ON THE EFFECTS OF GROUND REFLECTION AND ATMOSPHERIC ATTENUATION ON THE NOISE OF HELICOPTERS

L. M. B. C. Campos, J. M. G. S. Oliveira
CCTAE, Instituto Superior Técnico, Lisboa, Portugal

Keywords: *Helicopter noise, ground effect, atmospheric attenuation of noise*

Abstract

There is an extensive literature on the effects on noise of ground impedance and barriers [1-4] and atmospheric attenuation and wind [5-10] These effects are particularly important for helicopters, due to their ability to operate in limited spaces near populated areas; the full exploitation of the dynamical abilities of helicopters can be restricted by noise limits. Within the noise limits the ground and atmospheric effects can influence the operational envelope of the helicopter.

The present paper contains 6 models of helicopter noise addressing specifically the effects of sound reflection and atmospheric absorption.

The baseline model I is a single reflection point over flat ground and relies on the following assumptions: (i) isotropic, point source of sound emitting spherical waves (valid if the distance of observer is large relative to the helicopter size); (ii) static source (neglects Doppler effects for helicopter speed small relative to sound speed); (iii) flat, horizontal ground (excludes mountainous ground and obstacles, hence no multipath effects or wide area or multiple scattering); (iv) homogeneous atmosphere (neglects density and temperature, or sound speed, stratification, hence refraction effects); (v) atmosphere at rest (no wind or mean flow convection effects on sound); (vi) uniform ground impedance (same ground composition everywhere all the time, excludes different soils, humidity changes during the day, etc.)

The following extensions relax some restrictions of the baseline model to include: (i) non-flat ground including multiple paths in model II; (ii) wide area scattering by flat ground in model III; (iii) single scattering by mountainous ground in model IV. Besides effects of ground reflection, those of atmospheric absorption are also considered in the same instances: (i) atmospheric absorption over flat ground extends model II to model V; (ii) atmospheric absorption in the case of uneven ground, including wide area scattering extends model III to model VI. The application of all six models depend on the calculation of reflection points, which is done for: (i) reflection from flat ground, applicable to models I and V; (ii) reflection from a two-dimensional slice of ground, applicable to model II; (iii) reflection from a line or region over three-dimensional rough ground, applicable to models III, IV, and VI. For each of these models, formulas for the SPL (sound pressure level) variation and the phase shift of acoustic pressure are presented.

The methods presented are illustrated by applying models I and V to horizontal flight over flat ground with uniform impedance.

1 Sound reflection from flat or uneven ground

The ground effect on sound reflection is considered starting with flat ground, and then proceeding through mountainous ground in two and three dimensions to wide area scattering.

1.1 Baseline model I of reflection by flat ground

The baseline model I relies on the assumptions indicated in the introduction.

The Figure 1 shows the **source-observer coordinate** system: (i) the x-axis is horizontal and the z-axis vertical in the vertical plane passing through source S and observer O; (ii) the y-axis forms a right-handed triad, and the origin is any point on the intersection of the vertical plane with the ground. For definiteness the origin may be taken on the ground, in the vertical through the observer, in which case $x_O = 0$.

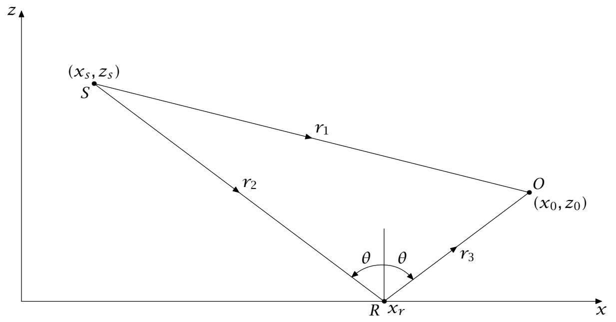

Fig. 1 Source-Observer coordinate system: sound sources at $S = (x_s, z_s)$ and observer at $O = (x_0, z_0)$.

Using the vertical plane through the source and observer, i. e. the line-of-sight plane, the direct received acoustic pressure is:

$$p_0 = \mathrm{e}^{ikr_1}/r_1 \tag{1}$$

where a constant amplitude is ommited, and r_1 is the distance from observer to source:

$$r_1 \equiv \left|(x_0 - x_s)^2 + (z_0 - z_s)^2\right|^{1/2}. \tag{2}$$

Line-of-sight reflection occurs at the reflection point $R \equiv (x_r, 0)$, such that the angles of incidence and reflection θ are the same,

$$(x_0 - x_r)/z_0 = \tan\theta = (x_r - x_s)/z_s. \tag{3}$$

This can be solved for x_r:

$$x_r = (x_0 z_s + x_s z_0)/(z_0 + z_s), \tag{4}$$

to specify the position of the point-of-reflection. The sound field reflected in line-of-sight:

$$p_r = R\frac{\mathrm{e}^{ik(r_2+r_3)}}{(r_2+r_3)}, \tag{5}$$

consists of: (i) a spherical wave from the sound source to the reflection point at distance:

$$r_2 \equiv \left|(x_s - x_r)^2 + (z_s)^2\right|^{1/2}, \tag{6}$$

and from the reflection point to the observer, at a distance:

$$r_3 \equiv \left|(x_0 - x_r)^2 + (z_0)^2\right|^{1/2}. \tag{7}$$

(ii) a reflection coefficient, which can have a modulus $|R| < 1$ and a phase $\arg(R)$ and depends on ground properties. The total acoustic field

$$p_I = p_0 + p_r = \frac{\mathrm{e}^{ikr_1}}{r_1} + R\frac{\mathrm{e}^{ik(r_2+r_3)}}{(r_2+r_3)}, \tag{8}$$

is the sum of the direct (1) and reflected (5) fields.

