AEROSPACE TRAJECTORY OPTIMIZATION: NOVEL VIEW ON A ROLE OF ATMOSPHERIC FLIGHT

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Abstract

The problem of launcher trajectory optimization by criterion of the injected-into-orbit payload mass is considered. The fundamental features of optimal solutions caused by the presence of aerodynamic forces are investigated on the basis of the rigorous approach using the maximum principle. The extremals behavior including the qualitative transitions is analyzed at variation of launcher aerodynamic configuration in view of constraints on a g-load and pitch trim.

Nomenclature

Symbols:

C_D	aerodynamic drag coefficient
C_{D0}	zero-lift drag coefficient
C_L	aerodynamic lift coefficient
C_L^{α}	$= \partial C_L / \partial \alpha$
C_m	pitching moment coefficient
$C_m^{C_L}$	$= \partial C_m / \partial C_L$
C_m^{α}	$= \partial C_m / \partial \alpha$
F_0	reference cross section area
g	gravity acceleration
h	altitude
L/D	lift-to-drag ratio
$L/D_{\max}$	$= \max\limits_{\alpha}(L/D)$
M	Mach number
m	vehicle mass
n_z	transverse loading
q	dynamic pressure
T	thrust value
$\mathbf{T}$	thrust vector
t	time
V	velocity value
$\mathbf{V}$	velocity vector
α	angle of attack
γ	path angle
θ	pitch angle

Subscripts:

$(\ )_{adm}$	admissible limit
$(\ )_f$	at the final point
$(\ )_i$	at the initial point
$(\ )_{\max}$	maximum value
$(\ )_{opt}$	optimal value

1 Introduction

Aerospace vehicles such as space and sub-orbital transportation systems, expendable or reusable ones, start in dense atmospheric layers. Traditionally, they are divided into two segregated conventional types: “rockets” and “airplanes”.

From flight mechanics point-of-view the vehicles differ in attitude to a role of the atmosphere. It is traditionally considered that the atmosphere for “rocket” type vehicles is exclusively a source of a backpressure at main engines, a drag and transverse loadings, which should be minimized. These reasons result in selection of conventional launcher layout and relevant nominal ascent trajectories that are characterized by the vertical start, near-zero angles of attack in dense atmospheric layers and quasilinear optimal pitch control programs. “Airplane” type vehicles, on the contrary, lean (in a direct and figurative sense) on the atmosphere. Their trajectories do not hurry to leave dense atmospheric layers, and the optimal pitch control laws have an oscillating structure.

Such division of vehicles into “rockets” and “airplanes”, although it is traditional, is dictated by departmental reasons, rather than scientific ones. Thus, a priori suppositions about the physical boundary between these vehicle types based on a “traditional” view can lead to a false choice of the control program structure and distort essentially an impartial vehicle efficiency assessment.

In the paper authors conduct the unbiased investigation of optimal launcher trajectories using a rigorous method of indirect optimization - the Pontryagin maximum principle.

The class of conventional “rocket” layouts with side-mounted boosters is considered. Their aerodynamic lift capabilities are smaller than the weight. Nevertheless, even such rocket layouts are able to reach a rather high aerodynamic lift-to-drag ratio L/D at “flat” arrangement of side-mounted boosters. According to previous results [3, 5] it is the value of L/D_{max} that characterizes the structure of the launcher optimal control law in the atmosphere. Excess of critical value of 1.5 – 2.0 leads to a qualitative reconstruction of the optimal pitch program from the classical quasilinear law [1, 2], obtained by Okhotsimskii-Eneev-Lawden for optimal rocket flight in the in-plane uniform gravity field without atmosphere, to oscillating ones.

As “airplane” extremals lie in more dense atmospheric layers it is of importance to investigate the ability to solve effectively the problems of aerodynamic loads and moments increase.

2 Qualitative features of optimal control laws for spacecraft injection into a low Earth orbit

The maximization problem of payload mass injection into a low orbit has well-known theoretical approximate solutions [1, 2] determining the optimal program of thrust vector orientation, which cause the pitch angle to be linear in time.

