05	1,05	1,05	1,05	0,00	18,59	3,23
Cost	55384,11	-0,26	0,71	0,42	0,42	-0,31	0,21	0,21	0,21	0,21	0,00	3,23	0,65

Table 7. System characteristics interaction, SCD

There is also the adjusted system characteristics which are where the elements in a row are normalized with respect to the diagonal elements. Furthermore, they are multiplied with the sign φ of the desired direction for a system characteristic, so that if a large value is desirable, or required, the φ = 1 and if a small value is desirable, or required, φ = -1.

$$ASCD_{ik} = \varphi_i \varphi_k SCD_{ik} / SCD_{ii} \tag{22}$$

This is an asymmetric matrix. Here the influence of system characteristics in the columns on the system characteristics in the rows is displayed. It can also be coloured so that highly negative interaction is marked with red, and highly positive interactions are green.



System Characteristics		Range	Liftoff distance	Landing distance	Takeoff weight	Required weight quotient	Optimal cruise speed	Landing speed	Liftoff speed	Stall speed	Emissions	MTBF	Cost
		5948.77	437.66	120.30	84442.66	0.99	147.43	26.48	31.52	52.54	19091.86	6769.23	55384.11
Range	5948.77	1.00	-0.23	-0.27	-0.02	-0.12	0.05	-0.09	-0.09	-0.09	-0.41	-0.53	0.11
Liftoff distance	437.66	-0.10	1.00	0.32	0.11	-0.13	-0.24	0.22	0.22	0.22	0.10	-0.97	0.13
Landing distance	120.30	-0.30	0.79	1.00	0.15	-0.08	-0.27	0.40	0.40	0.40	0.10	-0.97	0.19
Takeoff weight	84442.66	-0.19	1.81	1.00	1.00	-0.51	-0.50	0.50	0.50	0.50	0.70	-6.51	1.31
Required weight quotient	0.99	-1.24	-2.92	-0.70	-0.69	1.00	0.60	-0.50	-0.50	-0.50	0.69	7.71	-1.32
Optimal cruise speed	147.43	0.22	-2.38	-1.06	-0.29	0.26	1.00	-0.84	-0.84	-0.84	-0.21	1.90	-0.38
Landing speed	26.48	-0.47	2.43	1.79	0.33	-0.24	-0.96	1.00	1.00	1.00	0.23	-2.17	0.43
Liftoff speed	31.52	-0.47	2.43	1.79	0.33	-0.24	-0.96	1.00	1.00	1.00	0.23	-2.17	0.43
Stall speed	52.54	-0.47	2.43	1.79	0.33	-0.24	-0.96	1.00	1.00	1.00	0.23	-2.17	0.43
Emissions	19091.86	-1.00	0.56	0.23	0.23	0.16	-0.11	0.11	0.11	0.11	1.00	0.00	0.00
MTBF	6769.23	-0.07	-0.28	-0.11	-0.11	0.10	0.06	-0.06	-0.06	-0.06	0.00	1.00	-0.17
Cost	55384.11	0.40	1.08	0.64	0.64	-0.48	-0.32	0.32	0.32	0.32	0.00	-4.96	1.00

Table 8. Adjusted system characteristics dependencies, ASCD

With the *adjusted system characteristics dependencies* matrix (ASCD), It is possible to quickly assess how the system characteristics influence each other. For instance, looking at the first row, range is in conflict with emissions since a long range implies a larger plane for the same payload, which produce more drag and hence more emissions.

The *system characteristics dependencies* matrix can also be seen as a quantified "roof" to the QFD-matrix in Table 1. Together they form the well known "house of quality". Note that there really should be a full matrix rather than just a triangle since the ASCD matrix is asymmetric. Alternatively an extra row for the diagonal elements in the SCI matrix has be provided if, the symmetric SCD matrix is used instead.

Of course, it could also be possible to use the correlation-coefficient instead, but that is limited to the interval [-1,1], and it is symmetric, so there is no information regarding the dominant direction of dependency.

$$SCC_{ik} = \frac{\frac{1}{n}\sum_{j=1}^{n} k_{ij}^{0} k_{kj}^{0}}{s_i s_k} \tag{23}$$

And an adjusted correlation

$$ASCC_{ik} = \varphi_i \varphi_k \frac{\frac{1}{n}\sum_{j=1}^{n} k_{ij}^{0} k_{kj}^{0}}{s_i s_k} \tag{24}$$

Here the standard deviations in the sensitivities are:

$$s_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n} (k_{ij}^{0})^2} \tag{25}$$

7. Conclusions

In this paper a range of linked design analysis tools are described that together forms a model based QFD, and are useful in the design process. These range from a quantized house of quality, to design optimisation. This means that there is a transparent coupling between customer requirements down to design parameters. In particular, the aggregated normalized sensitivity matrix is an excellent tool to represent design dependencies in a complex design by introducing hierarchy and enable traceability between top level requirements down to component parameters. The system characteristics correlation matrix is also a very useful tool when negotiating requirements. Furthermore, it is shown how it is possible to manage the model fidelity and data uncertainty in such a way that the proper resources are allocated where they have the greatest effect of reducing design uncertainty, by using a robustness matrix.

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