AERO-STRUCTURAL MDO OF A SPAR-TANK-TYPE WING FOR AIRCRAFT CONCEPTUAL DESIGN

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Abstract

A study on a multidisciplinary design optimization tool for aircraft conceptual design is proposed. The wing of a conceptual aircraft is the baseline of this work. A tubular spar instead of a typical wing box structure is used to support the loads and to work as fuel tank. The aerodynamic analysis uses the Weissinger method, classic closed thin walled tubes stress analysis is employed to the structural evaluation and as the optimization code, a genetic algorithm was implemented. The aim of this study is to optimize the aerodynamics of the wing, minimize its structural weight and maximize the fuel volume, considering four basic flight conditions that are take off, climb, cruise and loiter.

1 Introduction

In order to raise the flight time or the range of an aircraft it is a common practice to install auxiliary fuel tanks, such as tip tanks, conformal tanks in the fuselage or under wing tanks in military aircraft. Under wing tanks have the advantage that they can be released after fuel is consumed, but their drag is generally grater than that of the conformal tanks and it uses a space that could be used for weapons (in military aircrafts). On the other hand, conformal tanks has a smaller signature in radar, but cannot be removed in flight which means more weight and drag after the fuel is consumed. Finally, tip tanks concentrates the load at the tip of the wing, what makes the structure heavier. The best choice for placing the fuel tank is inside the wing, as it is usually done, but considering it as a separate structure also makes the structure heavier.

Another important subject is the aero-structural optimization of wings, which has been treated successfully in several researches [1, 2, 3]. Although there are many efforts in MDO, very few of them are treated at the conceptual design stage[4]. The idea behind this work was born by adding aero-structural and fuel volume optimization at the conceptual design.

It is proposed a tubular concept of tank for the wing of a conceptual primary trainer aircraft, which also works as spar, as shown in Figure 1. This work consists on the optimization of the wing aerodynamics, the structural weight, the fuel volume of this spar-tank, considering its static strength as a constraint and four flight conditions (take off, climb, cruise and loiter) for the aerodynamic analyses. The aerodynamic variables are aspect ratio, taper ratio, twist and sweep angles for a given wing area. The flight conditions are defined by angle of attack and the mach number for the analyses performed by a code based on the Weissinger method [5].

The structural analysis is conducted with the matrix method [6]. The variables considered are the thicknesses of the spar sheets and the cross sections dimensions (Figure 2), all defined at the root and at the tip of the wing, fitted inside the aerodynamic shape. The internal volume of this spar is considered as the fuel tank, so that the fuel volume is defined after these variables. The structural weight is also calculated based on these variables.

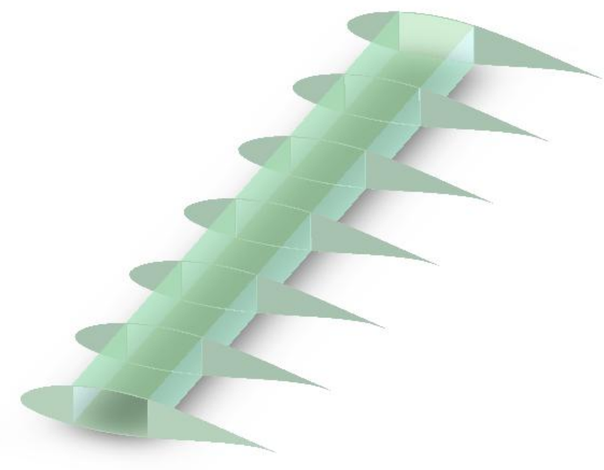

Fig. 1 Proposed spar-tank configuration.

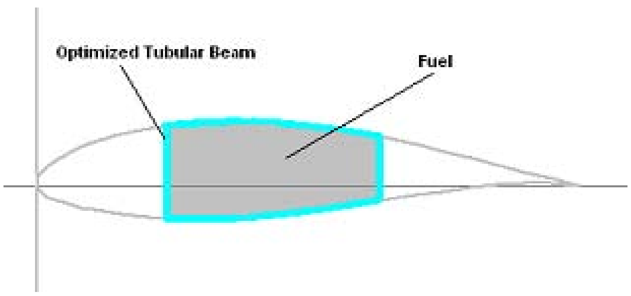

Fig. 2 Cross section of the wing.