As shown in [3, 5] the strategy of the optimal trajectory control is determined by the correlation of the thrust **(T)**, aerodynamic **(A),** inertial **(C)** and gravitational **(G)** forces. If the thrust dominates, the optimal control law is qualitatively in conformity with the traditional one [1, 2] obtained for uniform gravitational field under an assumption of the negligibility of aerodynamic forces. However, if

$|\mathbf{T}| >> |\mathbf{A}|$ and $|\mathbf{G}+\mathbf{C}| >> |\mathbf{A}|$, but

$$|\mathbf{T}+\mathbf{G}+\mathbf{C}| \sim |\mathbf{A}|, \qquad (1)$$

the effect of aerodynamic forces can change the structure of the optimal control law and generate multiplicity of extremals.

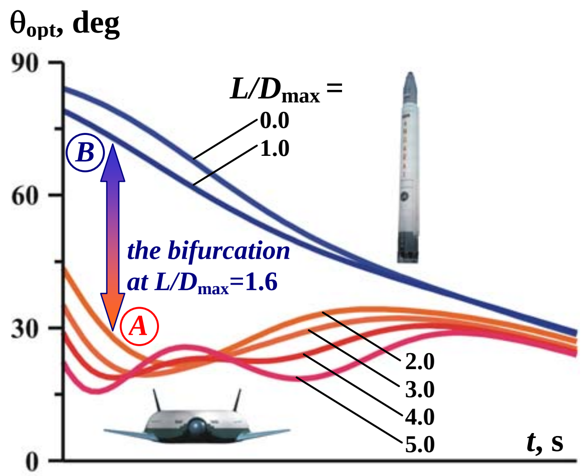

Fig. 1. Optimal pitch programs for launchers with various L/D_{max}.

According to the classification [3-5] possible extremals in the problem of optimal injection of spacecraft in the Earth atmosphere into low orbits can have one of three different types: *B* – ballistic, *A* – aerodynamic, *M* - intermediate.

***B*-type (“Ballistic”) extremals**:

- the optimal pitch angle programs are quasilinear (Fig. 1) to correspond the well-known “traditional” solutions [1, 2];
- the aerodynamic forces influence weakly on the optimal control law structure;
- the optimal start is nearly vertical;
- the atmosphere is “perceived” only as a medium with some drag;
- they are typical to be used in guidance algorithms for current space ballistic launchers;
- they provide the global optimum at low lift-to-drag ratios.

***A*-type (“Aerodynamic”) extremals:**

- the optimal pitch angle program during the atmospheric flight has a pronounced oscillatory nature (Fig. 1);
- an inclined and quasihorizontal start is optimal;

- the atmosphere is mainly perceived as a medium that produces a lift; the optimal trajectories pass into regions with higher dynamic pressures as compared with the *B*-type extremals;
- they provide the global optimum at lift-to-drag ratios that greater than some critical value ~ 1.5–2.0.

M-type ("interMediate") extremals:
- they do not provide a global optimum.

In this paper the attention is paid to the application of outcomes [4, 5] for a class of vertically started launchers with constraints on aerodynamic loadings and the condition of pitch trim being regarded.

3 The trajectory optimization

The launcher mass centre motion is considered in the coordinate system fixed to the start point:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},\mathbf{u},t), \mathbf{f} = \{\mathbf{V},\ \mathbf{T}/m + \mathbf{A}/m + \mathbf{g} + \mathbf{\Omega},\ -\mu\}^{\mathrm{T}}, \quad (2)$$

where $\mathbf{x}=\{\mathbf{r},\ \mathbf{V},\ m\}^{\mathrm{T}}$ is the state vector, $\mathbf{r}$ is the radius-vector, $\mathbf{u}$ is the control vector, $\mathbf{A}$ is the vector of aerodynamic forces, $\mathbf{g}$ is the gravitational acceleration vector, $\mathbf{\Omega}$ is the acceleration vector due to coordinate system noninertiality, μ is the mass flow rate.

The vector of aerodynamic forces is written in the form [5]:

$$\mathbf{A} = qF_0\Big(C_L^{\alpha}\mathbf{e}_{\tau} - \big(D_0 + \big(C_L^{\alpha} + D_{\alpha}\big)(\mathbf{e}_{\tau},\mathbf{e}_{\mathrm{v}})\big)\mathbf{e}_{\mathrm{v}}\Big), \quad (3)$$

where $\mathbf{e}_{\tau}$ is the unit vector directed along the vehicle's longitudinal axis, $\mathbf{e}_{\mathrm{v}}$ is the velocity unit vector. The following form of aerodynamic coefficients is used [5]:

$$C_L = C_L^{\alpha}\sin\alpha,\quad C_D = D_0 + D_{\alpha}\cos\alpha, \qquad (4)$$
$$C_m = C_m^{\alpha}\sin\alpha,$$

that is in accordance with the square aerodynamic polar

$$C_L \cong C_L^{\alpha}\alpha,\quad C_D \cong C_{D0} + k\alpha^2, \qquad (5)$$
$$D_0 = C_{D0} + 2k,\quad D_{\alpha} = -2k.$$

at a small angle of attack.