The effect on the acoustic energy of the ground reflection of the free acoustic field:

$$E_I = |p_I|^2/|p_0|^2 = |1 + p_r/p_0|^2 \tag{9}$$

in the present case is:

$$E_I = \left|1 + [r_1/(r_2+r_3)]\,R\mathrm{e}^{ik(r_2+r_3-r_1)}\right|^2. \tag{10a}$$

This corresponds to a change in SPL (sound pressure level) in a decibel (dB) scale:

$$A_I = 10\log E_I = 10\log\left\{1 + \left[\frac{r_1}{r_2+r_3}\right]^2 |R|^2 + \frac{2r_1|R|}{r_2+r_3}\cos\left[k(r_2+r_3-r_1) + \arg(R)\right].\right\} \tag{10b}$$

The phase of the acoustic pressure has a variation:

$$\Phi_I = \arg(p_I) - \arg(p_0) = \arg(p_I/p_0) = \operatorname{arccot}[\mathrm{Re}(p_I/p_0)/\mathrm{Im}(p_I/p_0)], \tag{11a}$$

and is given in the case (10b) by

$$\Phi_I = \operatorname{arc\,cot}\left\{ \cot[k(r_2+r_3-r_1)+\arg(R)] + \left[\frac{r_2+r_3}{r_1|R|}\right] \csc[k(r_2+r_3-r_1)+\arg(R)] \right\} \quad (11b)$$

The change in SPL in (10b) and the change in phase of the acoustic pressure (11b) are valid for arbitrary reflection factor R, which may involve an amplitude R and a phase $\arg(R)$.

1.2 Model II for multiple path in mountainous terrain

The extension to non-flat ground requires knowledge of the terrain profile

$$z = h(x), \quad (12)$$

in the plane of line-of-sight, i. e. in the source-observer coordinate system. The formula for the reflected (5) and hence for the total (8) acoustic pressure remains valid if the slope of the terrain is neglected:

$$\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)^2 \ll 1, \quad (13)$$

so that the angles of incidence and reflection are still measured from a vertical normal direction. The difference from the case of flat ground is that the location of the reflection point is no longer given by (3)≡(4), because the reflection point is no longer at zero height:

$$Q \equiv (x_r, h(x_r)). \quad (14)$$

Thus the condition (3) is replaced by:

$$\frac{[z_0 - h(x_r)]}{(x_r - x_0)} = \cot\theta = \frac{[z_s - h(x_r)]}{(x_s - x_r)}, \quad (15a)$$

which states again the equality of the angles of incidence and reflection. In (15a) the terms dependent on the reflection point x_r are separated on the r.h.s.:

$$\frac{x_0 z_s + x_s z_0}{z_0 + z_s} = x_r + \frac{x_0 + x_s - 2x_r}{z_0 + z_s} h(x_r). \quad (15b)$$

Given the source $S \equiv (x_s, z_s)$ and observer $O \equiv (x_0, z_0)$ position, the solutions of (15b) for x_r give the reflection(s) point(s) in line-of-sight. There may be one x_r or several x_{r_j}, with $j = 1, \ldots, M$, depending on the terrain (12). The total sound field:

$$p_{II} = \frac{\mathrm{e}^{\mathrm{i}kr_1}}{r_1} + \sum_{j=1}^{M} R_j \frac{\mathrm{e}^{\mathrm{i}k(r_{2_j}+r_{3_j})}}{r_{2_j}+r_{3_j}}, \quad (16)$$

involve a sum over all the reflection points, where the reflection factor R_j may vary with the reflection point and r_{2_j} and r_{3_j} are the distances from the source and observer, respectively, to the j-th reflection point. The effect on the acoustic energy is obtained substituting (16) in (9):

$$E_{II} = |p_{II}/p_0|^2 = \left| 1 + \sum_{j=1}^{M} R_j \left[r_1/(r_{2_j}+r_{3_j}) \right] \mathrm{e}^{\mathrm{i}k(r_{2_j}+r_{3_j}-r_1)} \right|^2, \quad (17a)$$

or, the change in SPL on a decibel scale:

$$\begin{aligned} A_{II} \equiv 10\log E_{II} = & \, 10\log\left\{ 1 + \sum_{j=1}^{M} \left[r_1/(r_{2_j}+r_{3_j}) \right]^2 |R_j|^2 + \right. \\ & 2\sum_{j=1}^{M} \frac{r_1}{r_{2_j}+r_{3_j}} \cos\left[k(r_{2_j}+r_{3_j}-r_1) + \arg(R_j) \right] + \\ & 2\sum_{j=1}^{M}\sum_{l=1}^{j-1} \frac{r_1}{(r_{2_j}+r_{3_j})} \frac{r_1}{(r_{2_l}+r_{3_l})} |R_j|\,|R_l| \times \\ & \cos\left[k\left(r_{2_j}+r_{3_j}-r_{2_l}-r_{3_l} \right) + \right. \\ & \left. \left. \arg(R_j) - \arg(R_l) \right] \right\}. \end{aligned} \quad (17b)$$