A genetic algorithm (GA) code is implemented for the optimization routine [7]. The GA optimization parameters are chosen after a statistical analysis to guarantee good evolution of the individuals.

This multidisciplinary optimization scheme is expected to find the best wing for the trainer aircraft, considering structural, aerodynamic and performance features.

2 Weissinger Method

Based on the potential theory, the Weissinger method [5] is similar to a vortex lattice method, but with just one panel element chordwise. It is capable of modelling swept, tapered and twisted wings with low or high aspect ratios, showing good results in subsonic flows. A code based on the Weissinger method was implemented to calculate the lift and moment coefficients distributions over the semi-span, from what the induced drag is inferred.

This method uses horseshoe vortices that extent to infinity satisfying the Helmholtz theorem, as shown in Figure 3. It calculates the strength of each vortex necessary to keep a tangent flow at the control point.

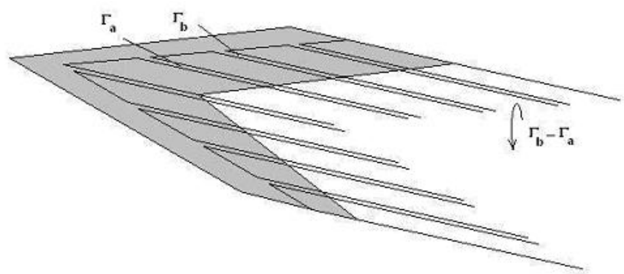

Fig. 3 Horseshoe vortices.

If the unitary strength vortex at the section j generates the downwash speed AIC_{ij} at the section i, then the linear system of equations representing the boundary conditions may be written as:

$$[AIC]\{\Gamma_i\} = U_\infty\{\alpha\} \tag{1}$$

where $\{\alpha\}$ represents the sections incidence angle semi-spanwise. If the airfoil is cambered the local angle of attack is referred to its zero lift angle.

The linear system may also be written as function of the angle at the root of the wing and the washout angle. Equation (2) shows $\{\alpha\}$ for wings with linear washout distribution, where $\{\alpha_r\}$ is a vector of the angle at the root for each element, y is the semi-span coordinate, θ is the washout angle.

$$\{\alpha\} = \{\alpha_r\} - \frac{2\theta}{by} \tag{2}$$

The wing circulation distribution may be written as the addition of two distributions as in equation (3).

$$\{\Gamma\} = [AIC]^{-1}\alpha_r l - [AIC]^{-1}\frac{2\theta}{by} \tag{3}$$

Since the section lift (per length unit semi-spanwise) is related to circulation as:

$$\{l\} = \rho U_\infty\{\Gamma\} \quad , \tag{4}$$

the lift distribution may be expressed as in the equation (5), where l_1 and l_2 are independent from the incidence angles, depending only on the planform of the wing.

$$\{l\} = \alpha_r l_1 + \theta l_2 \tag{5}$$

As the lift coefficient of the wing (C_L) is linearly related to the angle of attack, the lift distribution may be rewritten as:

$$\{l\} = C_L l_3 + \theta l_4 \quad . \tag{6}$$

The first term is known as the additional lift distribution and the second term is called the basic lift distribution and they are linearly related to the lift coefficient and the washout angle, respectively.

3 Structural Analysis

Knowing the load distributions, it is possible to determine the bending and torsion loads. The method used in this work to determine this loads is the matrix method. Next it is presented in its fundamental matrix notations.

