EFFECTS OF HIERARCHY ORDER ON NATURAL FREQUENCY AND VIBRATION TRANSMISSION OF SELF-SIMILAR TRUSSES

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Abstract
Hierarchical structures are of interest due to several advantages that they can offer specifically the low mass which is of great interest by the aerospace industry. Hierarchical solids contain structural elements which themselves own a kind of structure. For a hierarchical structure the number of levels of scale with recognized structure is defined as the hierarchy order "HO". The fractals are normally the interesting type of hierarchical structures Many natural structures have fractal geometry. The self-similar fractal is a special case of fractals which is widely used in trade studies of hierarchical structures. One of the main advantages of hierarchical structures, which has been widely investigated and reported, is the buckling response (i.e. both local and global buckling). However, what is of great importance when dealing with space structures is the natural frequency and capability of the design in damping or transmitting the vibration energy. These are the things which are less investigated about the hierarchical structures. In the present study the effects of hierarchy order "HO" on this aspect of hierarchical structures will be studied using a simple model.

1 Introduction

Hierarchical solids contain structural elements which themselves are made up of a structure. The Hierarchical Order (HO) of a structure or a material may be defined as the number of levels of scale with a recognized structure. For example, HO = 0 corresponds to a material viewed as a continuum which is used for the purpose of analysis of physical properties; HO = 1 (first order) could represent a latticework of continuous ribs or the atomic lattice of a crystal. The improved strength and toughness or higher natural frequencies, or unusual physical properties such as a negative Poisson's ratio can be considered as the benefits of hierarchical structures. There are evidences that suggest the continuous increase in levels of hierarchy could lead to lighter and more efficient performing structures [1]. For example, because of more distant distributed mass and therefore more effective distribution of mass, a tube is stronger than a solid rod of the same mass subjected to bending and torsion loadings. As a result, a structure constructed of tubes is stronger than a structure of solid rods of equal mass. In this way, topology optimization procedures often predict highly latticed solutions under different loading constraints and can predict hierarchical structures. But, more constraint equations or filtering techniques could be employed to enforce limits on minimum element size and therefore reduce the latticework level [2].

The Eiffel tower is a popular example of a hierarchical structure. In the Eiffel tower, the lowest hierarchy order building elements (0^{th} order) are rectangular or L-shaped cross-section bars. Where, the trusses with 1^{st} order hierarchy are formed from the mentioned elements to build up the columns. These columns are then tied together to build the legs of the tower which are of the 2^{nd} hierarchy order. Finally, the four legs are tied together to form a tower with 3^{rd} order hierarchy. The resulting structure possesses an unprecedented level of low effective density where the effective density is defined as ρ / ρ_0. Note, ρ is the mass per unit volume of the structure and ρ_0 is the density of the material of which the tower is made of it. Here, ρ / ρ_0 is just 1.2 x 10^{-3} times that of iron [3], which is weaker than the structural steel. The rationale for the use of small girders in such a large structure was attributed to ease of the construction [4], though it has also been suggested by Mandelbrot [5] that Eiffel perceives some other structural advantages. For comparison, we remark that the Pompidou Center (Paris), a first order hierarchy (i.e. HO=1) contains a volume fraction of structural steel [6] with ρ / ρ_0 = 5.7 x 10^{-3}. A more recent example is a proposal by Dyson [7] to construct hierarchical frameworks in outer space. Dyson presented scaling arguments to the effect that very large structures could be constructed with low mass; however no stress analysis was performed.

In deployable beam-like space structures, 1st order hierarchy is most common and is seen in structures built by AEC-Able Engineering [8]. Structures with 2nd order hierarchy are also common in the form of trusses built from tubes [8]. Mikulas et al. in their paper [9] considered 2nd order hierarchy structures for large scale telescope mirrors and offered a simple numerical procedure that illustrates the important trades involving areal density. In these structures, the 1st order hierarchy is a shell structure and the 2nd is a truss. As they have discussed, no existing space structures with 2nd order hierarchy and latticing at all levels are known [9].