The change of phase of the acoustic pressure:

$$\Phi_{II} = \arg(p_{II}) - \arg(p_0) = \operatorname{arc\,cot}\left[\mathrm{Re}(p_{II}/p_0)/\mathrm{Im}(p_{II}/p_0) \right], \quad (18a)$$

is given by

$$\Phi_{II} = \operatorname{arc\,cot}\left\{\left[1+\sum_{j=1}^{M}\frac{|R_j|r_1}{(r_{2_j}+r_{3_j})}\cos\left[k(r_{2_j}+r_{3_j}-r_1)+\arg(R_j)\right]\right]/\left[\sum_{j=1}^{M}\frac{|R_j|r_1}{(r_{2_j}+r_{3_j})}\sin\left[k(r_{2_j}+r_{3_j}-r_1)+\arg(R_j)\right]\right]\right\} \tag{18b}$$

In the case of a single reflection point $j = l = M = 1$, then (17b) and (18b) reduce respectively to (10b) and (11b).

1.3 Model III of scattering from a flat wide area

If reflection is considered from all points on the ground, then the preceding analysis needs extension, first in the cases of flat ground. Considering the sound source $R \equiv (x_s, y_s, z_s)$ and observer $O \equiv (x_0, y_0, z_0)$ at arbitrary positions over a flat ground, scattering is possible at all positions $Q \equiv (\xi, \eta, 0)$ on the ground and thus the total acoustic field (8) is now given by

$$p_{III} = \frac{e^{ikr_1}}{r_1} + \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, R(\xi,\eta)\frac{e^{ik(r_2+r_2)}}{r_2+r_3}, \tag{19}$$

where: (i) the distance from the observer to source is constant:

$$r_1 \equiv \left|(x_0-x_s)^2+(y_0-y_s)^2+(z_0-z_s)^2\right|^{1/2}; \tag{20}$$

(ii) the distance from the source to scattering point r_2 and from scattering point to observer r_3 are now

$$r_2 \equiv \left|(x_s-\xi)^2+(y_s-\eta)^2+(z_s)^2\right|^{1/2}, \tag{21}$$

$$r_3 \equiv \left|(x_0-\xi)^2+(y_0-\eta)^2+(z_0)^2\right|^{1/2}, \tag{22}$$

and depend on ξ, η; (iii) the reflection factor R does not depend on ξ, η for an homogeneous ground and is taken to be zero outside the reflection area. The effect on acoustic energy is obtained substituting (19) in (9):

$$E_{III} \equiv |p_{III}/p_0|^2 = \left|1+\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, R(\xi,\eta)\frac{r_1}{r_2+r_3}e^{ik(r_2+r_3-r_1)}\right|^2, \tag{23a}$$

or, the SPL on a decibel scale,

$$\begin{aligned} A_{III} = 10\log E_{III} = 10\log\Bigg\{ & 1+2\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, \frac{r_1}{r_2+r_3}|R(\xi,\eta)|\times \\ & \cos\{k(r_2+r_3-r_1)+\arg[R(\xi,\eta)]\}+ \\ & \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\xi' \int_{-\infty}^{+\infty} d\eta' \Bigg[\frac{r_1}{r_2+r_3}\times \\ & \frac{r_1}{r_2'+r_3'}|R(\xi,\eta)|\,|R(\xi',\eta')|\times \\ & \cos\Big(k(r_2+r_3-r_2'-r_3')+ \\ & \arg R(\xi,\eta)-\arg R(\xi',\eta')\Big)\Bigg]\Bigg\} \end{aligned} \tag{23b}$$

where r_2', r_3' are the same functions of ξ', η' as r_2, r_3 are of ξ, η in (21) and (22). The phase difference of the acoustic pressure is given by:

$$\begin{aligned} \Phi_{III} = \arg(p_{III}/p_0) = \operatorname{arc\,cot}\Bigg\{ & \Bigg[1+\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, \frac{r_1}{r_2'+r_3'}|R(\xi,\eta)| \\ & \cos\Big(k(r_2+r_3-r_1)+\arg R(\xi,\eta)\Big)\Bigg]\Bigg\}/ \\ \Bigg\{ & \Bigg[1+\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, \frac{r_1}{r_2'+r_3'}|R(\xi,\eta)|\times \\ & \cos\Big(k(r_2+r_3-r_1)+\arg R(\xi,\eta)\Big)\Bigg]\Bigg\} \end{aligned} \tag{24}$$

In the case of single-point reflection, the integrations in (23b) and (24) are omitted, with $\xi = \xi'$, $\eta = \eta'$ and hence $r_2' = r_2$, $r_3' = r_3$, and respectively (10b) and (11b) are obtained.