The aircraft wing resembles a cantilever beam, as shown in Figure 4. If the loads L1 and L2 representing the air load are applied, the beam bends upwards. The beam must be structurally capable of resisting the ultimate bending load and stresses which vary along the beam length. In this work, the shear stresses will not be considered. The shear force on the beam in Figure 4 is defined as the sum of the vertical forces acting on the wing. The shear force V3 at point 3 is the sum of L1 and L2, as presented in equation 7.

$$\begin{aligned} V2 &= L1 + 0 \\ V3 &= L1 + L2 \end{aligned} \tag{7}$$

Equation (8) shows the shear load in matricial form, where $\{V\}$ and $\{L\}$ are column matrices and $[A]$ is a square matrix.

$$\{V\} = [A]\{L\}$$
$$[A] = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{8}$$

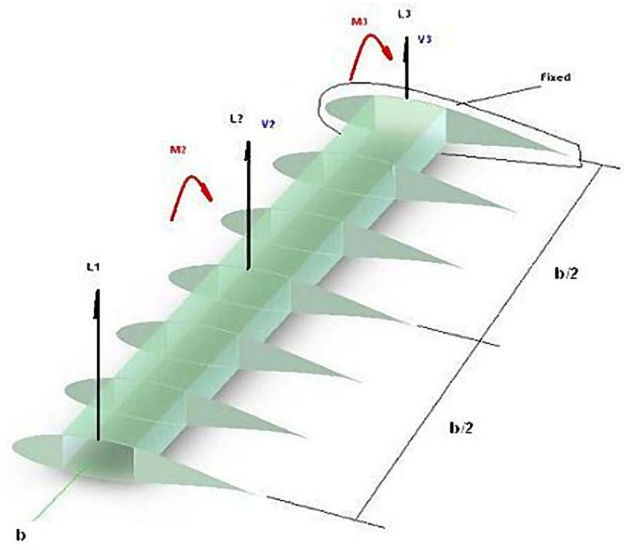

Fig. 4 Details of structural modelling.

The bending moment at point 2 (Figure 4), M2, is:

$$M2 = L1\frac{b}{2} \tag{9}$$

and M3, at point 3, is:

$$M3 = 2L1\left(\frac{b}{2}\right) + L2\left(\frac{b}{2}\right) \quad . \tag{10}$$

In matrix notation:

$$\{M\} = [B]\{L\} \quad , \tag{11}$$

where $\{M\}$ is the bending moment array, $[B]$ is the positions matrix and $\{L\}$ is the shear forces array (local lift).

Similarly, the torsion moment can be written as:

$$\{T\} = [C]\{M_A\} \quad , \tag{12}$$

where $\{T\}$ is the torsion moment array, $[C]$ is the positions matrix and $\{M_A\}$ is the aerodynamic local pitching moment array.

With bending and torsion moment distributions it is possible to determine if the spar caps and shear webs thicknesses along the spar-tank are enough to resist the loads. For given thicknesses, the tensile and compressive stresses are determined and the Von Mises equation for combined stresses is applied:

$$\sigma_v = \sqrt{\sigma_{bend}^2 + 3\tau_{tors}^2} \quad , \tag{13}$$

where σ_v is the Von Mises stress, σ_{bend} is the bending stress and τ_{tors} is the torsion stress.

The safety margin is then calculated for each section of the spar-tank:

$$SM = \left(\frac{\sigma_{all}}{1.5\sigma_v} - 1\right) \quad , \tag{14}$$

where σ_{all} is the allowable stress and the factor 1.5 is a safety factor.

4 Optimization

The optimization applied in this work is a genetic algorithm code based on the implementation shown by Goldberg [7]. There are eight variables in the string (chromosome) which are aspect ratio, taper ratio, sweep angle, twist angle, percentual width of the spar-tank section (beginning at 12% of the airfoil), thickness of superior cap sheet, thickness of the web sheets and thickness of inferior cap sheet. Ten sections are used to model the wing for the aerodynamic and structural analyses.

Flight conditions are defined by Mach number and angle of attack (α) as shown in Table 1. Each condition takes an estimated time (T) inside the mission considered, also shown in that table, which are used as weight factors in the fitness function. This function also takes into account the relation $\frac{C_L}{C_D}$ for each flight condition.

Table 1 Flight conditions.