From a structural performance perspective, it is important to consider how great the advantages of using hierarchical structures are; however, there are few studies that compare the performance of space structures with hierarchy. Mikulas compared the mass efficiency of several column configurations in his paper on the efficiency of long lightly loaded columns, but he was not specifically looking at hierarchy [10]. In reference [2], Lakes provides a review of the work in this area and cites 57 records. In reference [1], Murphy considered stiffness, bending strength, mass and dimensions of latticed hierarchical structures with respect to order of hierarchy (HO).

In the present study the effects of HO and number of cells at specified hierarchy level are considered over the natural frequencies of the resulting structures constrained to the equal mass.

2 Geometric Modeling

In limit, self-similar hierarchical geometry is led to fractal which is the geometry of nature [5]. In usual engineering, a few order of hierarchy can be possible and also would be useful. There are some methods for generating hierarchical (or fractal) shapes. One of these methods is the iteration process [11] as shown in Fig. 1.

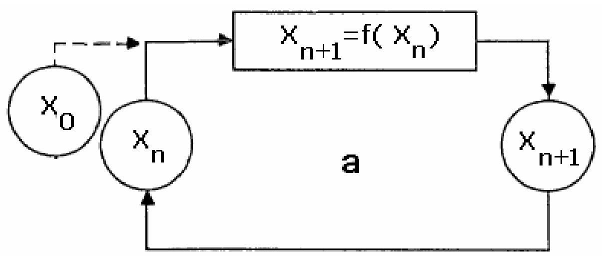

Fig.1. Iteration Process Diagram

Output from previous iteration will be used as an input for the next iteration. Therefore, this process consists of an initial object and a generation function or a set of functions. Fig. 2 shows how the Koch curve is generated with this process. In this example, in each iteration, an edge of current object (initial object is the squire) is replaced by a set of 7 edges called generator. Therefore, a combination of transformations (i.e. scaling, rotation, and translation) is done on a generator to build up the next hierarchy

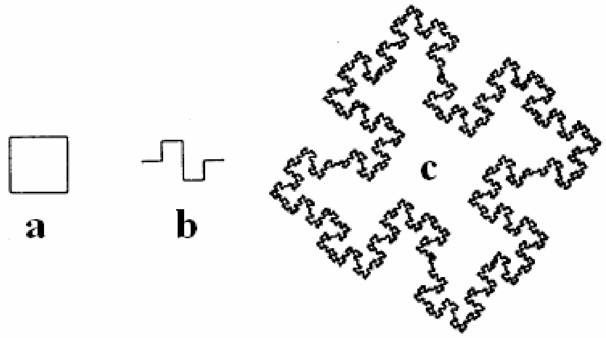

Fig.2. Iteration process example 1; a-initiator, b-generator, c-Koch curve

Fig. 3 shows another example. In this example current object roles as a generator. There is a governing pattern constructed from a set of transformations. In each iteration, governing pattern acts on the last hierarchy. In this example, the pattern role is to scale and reproduce objects of three times of the last hierarchy and place them at appropriate position.

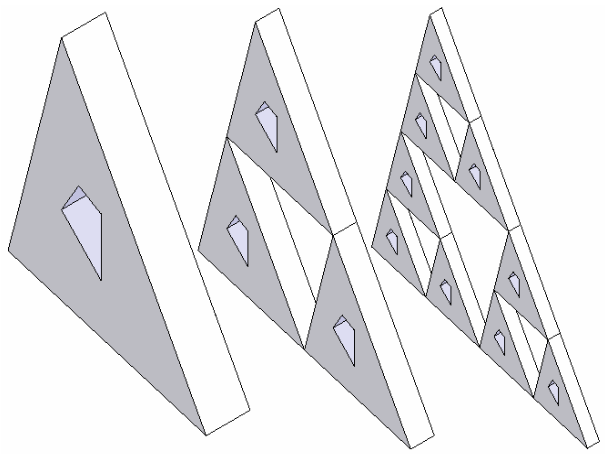
Fig.3. Iteration process example 2

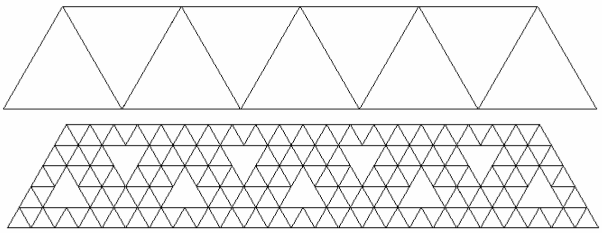
Fig.4. Example of not desirable hierarchical truss