The maximum lift-to-drag ratio $L/D_{\max}$ is defined by formula

$$L/D_{\max} = \frac{C_L^{\alpha}}{\sqrt{C_{D0}(C_{D0}+4k)}}.$$

Coefficients C_{D0} and k remain constant under $L/D_{\max}$ variations while C_L^{α} changes in accordance with equality:

$$C_L^{\alpha} = L/D_{\max}\sqrt{C_{D0}(C_{D0}+4k)}.$$

The thrust is constrained by minimum and maximum values:

$$T_{\min} \le T \le T_{\max}.$$

Thrust orientation is set as follows:

1) Without taking into account the pitch trim the thrust vector is directed along the launcher longitudinal axis:

   $$\mathbf{T} = T\mathbf{e}_{\tau}.$$

2) In view of aerodynamic moment the thrust vector deflects from longitudinal axis in pitch plane at the angle δ that is determined by the condition of pitch trim:

   $$Tl_T\sin\delta = \frac{1}{2}\rho V^2 F_0 l_0 C_m^{\alpha}\sin\alpha, \qquad (6)$$

where: l_T is the distance between the thrust application point and launcher nose;
l_0 is the reference length;
C_m is the pitching moment coefficient relative to vehicle mass center.

Launcher parameters and boundary conditions are set up in Appendix A.

The initial vertical climb leg ($\gamma = 90°$, $\theta = 90°$) is ended reaching the velocity 30 m/s. After the vertical climb a short duration turn takes place with a fixed angle of attack α_{v} (here $\alpha_{\mathrm{v}} = -5°$). The turn is finished when a current orientation of the thrust vector coincides with optimal one. The subsequent flight is optimal.

The optimization problem is to find the admissible control

$$\mathbf{u} = \{\mathbf{e}_{\tau}, \mathbf{T}\}^{\mathrm{T}} \in \mathbf{U}$$

to transfer the vehicle from the initial point to a specified Earth orbit with the minimum mass consumption that corresponds to maximization of the final vehicle mass:

$$\Phi \equiv m_f \Rightarrow \max_{\mathbf{u}\in\mathbf{U}}. \qquad (7)$$

A number of constraints are taken into account. They depend on a purpose of the investigation. In particular there are taken into account:

1) The constraint on the angle of thrust vector deviation from the launcher longitudinal axis:

$$|\delta| \le \delta_{adm}. \tag{8}$$

Using the pitch trim condition (6) this constraint reduces to the angle-of-attack constraint:

$$\sin|\alpha| \le \left|\frac{2Tl_T \sin\delta_{adm}}{\rho V^2 F_0 l_0 C_m^{\alpha}}\right|.$$

2) The constraint on the transverse g-load

$$\max_t \left|\frac{N}{mg}\right| < n_{z\,adm}\,, \tag{9}$$

where N is the resultant force acting transversally to the launcher longitudinal axis.

According to the Pontryagin maximum principle [6], the optimal control is found from the condition:

$$\{\mathbf{e}_\tau, \mathbf{T}\}_{opt} = arg\max \mathcal{H}\,,$$

where

$$\mathcal{H} = \mathbf{\Psi}^{\mathrm{T}}\mathbf{f}$$

is the Hamiltonian. The adjoint vector $\mathbf{\Psi}$ satisfies the equation

$$\dot{\mathbf{\Psi}} = -\left(\frac{\partial \mathcal{H}}{\partial \mathbf{x}}\right)^{\mathrm{T}} \tag{10}$$

and transversality conditions. Thus, the reference optimization problem reduces to a multipoint boundary-value problem (BVP) for equations (2), (10).

The numerical solution is found using the ASTER package [7]. It realizes the practically regular procedure of the BVP solution due to application of the modified Newton method, parameter continuation method and local extremal selection [7].