1.4 Model IV on single scattering from terrain

The results (23b) and (24) also apply to the single scattering from terrain with altitude profile:

$$z = H(x,y), \tag{25}$$

bearing in mind that the coordinate of the scattering points are now:

$$Q \equiv \left(\xi, \eta, H(\xi,\eta)\right), \tag{26}$$

and thus the distances to source r_2 and observer r_3 become

$$r_2 \equiv \left|(x_s-\xi)^2 + (y_s-\eta)^2 + \left(z_s - h(\xi,\eta)\right)^2\right|^{1/2}, \tag{27a}$$

$$r_3 \equiv \left|(x_0-\xi)^2 + (y_0-\eta)^2 + \left(z_0 - h(\xi,\eta)\right)^2\right|^{1/2}. \tag{27b}$$

The restriction to single scattering excludes multiple reflections from the terrain.

2 Atmospheric absorption over smooth or mountainous ground

The effects of atmospheric absorption are considered first for sound propagation and reflection by flat ground, and then extended to mountainous ground and wide-area scattering.

2.1 Model V for the effects of atmospheric attenuation

It is assumed that the atmosphere is homogeneous and at rest, so that the attenuation $\delta(r)$ depends only on the distance of propagation, *viz.* (8) is replaced by:

$$p_V = \frac{e^{ikr_1-\delta_1}}{r_1} + R\frac{e^{ik(r_2+r_3)-\delta_2-\delta_3}}{r_2+r_3}, \tag{28}$$

where in the case of uniform atmospheric absorption per unit length:

$$\varepsilon = \text{const.}: \qquad \{\delta_1,\delta_2,\delta_3\} = \varepsilon\{r_1,r_2,r_3\}. \tag{29}$$

The effect of ground reflections

$$p_V = \frac{e^{ikr_1-\delta_1}}{r_1}F, \tag{30}$$

is equivalent to multiplication by a factor

$$F = 1 + RG, \tag{31a}$$

Which differs from unity on account: (i) of the geometrical factor

$$G \equiv \frac{r_1}{r_2+r_3}e^{ik(r_2+r_3-r_1)+\delta_1-\delta_2-\delta_3}, \tag{31b}$$

which depends only on observer and source position; (ii) the reflection factor R, which depends on ground properties. The effect on acoustic energy (9) is now:

$$E_V = \left|p_V\, e^{\delta_1}/p_0\right|^2 = |1+RG|^2 = \left|1 + R\left[r_1/(r_2+r_3)\right]e^{ik(r_2+r_3-r_1)+\delta_1-\delta_2-\delta_3}\right|^2 \tag{32a}$$

i. e. the change in SPL is:

$$A_V = 10\log E_V = 10\log\left\{1 + \left[\frac{r_1}{r_2+r_3}\right]^2 |R|^2 e^{2\delta_1-2\delta_2-2\delta_3} + 2\frac{r_1}{r_2+r_3}|R|\exp(\delta_1-\delta_2-\delta_3)\times \cos\left[k(r_2+r_3-r_1)+\arg(R)\right]\right\} \tag{32b}$$

The change in phase of the acoustic pressure is:

$$\Phi_V = \arg(p_V e^{\delta_1}/p_0) = \operatorname{arc\,cot}\left\{\cot[k(r_2+r_3-r_1)+\arg(R)] + \frac{r_2+r_3}{r_1|R|}e^{\delta_2+\delta_3-\delta_1}\csc[k(r_2+r_3-r_1)+\arg(R)]\right\} \tag{33}$$

In the absence of atmospheric attenuation $\delta_1 = \delta_2 = \delta_3 = 0$ then (32b) and (33) reduce respectively to (10b) and (11b).

The simplest form of the reflection coefficient R is:

$$R = \frac{1-R_0}{1+R_0}, \tag{34}$$

for an homogeneous ground of density ρ_1, generally much higher than the air density ρ_0:

$$R_0 = \rho_0\kappa'/\rho_1\kappa \tag{35}$$

where the vertical wavenumbers of incidence κ and transmission κ' are given by

$$\kappa = \frac{\omega}{a_0}\cos\theta, \tag{36a}$$

$$\kappa' = \frac{\omega}{a_1}\cos\theta', \tag{36b}$$

where a_0, a_1 are the sound speeds in air and ground, and the angles of incidence θ and transmission θ' are related by Snell's law:

$$\frac{\sin\theta}{a_0} = \frac{\sin\theta'}{a_1}, \tag{37}$$

stating the continuity of the transverse wavenumber. Substituting (37) in (36) and then into (35) leads to

$$R = \frac{\rho_0 a_0}{\rho_1 a_1}\sqrt{\sec^2\theta - (a_1/a_0)^2\cot^2\theta}, \tag{38}$$

which specifies the reflection factor (31b) in terms of the angle θ in (3) or (15a).

Many more sophisticated forms of reflection coefficients exist to represent different types of ground.