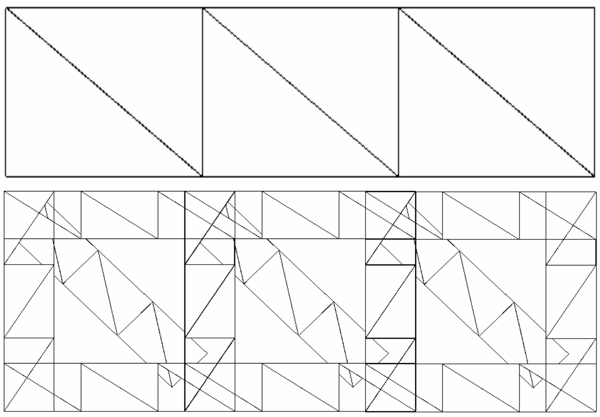
Fig.5. Example of unacceptable hierarchical truss

Figs. 4 and 5 show two undesirable and unacceptable hierarchies. These results appear because line object in the base pattern is replaced with a geometric object which has a thickness. Therefore, the hierarchical truss can not be build up from any pattern and an appropriate modification must be performed in generating a new hierarchy.

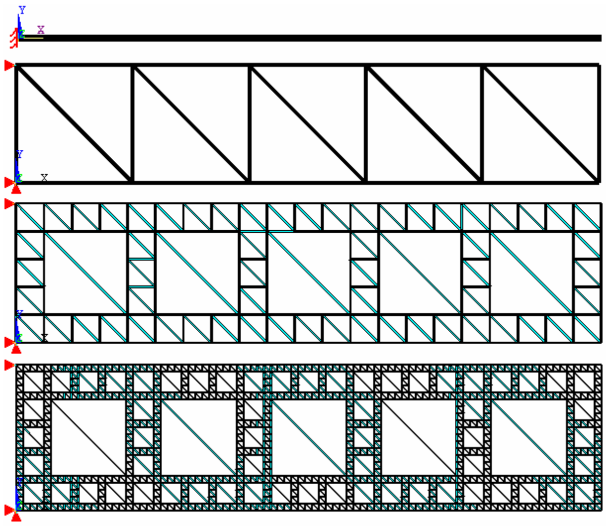
Fig.6. Samples of the intended model of this paper with increasing hierarchy (HO = 0 to 3)

The intended model of this research work is illustrated in Fig.6 with its boundary condition. An iterative process designed for this model is as follows:

1- Back-up base model as 'BM' (base model is the initial model or the result of previous iteration)
2- Reproduce 'BM' with the transformation vector (0, b-d), where 'b' is equal to the length of base model, and 'd' is its thickness. It will generate the upper link.
3- Reproduce 'BM' with mirror and rotate it 90 degree to generate vertical link and Backup the result as 'BackWall'.
4- Reproduce 'BackWall' with transforming it with the vector (b-d, 0).
5- Draw diagonal line between points (b-d, d) and (d, b-d).
6- Select all lines except 'BackWall', then Reproduce selected lines 'n-1' times with the transformation step of (a-d,0), where n is the number of cells in the truss pattern.
7- Select all lines and merge overlapped points and lines.

8- Scale the result with the scaling factor of $s = \dfrac{b}{n(b-d)+d}$. Overall length of the new hierarchy will be the same as the previous one, but its thickness will be 's.b'.

By iterating this process the models of all hierarchies with the intended number of cells can be generated.