4 Optimal control laws in aerodynamic lift absence

Let's consider the optimal injection trajectories (see Fig. 2) in two imaginary situations when a launcher does not have aerodynamic lift capabilities:

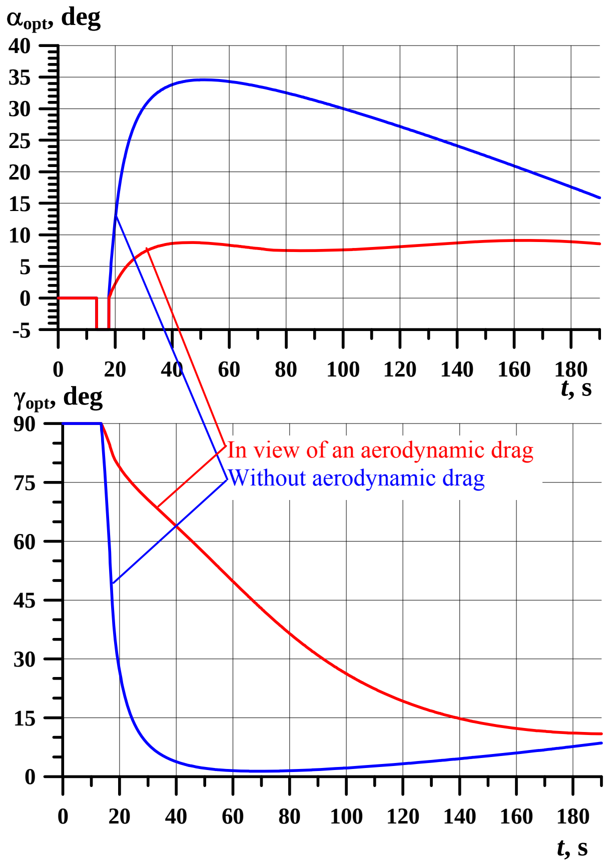

Fig. 2. Optimal time-relationships of the angle of attack and trajectory angle in view of an aerodynamic drag and without it.

1. In view of an aerodynamic drag.
2. Without aerodynamic drag.

Launcher characteristics correspond to data of Tabs. A.1 and A.2 at $T_i/m_i g_i = 1.2$. To eliminate the effect of the aerodynamic lift it was supposed $L/D_{\max} = 0$.

As it is seen in Fig. 2 optimal angles of attack are non-zero even in absence of an aerodynamic lift.

5 The aerodynamic lift effect on the injected mass

To investigate the aerodynamic lift effect on the optimal control laws and the injected mass let's introduce the specific criterion as a relative injected mass $\bar{m}_f$. This criterion equals to the ratio of the injected mass at the optimal through trajectory to the injected mass at the optimal trajectory with the gravitational turn at the first launcher stage.

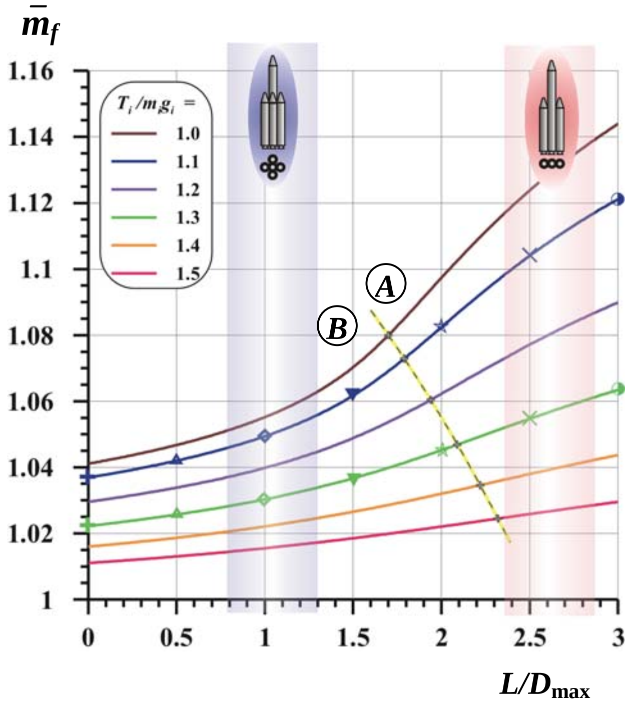

Fig. 3. Dependencies of the relative optimal injected mass on the maximum lift-to-drag ratio at various thrust-to-weight ratios

Variations of the launcher lift properties are modeled by changing L/D_{max} with C_{D0} and k remaining constant (see Eqs. (3) - (5)).