2.2 Model VI on the wide area effect of atmospheric attenuation

The effect of atmospheric attenuation can be extended from line-of-sight reflection (28) to scattering from terrain (19), *viz.*:

$$p_{VI} = \frac{e^{ikr_1-\delta_1}}{r_1} + \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, R(\xi,\eta)\frac{e^{ik(r_2+r_3)-\delta_2-\delta_3}}{r_2+r_3}, \tag{39}$$

where, since the attenuation depends on distance $\delta(r)$, then $\delta_1 = \delta(r_1)$ is constant, but $\delta_2 = \delta(r_2)$ and $\delta_3 = \delta(r_3)$ depend on the scattering point. This leads by (9) to the effect on acoustic energy:

$$E_{VI} = \left|p_{VI}e^{\delta_1}/p_0\right|^2 = \left|\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, R(\xi,\eta)\frac{r_1}{r_2+r_3}\times \exp\left[ik(r_2+r_3-r_1)+\delta_1-\delta_2-\delta_3\right]\right|^2, \tag{40a}$$

or hence to the SPL change on a decibel scale:

$$\begin{aligned} A_{VI} = 10\log E_{VI} = {} & 10\log\left\{1+\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, \frac{r_1}{r_2+r_3}\,|R(\xi,\eta)|\times\right. \\ & e^{\delta_1-\delta_2-\delta_3}\cos\{k(r_2+r_3-r_1)+\arg[R(\xi,\eta)]\}+ \\ & \int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta \int_{-\infty}^{+\infty} d\xi' \int_{-\infty}^{+\infty} d\eta' \left[\frac{r_1}{r_2+r_3}\frac{r_1}{r_2'+r_3'}\times\right. \\ & |R(\xi,\eta)|\,|R(\xi',\eta')|\exp(\delta_2'+\delta_3'-\delta_2-\delta_3)\times \\ & \cos\left[k(r_2+r_3-r_2'-r_3')+\right. \\ & \left.\left.\left.\arg R(\xi,\eta)-\arg R(\xi',\eta')\right]\right]\right\} \end{aligned} \tag{40b}$$

where r_2', r_3', δ_2', δ_3' are functions of ξ', η' as r_2, r_3, δ_2, δ_3 are functions of ξ, η. The change in phase of the acoustic pressure is given by:

$$\begin{aligned} \Phi_{VI} = \arg(p_{VI}e^{\delta_1}/p_0) = {} & \operatorname{arc\,cot}\left\{\left\{1+\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, |R(\xi,\eta)|\times\right.\right. \\ & \left. e^{\delta_1-\delta_2-\delta_3}\cos[k(r_2+r_3-r_1)+\arg(R(\xi,\eta))]\right\}/ \\ & \left\{\int_{-\infty}^{+\infty} d\xi \int_{-\infty}^{+\infty} d\eta\, |R(\xi,\eta)|\, e^{\delta_1-\delta_2-\delta_3}\times\right. \\ & \left.\left.\sin[k(r_2+r_3-r_1)+\arg(R(\xi,\eta))]\right\}\right\}. \end{aligned} \tag{41}$$

In the case of point reflection, since all primed and unprimed quantities coincide, (40b) and (41) reduce to (32b) and (33).

3 Determination of the coordinates of reflection points

The calculation of the effect of ground reflection and atmospheric absorption on helicopter noise depends on the location of the reflection point(s). The latter affects the length of the ray path, and hence the amplitude decay and phase shift. The location of the reflection point(s) is calculated in the microphone reference system in three cases: (§3.1) flat ground; (§3.2) two-dimensional slice of rough ground; (§3.3) three-dimensional rough ground.

3.1 Reflection from flat ground

Each of the six preceding models leads to a formula for the effects of ground reflection and atmospheric absorption on the acoustic pressure p, which specifies: (i) the difference in acoustic energy or difference in SPL in dB:

$$A = 10\log_{10}|E| = 20\log_{10}|p/p_0|; \tag{42}$$

(ii) the phase shift of the acoustic pressure:

$$\Phi = \arg p/p_0 = \arg(p) - \arg(p_0). \tag{43}$$

These outputs depend on the calculation of the reflection point(s). The simplest case is a flat ground, for which there is a single reflection point.

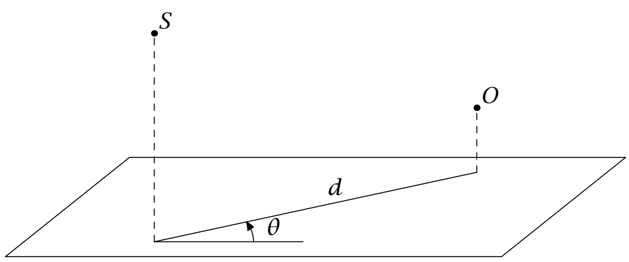

Fig. 2 Relative position of source and observer over flat ground.

The source $R \equiv (X_s, Y_s, Z_s)$ and observer $O \equiv (X_o, Y_o, Z_o)$ positions in the microphone reference system are given (Figure 2); their horizontal projections are at distance d; the line joining the horizontal projections makes an angle θ with the X-axis. Making a section by a vertical plane passing through the source and observer positions and choosing cartesian coordinates with Ox-axis on the ground and Oy-axis passing through the observer, leads to coordinates in the source-observer reference system:

$$x_o = 0 \qquad y_o = Z_o, \tag{44}$$

as observer coordinates, and

$$y_s = Z_s \quad x_s = d \equiv \left|(X_s - X_o)^2 + (Y_s - Y_o)^2\right|^{1/2} \tag{45}$$

as source coordinates. The location of the reflection point and effects on acoustic energy follow as in (§1.1) of for model I in (§1.1) and its extension to include atmospheric absorption in model IV in (§2.1).