3 FEM Analysis

For FEM analysis of the model it is necessary to assign appropriate material, element type, and cross section to the lines. In this paper it is assumed that the material is linear isotropic with the given properties of:

$$E = 10^{11}\, pa,\ \rho = 1000 Kg/m^3,\ \nu = 0.3.$$

From the classical vibrations theory it can be shown that the natural frequency is proportional to $\sqrt{E/\rho}$, see Eqs. (1) and (2);

$$(-\omega^2[M]+[K])\{X\} = \{0\} \qquad (1)$$

Can be rewritten as:

$$(-(\frac{\omega}{\sqrt{E/\rho}})^2[\overline{M}]+[\overline{K}])\{X\} = \{0\} \qquad (2)$$

Where $\overline{K}$ and $\overline{M}$ are independent of E and ρ. Since the Poisson's ratio of engineering materials has a little variation, so it has a minor effect on the results. Therefore, what is concluded here for a sample material; would be extended to other linear isotropic materials.

What is important here is that by fixing the material properties, just the material volume can affect the results. Now by fixing the material volume in all hierarchies, just the effects of the hierarchies' geometry will be seen in the results. In this paper the effect of hierarchical geometry will be investigated for various orders of material volume and the natural frequencies are calculated for the volumes of: 0.0001, 0.001, 0.01, 0.1, and 1 cubic meter. Also it's assumed that the length of all models is equal to 1m, and all cells are square.

Regarding the selected FEM element type, some further information about cross section of the elements is required. Candidate element types here are: 2D link (truss) element and 2D beam element. The link element represents an ideal truss with ideal hinges. However, it can not model the lateral vibration of the truss members. Fig. 7 shows that with this element the obtained results are independent of the amount of material. However, the beam element seems to be more close to what physically happen and therefore this element is selected for the analysis. Here, it is assumed that the elements are of solid rod type.

4 Analysis and Results

The analysis performed here for zero order to fourth order of hierarchies, for the cells 1 to 10 (1 to 6 cells for last hierarchy). Results are given in Table 1. Fig.8 shows the effect of number of cells on natural frequency for HO=1. Also, Fig.9 shows the effect of material volume for HO=1. Figs. 10 and 11 graph the best of each hierarchy in various material volumes in log-log and semi-log scale. Fig. 12 shows optimum number of cell versus material volume for HO=1.

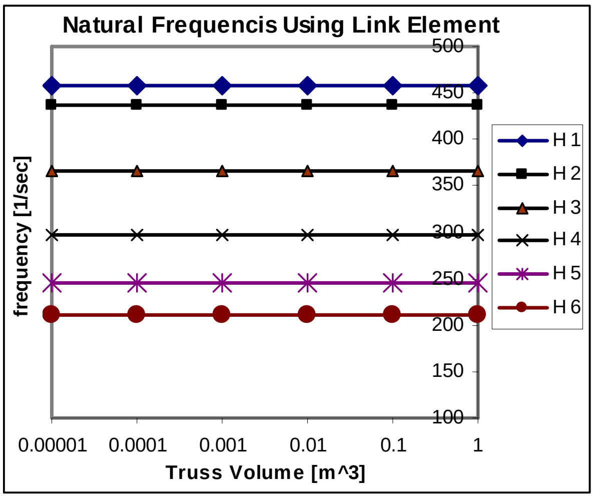

Fig.7. Results of analysis using link element

5 Conclusion

From the results it is seen that at HO=0, there is a linear relationship between the natural frequency and the amount of material volume in log-log scale (Fig. 10). But, for higher hierarchy orders, this relationship remains linear in semi log scale, as well (see Fig. 11). Therefore, there is a point in which superiority switches between zero-order and high-order hierarchies. As material volume decreases, the effect of higher order hierarchies shows itself. In moving from HO=0 to HO=1 in low volumes, a great increase

in natural frequency occurs. However, with more increase in the hierarchy orders i.e. HO=2&3, a little improvement is seen. In HO=4 the frequencies begin to decrease.
In each hierarchy (e.g. Fig. 9) with increasing the material volume, natural frequency increases linearly in log-log scale (similar to what happens in HO=0), until a saturation region is reached. After that, the natural frequency exhibits a little sensitivity to it. Finally, for the investigated model, HO=1 would be a good design for volumes less then 0.01 cubic meter. In low volumes, the optimum number of cells decreases almost linearly with respect to the logarithm of material volume (Fig. 12).