The initial thrust-to-weight ratio is varied over the range $1.0 \leq T_i/m_i g_i \leq 1.5$ at the cost of the initial mass at constant initial thrust.

According to Fig.3 the injected mass grows due to an increase in aerodynamic lift. It could be explained by the following physical reasons:

1. At fixed angle of attack the presence of the lift allows to change a direction of a resultant force and approach it to the optimal one.
2. The lift allows compensating partially the gravity. Thus, gravitational losses of the characteristic velocity decrease, but they exceed characteristic velocity losses on the aerodynamic drag in order.
3. The aerodynamic drag increment is a higher-order value as compared to the lift at a small angle of attack.

The effect of the optimal use of launcher carrying properties increases at reduction of $T_i/m_i g_i$ that is connected with increasing a share of aerodynamic forces in the resultant of forces (1) acting on the launcher.

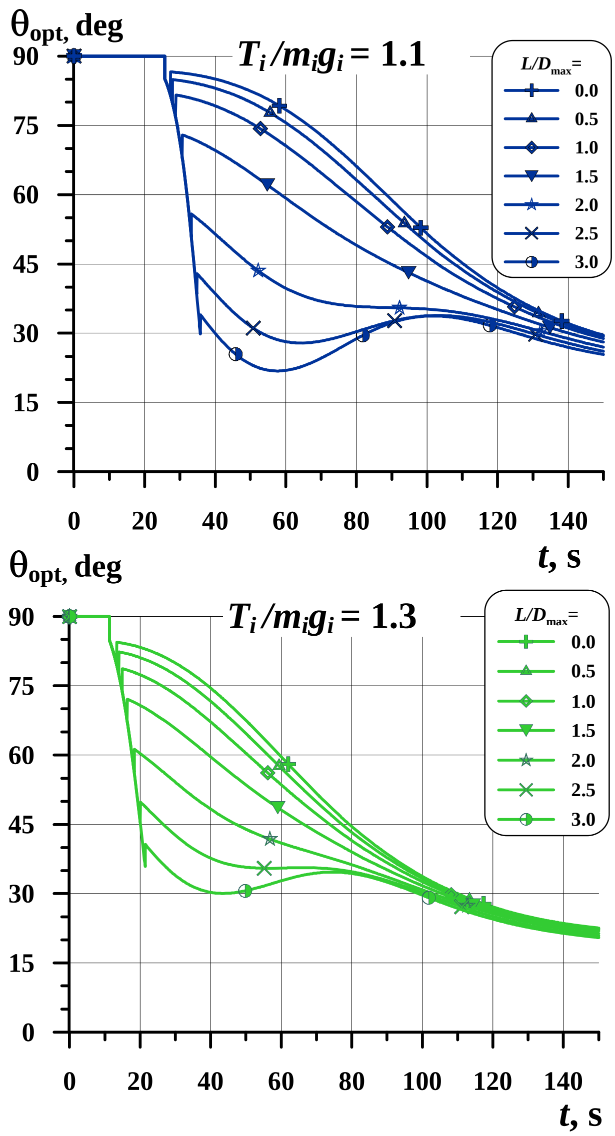

Fig. 4. Optimal time-relationships of the pitch angle at various L/D_{max}.

Time-relationships of the pitch angle on the initial segment of the optimal trajectory for various L/D_{max} are given in Fig. 4. At increase in L/D_{max} these relationships alter, passing from quasilinear that are typical for *B*-type extremals to oscillatory, typical for type *A*.

The boundary line that divides conditionally the areas in which optimal control laws belong to *A* and *B* types is shown in Fig. 3. Let's show this boundary line lies in range of L/D_{max} values implemented for typical "rocket" layouts. We shall consider the launcher with four and two side-mounted boosters. Their aerodynamic characteristics are borrowed from the monograph [8] by K.P. Petrov, a known TsAGI scientist in the field of an experimental investigation