3.2 Two-dimensional slice of rough ground

Let the height of the rough ground be given by:

$$Z = H(X, Y). \tag{46}$$

The two-dimensional slice made as before (§3.1) leads for an arbitrary point $P \equiv (X, Y, Z)$ to an x-coordinate in the source-observer coordinate system:

$$x = \left|(X - X_o)^2 + (Y - Y_o)^2\right|^{1/2}, \tag{47}$$

and angle φ with the x-axis:

$$\tan\varphi = \frac{Y - Y_o}{X - X_o}. \tag{48}$$

Using the transformation:

$$X = X_s + x\cos\varphi \tag{49a}$$

$$Y = Y_s - x\sin\varphi \tag{49b}$$

the two-dimensional slice through the rough ground is specified in the source-observer coordinate system by:

$$y = h(x) = H(X_s + x\cos\varphi, Y_s - x\sin\varphi). \tag{50}$$

This specifies the terrain profile function (12) used in model II in (§1.2) and its extensions to include atmospheric absorption in model V in (§2.1).

3.3 Three-dimensional rough ground

The case of three-dimensional rough ground can be treated without the assumption made in (§3.1) that the reflection point lies in the vertical plane through the source and observer. Let a reflection point be $R \equiv (X_r, Y_r, Z_r \equiv H(X_r, Y_r))$. The unit vector in the direction from the source to reflection point is

$$\nabla(SR) = \{X_s - X_r, Y_s - Y_r, Z_s - H(X_r, Y_r)\} \times \left|(X_s - X_r)^2 + (Y_s - Y_r)^2 + [Z_s - H(X_r, Y_r)]^2\right|^{-1/2} \tag{51a}$$

and the unit vector from reflection point to observer is:

$$\nabla(RO) = \{X_r - X_o, Y_r - Y_o, H(X_r, Y_r) - Z_o\} \times \left|(X_r - X_o)^2 + (Y_r - Y_o)^2 + [H(X_r, Y_r) - Z_o]^2\right|^{-1/2} \tag{51b}$$

The normal direction at the reflection point is specified by:

$$\vec{N} = \nabla Z - H(X, Y) = \{-\partial H/\partial X, -\partial H/\partial Y, 1\}. \tag{52}$$

The reflection point satisfied the condition that the observer and source directions make the same angle with the normal:

$$\vec{N} \cdot \nabla(SR) = \vec{N} \cdot \nabla(RO). \tag{53}$$

Substituting (51a,b) and (52) in (53) yields the condition:

$$\left|(X_r - X_o)^2 + (Y_r - Y_o)^2 + [H(X_r, Y_r) - Z_o]^2\right|^{-1/2} \times \left\{H(X_r, Y_r) - Z_o + (Y_r - Y_o)\frac{\partial H}{\partial Y_r} - (X_r - X_o)\frac{\partial H}{\partial X_r}\right\} = \left|(X_s - X_r)^2 + (Y_s - Y_r)^2 + [Z_s - H(X_r, Y_r)]^2\right|^{-1/2} \times \left\{Z_s - H(X_r, Y_r) + (Y_s - Y_r)\frac{\partial H}{\partial Y_r} - (X_r - X_s)\frac{\partial H}{\partial X_r}\right\}. \tag{54}$$

This condition is solved for (X_r, Y_r) to specify the reflection points. If they lie along a curve:

$$Y_r = g(X_r), \tag{55}$$

its arc length is:

$$dl = \left|1 + \left(\frac{dg}{dX_r}\right)^2\right|^{1/2} dX_r. \tag{56}$$

It may happen that the reflection points lie in a two-dimensional region of coordinates (ξ, η). the case of reflection points a curve (55) is $\eta = g(\xi)$. The location of the reflection points is then used for wide-area scattering model III in (§1.3) ans its extension to include atmospheric absorption as model VI in (§2.2).

4 Horizontal flight over flat impedance

The methods presented are illustrated by applying models I and V to horizontal flight over flat impedance. Model I is illustrated in Figure 3 by sound sources $S = (x_s, z_s)$ and observer $O =$

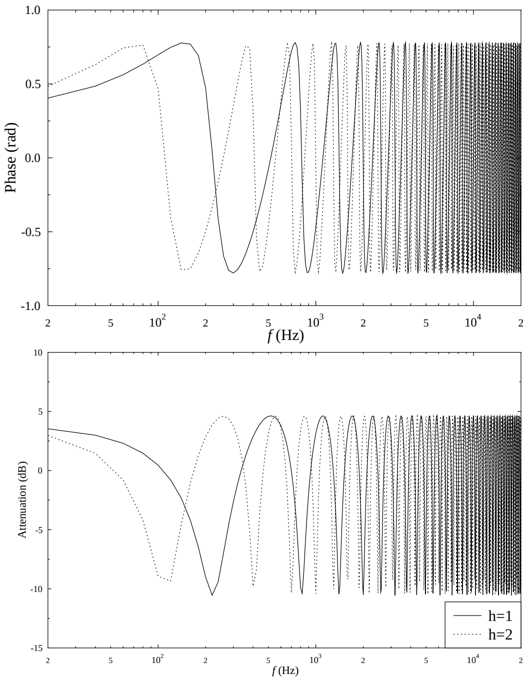

Fig. 3 Sound attenuation and phase shift of acoustic pressure for a ground impedance $R = 0.5 + i\,0.5$, a horizontal distance between source and observer $\Delta x = x_0 - x_s = 100\,$m, a source at a height of $z_s = 30\,$m, as a function of sound frequency.