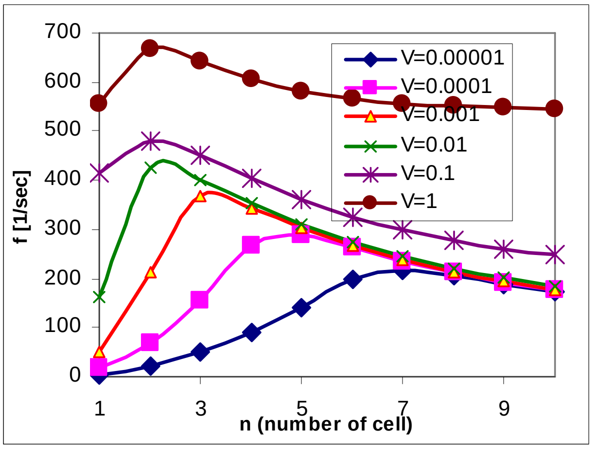

Fig.8- Effect of number of cell on natural frequency for HO=1

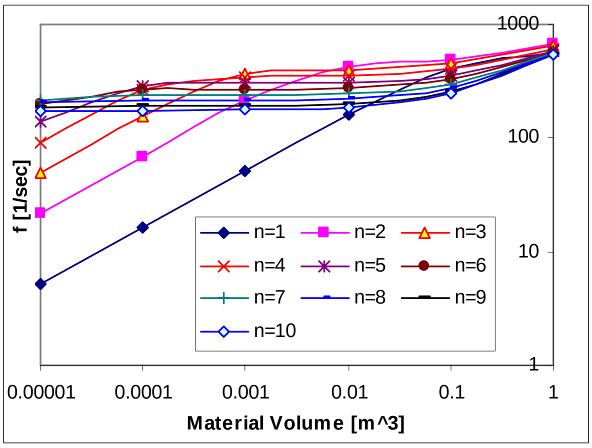

Fig.9- Effect of material volume on natural frequency for HO=1

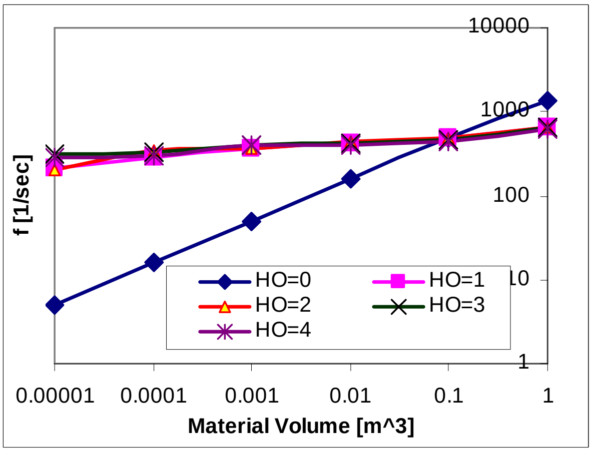

Fig.10- Best of each hierarchy versus material volumes (log-log graph).

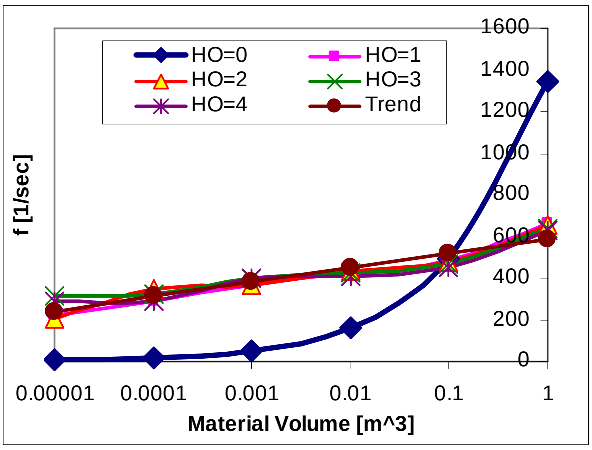

Fig.11- Best of each hierarchy versus material volumes (semi-log graph).