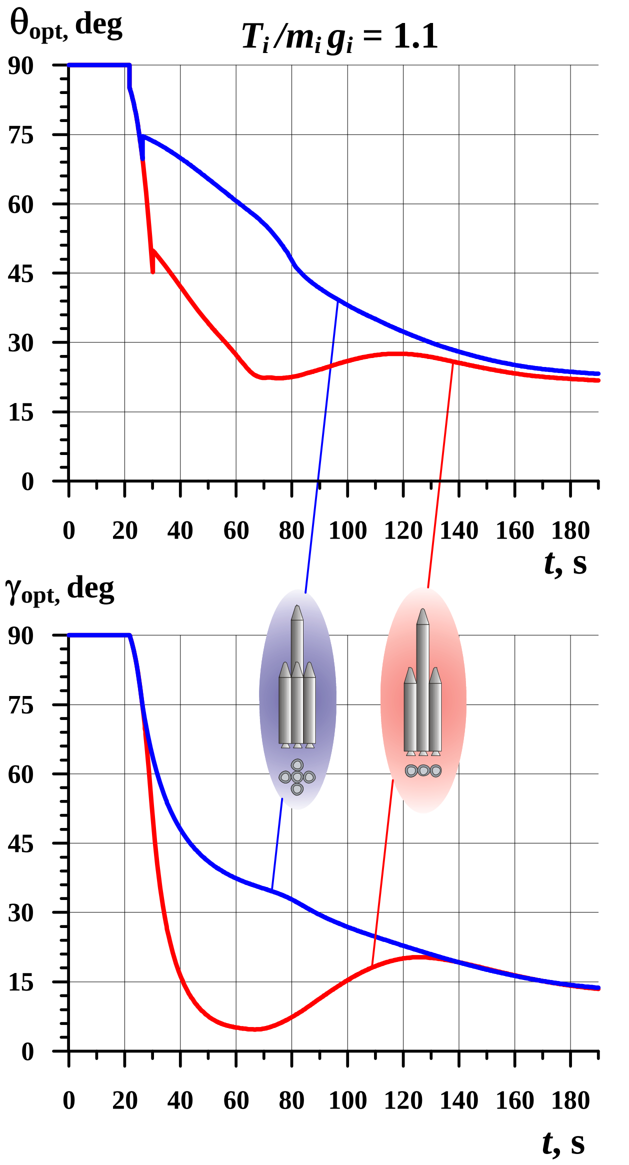

Fig. 5. Optimal time-relationships of pitch and trajectory angles for launchers with four and two side-mounted boosters.

of aerodynamic characteristics of rockets. The corresponding data are presented in Appendix A.

For the launcher with four side-mounted boosters averaged maximum lift-to-drag ratio (depending on the initial thrust-to-weight ratio) lies in the range $0.8 < L/D^*_{max} < 1.3$. The similar range for the launcher with two side-mounted boosters is $2.4 < L/D^*_{max} < 2.9$. In Fig.3 these ranges shown by color bands. Here the averaged maximum lift-to-drag ratio denote such constant on Mach numbers L/D_{max} value, at which the criterion $\bar{m}_f$ is the same as at variable aerodynamic characteristics on Mach numbers according to [8].

Time-relationships of pitch and trajectory angles for optimal control program of these launchers are presented in Fig. 5. One can see that structures of these relations for the launcher with four side-mounted boosters correspond to *B*-type extremals, and for the launcher with two side-mounted boosters do to *A*-type extremals.

Thus, variations of layout parameters inside the conventional ballistic launcher class can result in qualitative change of optimal control laws.

6 The aerodynamic moment effect

An aerodynamic pitching moment, arising from the use of non-zero angles of attack at the ascent trajectory, is usually compensated by deviation of the main engine thrust vector. For typical “rocket” layouts at $\alpha > 0$ the thrust vector should deflect from the launcher long axis making an upward component. Thus, the thrust vector is closing to the “ideal” optimal direction (see Section 4) that increases the launcher mass efficiency.

Let us test this supposition by the example in view of the constraint on the angle δ of thrust vector deviation (8). Take for distinctness