Condition	Mach	α $[^o]$	T $[s]$
Take-off	0.1	4	30
Climb	0.12	7	600
Cruise	0.3	1	16500
Loiter	0.15	2	900

The structural analysis evaluates the safety margin at each of the ten sections and the equation:

$$\begin{aligned} SM_{localfit} &= 100 - 100SM_{local} \quad &, SM_{local} \geq 0 \\ SM_{localfit} &= -1000 \quad &, SM_{local} < 0 \\ SM_{tot} &= \textstyle\sum_{i=1}^{10} SM_{localfit}(i) \end{aligned} \tag{15}$$

shows how it manipulates these values, aiming for an optimally sized structure. The aerodynamic loads considered in this evaluation considers Mach $= 0.3$ and $\alpha = 7^o$, that was found to be the most critical condition for the mission.

The weight of the structure (mass - M) is calculated in the structural analysis simply by integrating the tank sectional area spanwise and multiplying by the aluminium density. It is inserted in fitness function as:

$$Fit_{mass} = \frac{10}{M} \quad . \tag{16}$$

A similar procedure is used to calculate the fuel volume (V), applied at the fitness function as:

$$\begin{aligned} Fit_{vol} &= 10^2 V \quad &, V \geq 0.1\ m^3 \\ Fit_{vol} &= -150 \quad &, V < 0.1\ m^3 \end{aligned} \quad . \tag{17}$$

At last, all the parameters are used in the fitness funcion:

$$\begin{aligned} Fit_1 &= \frac{T_{to}\left(\frac{C_L}{C_D}\right)_{to} + T_{cli}\left(\frac{C_L}{C_D}\right)_{cli} + T_{cru}\left(\frac{C_L}{C_D}\right)_{cru} + T_{lo}\left(\frac{C_L}{C_D}\right)_{lo}}{T_{tot}} \\ Fit_2 &= SM_{tot} + Fit_{mass} + Fit_{vol} \quad . \\ Fitness &= Fit_1 + Fit_2 \end{aligned} \tag{18}$$

For a better understanding of the implementation Figure 5 shows a flowchart with the steps followed.

5 Results and Discussion

The parameters chosen for the GA optimization are shown in Table 2.

It was defined a range for each of the variables, shown in Table 3.

The evolution obtained from the optimizer in 80 generations is presented in Figure 6. In this

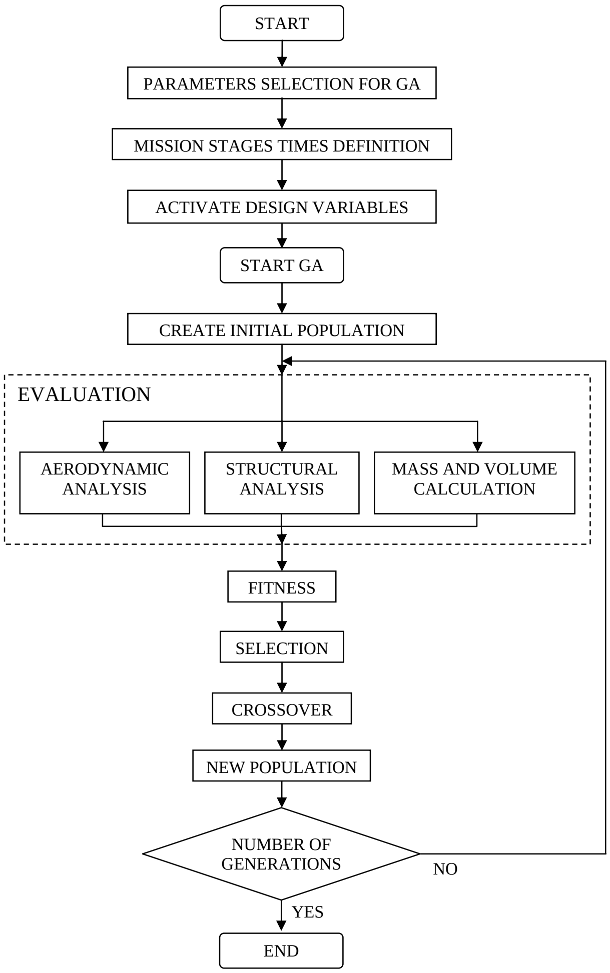

Fig. 5 Flowchart of the implementation.