(x_0, z_0) at arbitrary positions over a flat ground. The sound attenuation (that is, the SPL variation) and the phase shift of acoustic pressure is shown for a dimensionless ground impedance $S = Z/\rho c = 0.5 + \mathrm{i}\, 0.5$, a horizontal distance between source and observer $x = x_0 - x_s = 100\,\mathrm{m}$, a source at a height of $z_s = 30\,\mathrm{m}$, and for two values of the height of the observer $z_0 \equiv h = 1, 2\,\mathrm{m}$. It is clear that the sound attenuation depends on both the observer position and the sound frequency.

Figures 4–6 show the sound attenuation and phase shift as a function of observer height h, source height z, and observer-source distance x, for a sound frequency $f = 1\mathrm{kHz}$ and for hard flat ground $S = 1$.

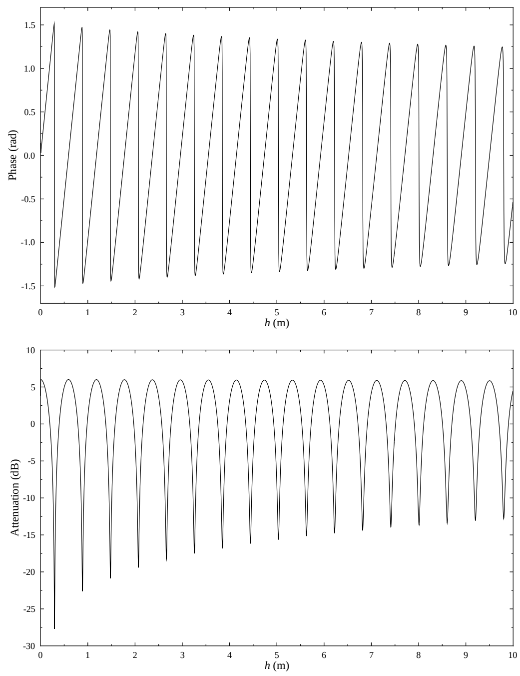

Fig. 4 Sound attenuation and phase shift as a function of observer height h, for hard ground $S = 1$. The sound frequency is $f = 1\mathrm{kHz}$, the observer is at horizontal distance $x = 100\mathrm{m}$ from the source, and the source is at a height $z = 30\mathrm{m}$.

It is interesting to note that there are zones of “silence”, that is, of increased sound attenuation, in each case. They are evenly spaced for changes in the height of the observer position as shown in Figure 4. As the source height increases, Figure 5 shows that the distance between the zones of silence also increases. The same effect is shown in Figure 6: as the distance from the source increases, the distance between the zones of “silence” is greater. In this case these zones are also more pronounced and the sound absorption is greater as x increases.

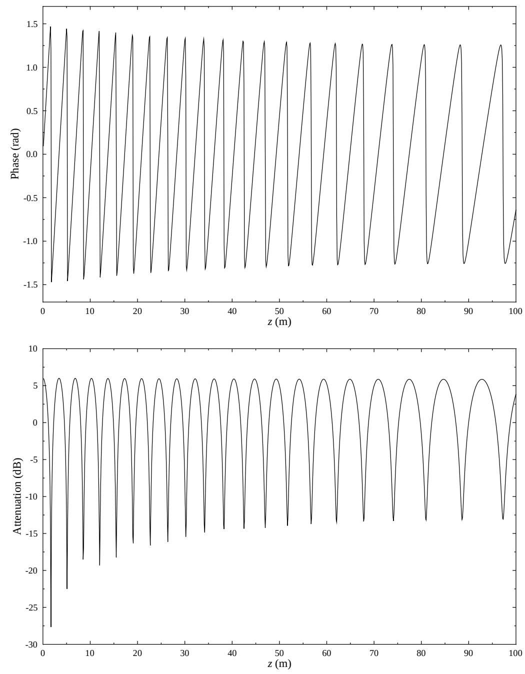

Fig. 5 Sound attenuation and phase shift as a function of source height z, for hard ground $R = 1$. The sound frequency is $f = 1\mathrm{kHz}$, and the observer is at horizontal distance $x = 100\mathrm{m}$ from the source and at a height $h = 5\mathrm{m}$.

Figure 7 shows the additional sound attenuation due to ground effect as a function of atmospheric absorption coefficient per unit length ε. This attenuation is to be added to the attenuation of direct sound due to atmospheric absorption.