$$|\delta| \le 5°. \qquad (11)$$

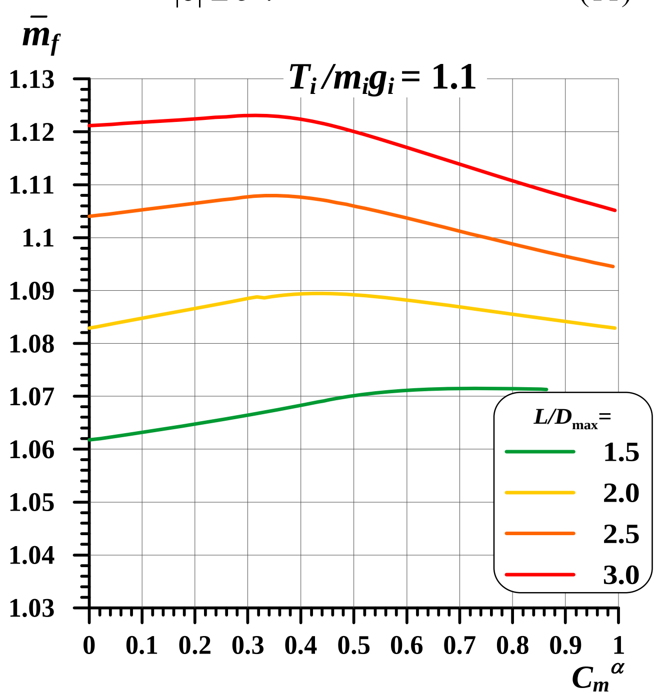

Fig. 6. The dependencies of $\bar{m}_f$ on the C_m^{α} (derivative of aerodynamic moment due to angle of attack) for various values of L/D_{max}.

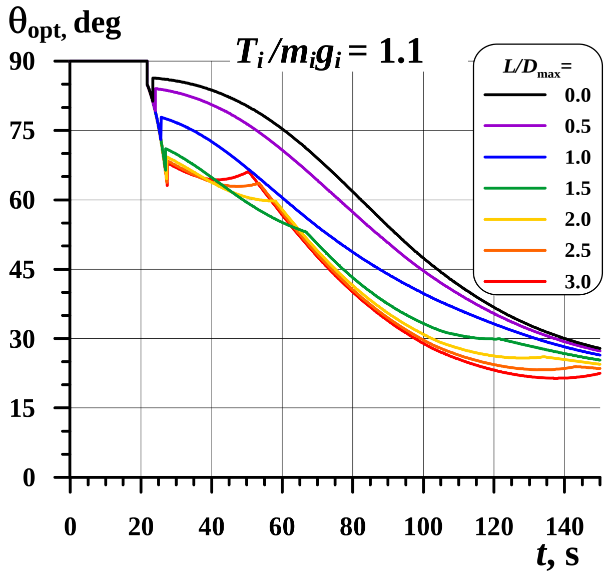

Fig. 7. Optimal time-relationships of the pitch angle at various $L/D_{\max}$ in view of the constraint (11).

The relationships between $\bar{m}_f$ and $C_m{}^{\alpha}$ for various values of $L/D_{\max}$ are shown in Fig. 6. While $L/D_{\max}$ is varying the derivative $C_m{}^{\alpha}$ is assigned by the condition that $C_m^{C_L}$ is fixed (here, $C_m^{C_L} = 0.24$).

Note, that the dependency $\bar{m}_f(C_m{}^{\alpha})$ has the maximum. The existence of the maximum point $C_m{}^{\alpha}{}_{\text{opt}}$ means that the aerodynamic moment (in certain limits) can help to improve the mass efficiency of launchers.

All the maximum points $C_m{}^{\alpha}{}_{\text{opt}}$ correspond to values at which constraint (11) is in force. The optimal pitch angle time-programs taking into account the constraint (11) are shown in Fig. 7.

7 The transverse load constraint effect

The optimal ascent trajectory lies in denser atmospheric layers then the optimal gravitational turn. How the admissible transverse g-load $n_{z\text{ adm}}$ effects typically on the relative inserted mass is shown in Fig.8 (the calculations are fulfilled with the thrust vector angle constraint (11)). The bolded points correspond to the maximal transverse g-load $n_{z\max}$, which is reached at the optimal trajectory, calculated without the transverse g-load constraint. Evi-