Table 2 GA parameters.

Parameter	Value
Number of individuals	40
Number of generations	80
Crossover points	10
Mutation probability	5%
Elitism percentual	10%

Table 3 Variables limits.

Variable	Min	Max	Step
Aspect ratio	5	11	1
Twist angle	-5^o	5.27^o	0.73^o
Taper ratio	0.4	1	0.1
Sweep angle	-4^o	11^o	1^o
Percentual width	1%	64%	1%
Superior cap thickn.	1 mm	16 mm	1 mm
Web thickness	1 mm	16 mm	1 mm
Inferior cap thickn.	1 mm	16 mm	1 mm

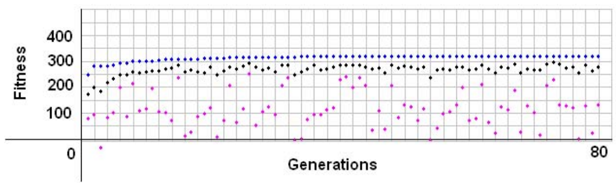

Fig. 6 Evolution of population in 80 generations.

figure it can be seen the best (blue), the average (black) and the worst (magenta) individuals at each generation. The behaviour shown by the worst individuals indicates a good search through the solution domain. In other words, it indicates that probably, the optimum found is global, not local.

Table 4 lists the variables of the optimal solution found.

Table 4 Optimal solution variables.

Variable	Value
Aspect ratio	5.0
Twist angle	1.6^o
Taper ratio	0.4
Sweep angle	11^o
Percentual width	76%
Superior cap thickness	1 mm
Web thickness	2 mm
Inferior cap thickness	16 mm

Figure 7 shows the planforms of the best, the average and the worst individuals in the 1^{st}, the 40^{th} and the 80^{th} generations, while Figure 8 shows a detail of the optimal solution and the spar-tank fitted inside (in blue).

Results presented shows that some features apearently good are not as good when other disciplines are considered. For instance, some aerodynamically good solutions are the worst among a generation, as is the case shown in Figure 7

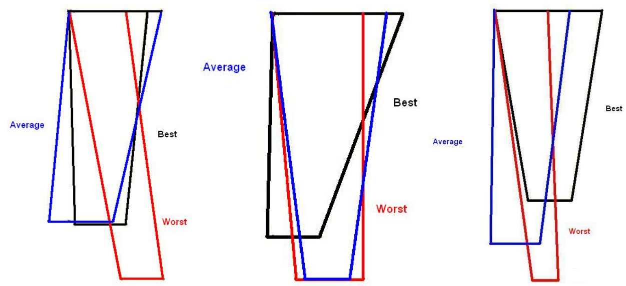

Fig. 7 Comparison of individuals in in the first (left), the 40^{th} (middle) and the 80^{th} generations (right).

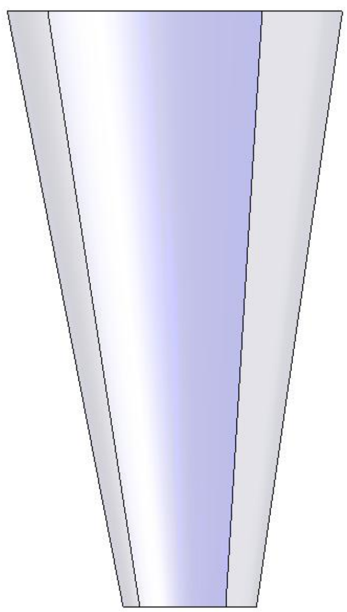

Fig. 8 Optimal solution and spar-tank (in blue).

in the last generation. This seems coherent for a multidisciplinary study, where trade-offs are wanted.

6 Conclusions

The MDO implemented was capable of determining a better solution for the conceptual design in study. The work shows that it is possible to leave the conceptual stage of a design with better solutions, just by applying low cost (low fidelity) numerical methods. It was evident by the results that the MDO took every variable into account, but from this experience one major subject to take care is the creation of the fitness function, where all the optimization process depends on.

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