Ground effect leads to an additional attenuation that does not strongly depend on the atmospheric absorption coefficient per unit length. Figure 7 shows that this additional attenuation is

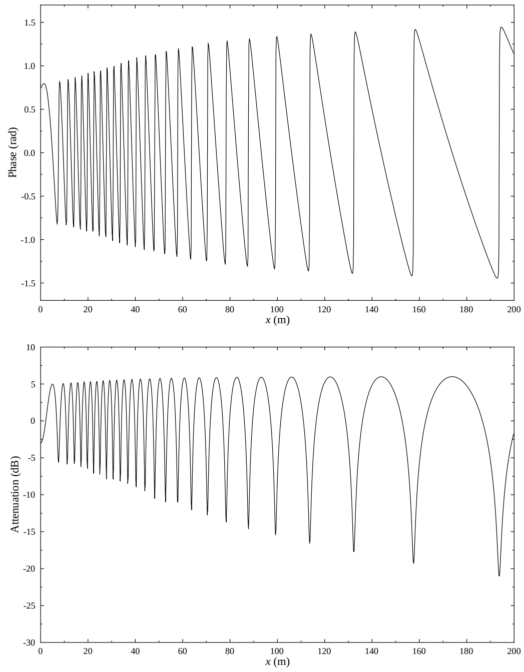

Fig. 6 Sound attenuation and phase shift as a function of observer-source distance x for hard ground $R = 1$. The sound frequency is $f = 1$kHz, the observer is at a height $h = 5$m, and the source is at a height $z = 30$m.

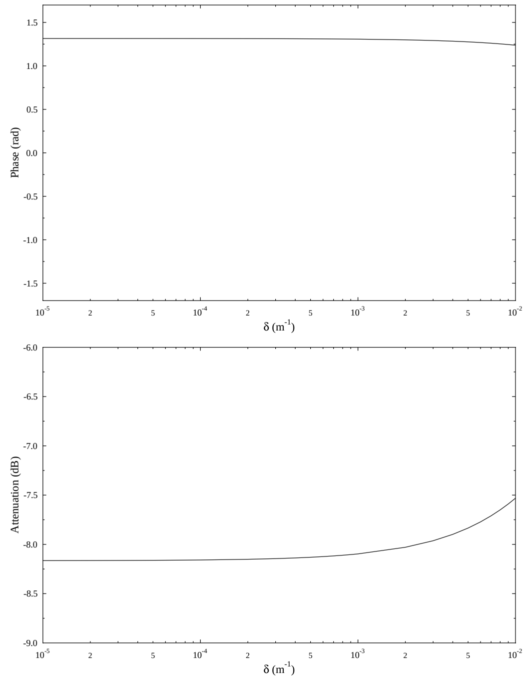

Fig. 7 Sound attenuation and phase shift due to ground effect as a function atmospheric absorption per unit length ε, for hard ground $R = 1$. The sound frequency is $f = 1$ kHz, the observer is at a distance of $x = 100$ m and a height $h = 5$ m, and the source is at a height $z = 30$m.

around -8dB for atmospheric absorption coefficient per unit length in the range 10^{-5}–10^{-2} m^{-1}.

5 Conclusion

The problem of ground effect and atmospheric attenuation on helicopter noise has been addressed by a sequence of six progressively more sophisticated models. The models evolve from (i) a single reflection from flat ground, through (ii) multiple reflection from rough ground and (iii) to wide area scattering from rough ground; atmospheric absorption can be included with (i) uniform or (ii) non-uniform attenuation. The ground may be (i) rigid or have (ii) a uniform impedance or (iii) a reflection coefficient with specified amplitude and phase. Some of these many possibilities were illustrated.

Acknowledgements

This work was supported by project Friendcopter of the EU Aeronautics Program.

Copyright Statement

References

[1] K. M. Li. "A High-frequency approximation of sound propagation in a stratified moving atmosphere above a porous ground surface", *J. Acoust. Soc. Am.* **95**, 1994, pp. 1840-1852.

[2] A. L'Esperance, P. Herzog, G. A. Daigle, J. Nicolas. "Heuristic model for outdoor sound propagation based on an extension of the geometrical ray theory in the case of a linear sound field profile", *Appl. Acoust.* **37**, 1992, pp. 111-139

[3] R. Raspet, S. W. Lee, E. Kuester, D. C. Chang, W. F. Richards, R, Gilbert, N. Bong. "A fast-field program for sound propagation in a layered atmosphere above an impedance ground.", *J. Acoust. Soc. Am.* **77**, 1985, pp. 345-352.

[4] R. Raspet, G Baird, W. Wu. "Normal mode solution for low-frequency sound propagation in a download refracting atmosphere above a complex impedance plane.", *J. Acoust. Soc. Am.* **91**, 1992, pp. 1341-1352.

[5] L. C. Sutherland, G. A. Gaigle. "Atmospheric Sound Propagation", Chap. 28 *in* M. J. Crocker (Ed.) *Handbook of Acoustics*, Wiley, New York, 1998.

[6] E. H. Brown, F. F. Hall, Jr. "Advances in atmospheric acoustics", *Rev. Geophys. Space Phys.* **16**, 1978, pp. 47-110.

[7] J. E. Piercy, T. F. W. Embleton, L. C. Sutherland. "Review of sound propagation in the atmosphere", *J. Acoust. Soc. Am.* **61**, 1977, pp 1403-1418.

[8] M. E. Delany. "Sound propagation in the atmosphere, a historical review", *Proc. Inst. Acoust.* **1**, 1978, pp 32-72.

[9] H. E. Bass, L. C. Sutherland, A. J. Zuckerwar, D. T. Blackstock, D. M. Heste. "Atmospheric absorption of sound: further developmnents", *J. Acoust. Soc. Am.* **97**, 1995, pp 680-683.

[10] K. Attenborough *et al.* "Benchmark cases for outdoor sound propagation", *J. Acoust. Soc. Am.* **97**, 1995, pp 173-191.