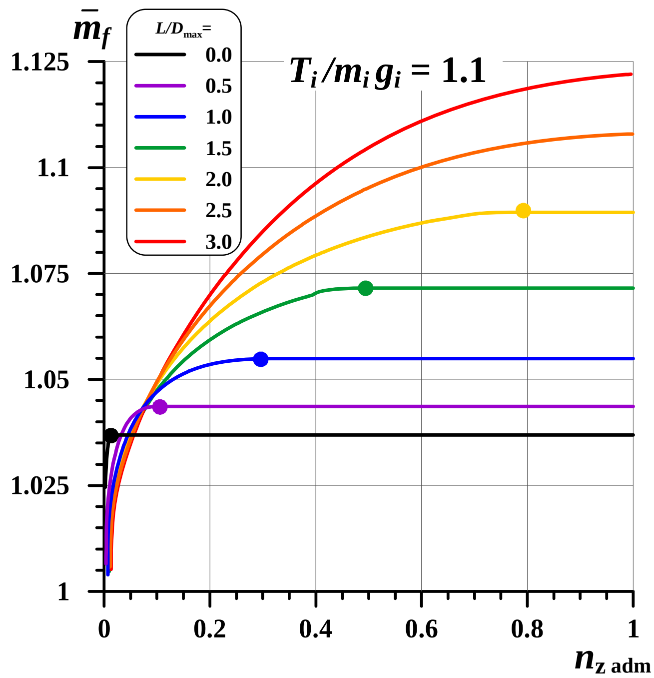

Fig. 8. The dependencies of $\bar{m}_f$ on the $n_{z\text{ adm}}$ for various values of $L/D_{\max}$.

dently the increase of $n_{z\text{ adm}}$ above $n_{z\max}$ does not cause any change of $\bar{m}_f$. The results show that the optimization of the atmospheric ascent leads to a noticeable gain in the functional even under strong constraints on the admissible transverse aerodynamic g-load.

The considered range of $n_{z\text{ adm}}$ can be conventionally divided on two parts. The $L/D_{\max}$ rise provides the payload increase at $n_{z\text{ adm}} > \tilde{n}_{z\text{ adm}} \approx 0.1$ and the payload decrease at $n_{z\text{ adm}} < \tilde{n}_{z\text{ adm}}$.

Conclusions

The use of the rigorous method of the trajectory optimization based on the Pontryagin maximum principle makes it possible to provide efficiently the control law optimization and reveal the qualitatively new solutions. The dependency of the maximum mass injected into a low Earth orbit on launcher parameters can have essentially nonlinear character. The nature of such behavior is stipulated by a possibility of changing the optimal control law structure at launcher parameters variation. Ballistic launchers with two lateral boosters such as “Delta IV”, “Atlas V”, “Ariane 5” have large enough lift-to-drag ratio that can result in the qualitative change of optimal control laws at dense atmospheric layers that makes them similar to typical laws for aircraft.

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Appendix A

Task parameters

The three-stage launcher with four side-mounted boosters (Table A.1) is chosen as primary.

Table A.1. Characteristics of launcher with four side-mounted boosters

Ratio of vehicle length to diameter of central booster	7
Ratio of diameter of side-mounted boosters to diameter of central booster	0.4
Ratio of length of side-mounted boosters to total vehicle length	0.525
Ratio of nose conical section length of central and side-mounted boosters to diameter of the corresponding booster	2.5
Reference area F_0	33m^2
The distance between the vehicle mass center and launcher nose divided by the reference length	2/3
Specific mass flow rate	$\mu g_i / T_i = 3.0 \cdot 10^{-3}$ c^{-1}

Aerodynamic characteristics of the launcher are determined based on data obtained for this layout in [8, p. 103]. At the parametric analysis the following average values are used:

Table A.2. Characteristics of the launcher at the parametric analysis

C_{D0}	0.31
k	4.55
$L/D_{\max}$	0.0 – 3.0
T_i/m_i	1.0 – 1.5

Variation of the initial thrust-to-weight ratio $T_i/m_i g_i$ is performed by the change of the initial mass. Characteristics of the launcher with two side-mounted boosters are presented in Table A.3.

Table A.3. Characteristics of the launcher with two side-mounted boosters

Ratio of vehicle length to diameter of central booster	10
Ratio of diameter of side-mounted boosters to diameter of central booster	1
Ratio of length of side-mounted boosters to total vehicle length	0.7
Ratio of nose conical section length of central and side-mounted boosters to diameter of the corresponding booster	1.54
Reference area F_0	20m^2
Distance between vehicle mass center and launcher nose divided by reference length	2/3
Specific mass flow rate	$\mu g_i / T_i = 3.0 \cdot 10^{-3}$ c^{-1}

Aerodynamic characteristics of launcher with two side-mounted boosters are determined based on the data obtained for this layout in [8, p. 93].

Boundary conditions are presented in Table A.4.

Table A.4. Boundary conditions

Initial conditions	$V_i = 0$ $h_i = 0$ $\gamma = 90°$ $\theta = 90°$
Final orbit	circular, $h_{orb} = 200$ km

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