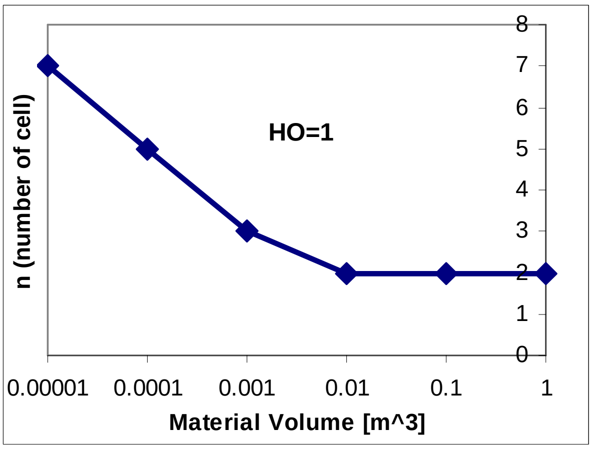

Fig.12- Optimum number of cell versus material volume for HO=1

Table 1 – Results of analysis

	ncell	Volume					
		1E-5	1E-4	0.001	0.01	0.1	1
HO=0	1	4.99	15.79	49.91	157.6	**490.2**	**1345**
HO=1	1	5.19	16.42	51.88	162.3	413.6	556.1
	2	21.63	68.37	214.5	**425.7**	**481**	**666.7**
	3	49.69	156.8	**366.7**	400.6	449.8	642
	4	89.41	267.9	342.2	353.7	403.5	605.8
	5	140.5	**288.1**	302.1	310.9	360.3	581.3
	6	197.4	262.2	266.7	275.1	325.2	566.1
	7	**217.5**	235.2	237.4	245.8	297.9	556.7
	8	205.6	211.7	213.2	221.9	276.8	550.7
	9	189.3	191.9	193.2	202.1	260.4	547
	10	173.9	175.2	176.4	185.7	247.5	544.6
HO=2	1	5.19	16.42	51.88	162.3	413.6	556.1
	2	36.94	116.8	356	**436.3**	**476.5**	**651.5**
	3	171.8	**351.9**	**365.8**	375.3	430	589.9
	4	173.4	295	298.3	311.2	367.6	522.8
	5	198.1	245.9	248.8	264.9	322.3	473.7
	6	**204.7**	210.7	214.3	232.5	293.1	434.7
	7	182.8	184.7	189.2	209.2	271.7	404
	8	163.4	164.6	170.2	192.2	253.5	380.4
	9	147.5	148.8	155.3	179.5	237	362.1
	10	134.4	135.9	143.4	169.6	222	348
HO=3	1	5.19	16.42	51.88	162.3	413.6	556.1
	2	55.28	174.8	**398.3**	**424.1**	**464.1**	**638.2**
	3	**316.6**	**325**	327.3	341.2	412.1	570.8
	4	172.4	250.2	254.5	280.1	357	495.2
	5	154.6	201.2	209.1	241.1	320.3	429.2
	6	162.4	170.5	182	219.9	289.3	380.2
	7	145.8	150.1	165.1	206.2	263	343
	8	130.2	135.8	154.8	193.4	241.8	313.9
	9	118.2	125.4	148	181	224.2	290.6
	10	108.7	117.8	142.2	169.7	209.1	271.4
HO=4	1	5.19	16.42	51.88	162.3	413.6	556.1
	2	76.13	240.7	**397.1**	**410.4**	**452.3**	**626**
	3	**292**	**292.8**	295.7	318.1	404.6	560.6
	4	130.4	214.4	224.6	266.9	360	471.6
	5	101.6	168.2	186.3	236.6	310.3	395.8
	6	106.7	143.7	168.6	213.3	270.7	344.5

6 Reference

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