DEVELOPMENT AND VALIDATION OF AN INVESTIGATION TOOL FOR NONLINEAR AEROELASTIC ANALYSIS

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Abstract

This paper presents a design tool based on computational methods for the aero-structural analysis of aircraft layouts at the conceptual design stage. Multi-disciplinary optimization (MDO) pushes the design towards the absolute optima. So far it has become one of the unquestioned primary targets for aircraft design tools, to obtain the best performances at the lowest possible cost. The structural weight tends to be continuously reduced in favor of improved performances. As a consequence, the analysis of increasingly flexible aircraft requires dedicated tools that allow to investigate the effects of details like material anisotropy and geometrical nonlinearities when composites materials and aeroelastic tailoring is exploited. ***SMARTCAD*** *(**S**implified **M**odels for **A**eroelasticity in **C**onceptual **A**ircraft **D**esign) allows the creation of low-order, high fidelity models. They can be designed to take into account most of the higher order/nonlinear effects and couplings of the aircraft under development. SMARTCAD can be used within an MDO framework to drive the optimization tool into the most appropriate direction; aeroelastic performances for non-conventional aircraft can be evaluated, potential couplings can be highlighted and their propitious or adverse nature can be investigated. The code has been developed under the research project named SimSAC (Simulating Aircraft Stability And Control Characteristics for Use in Conceptual Design), partially funded by the EC within the 6th European Research Framework.*

1 Introduction

SMARTCAD represents a design tool for aero-structural analysis to be used in the conceptual phase of fixed wing aircraft design. It allows to consider aeroelastic aspects from this early stage. Solving adverse aeroelastic issues like divergence, control surfaces reversal, flutter, increased drag at cruise speed due to structural deformability may require considerable changes in the structural design, limitations in flight envelope or weight penalties. The late discovery of this type of issues may result in significant cost increases and, in some cases, it may even require to actually close the project. In order to overcome the insurgence of these issues, the influence of deformability on flight and handling performances, on structural weight and on design costs needs to be taken into account as early as possible in the design process. Rapid and reliable methods need to be used in the initial steps of conceptual design, when many parameters have not been established yet. The enhancement of this phase with explicit multi-level design oriented numerical tools having physical basis rather than relying on implicit statistics of existing aircraft, prevents the design from being excessively modified during the detailed design phase if deficiencies are found, and guarantees the capability to study unconventional architectures (joined wings and blended wing-body aircraft) and new attractive technologies like composite materials.

SMARTCAD adopts a mixture of semi-empirical, computational and analytical tools (see Fig. 1) which allows, starting from a detailed

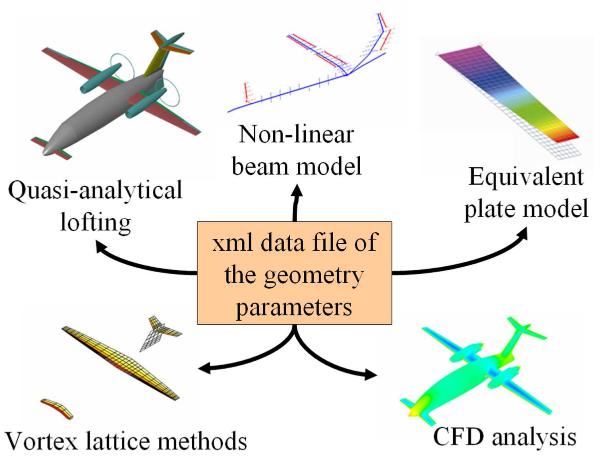


Fig. 1 Layout of the proposed aero-structural tool.

description of the external geometry, to:

- determine a first reliable distribution of structural stiffness for the complete airframe satisfying local and global buckling, compressive yield strength and ultimate tensile strength;
- have a better prediction on a physical basis of the airframe, overcoming the lacks of statistical methods which may not be even available for unconventional layouts;
- evaluate the static/dynamic aeroelastic performances of the sized airframe by means of beam models and linear/nonlinear aerodynamic methods.

Different kind of analysis can be carried out:

- vibration modes calculations and flutter analysis (see Fig. 5,8);
- linear/non-linear static aeroelastic analysis, trimmed calculation for a free-flying rigid or deformable aircraft (see Fig. 7,10);
- steady and unsteady aerodynamic analysis to extract derivatives for flight mechanics applications.

In the following sections, the proposed aeroelastic tools are outlined and applied to the Boeing 747-100 large transport aircraft.

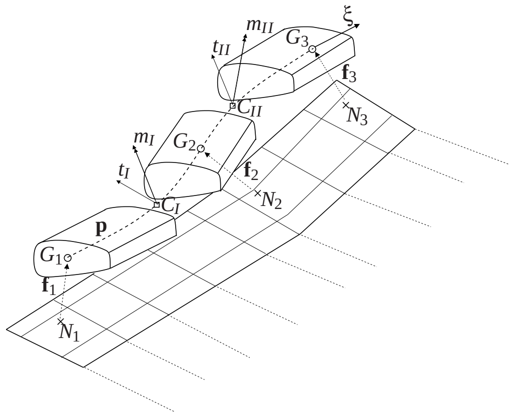


Fig. 2 Finite Volume three-node beam coupled to a classic lifting surface method.

2 Beam formulation

SMARTCAD adopts a three-node linear/non linear finite-volume beam, originally proposed in [11], which proved to be intrinsically shear-lock free. The finite-volume approach leads to the collocated evaluation of internal forces and moments, as opposed to usual variational principles which require numerical integration on a one-dimensional domain. As sketched in Fig. 2, each beam element is divided in three parts. Each part is related to a reference point G_i: the mid- and the two endpoints. They are referred to geometrical nodes N_i by means of offsets $\mathbf{f}_i$. This allows the elastic axis of the beam to be offset from the center of mass. Every node is characterized by a position vector and a rotation matrix. A reference line $\mathbf{p}$ describes the position of an arbitrary point $\mathbf{p}(\xi)$ on the beam section. Parabolic shape functions are used to interpolate displacements and rotation parameters of the generic point $\mathbf{p}(\xi)$ as functions of those of the reference nodes. The derivatives of the displacements and the rotation parameters at the two collocation points C_j (laid at $\xi = \pm 1/\sqrt{3}$ to recover the exact static solution for a beam loaded at the end points) are used to evaluate the strains and the curvatures. The latter are used to compute the internal forces and moments, which must balance the external forces $\mathbf{m}_i$ and moments $\mathbf{t}_i$.

3 Structural sizing and stick model generation for aeroelastic analysis

As far as the aero-structural design process is concerned, SMARTCAD is coupled to a pre-processor sizing tool named **GUESS** (**G**eneric **U**nknowns **E**stimator for **S**tructural **S**izing). It is derived from the *Analytical Fuselage and Wing Weight Estimation (AFaWWE)* method [2], further extended to the sizing of horizontal and vertical tail planes. This method results in a weight estimate which is directly driven by material properties, load conditions, and vehicle size and shape. Thus it is not confined to an existing data base, but it is rather independent of classic statistical formulas currently used in aircraft design to estimate the weight of airframe components. GUESS determines the distribution of stiffness and non-structural mass of the airframe subjected to different types of maneuvers (pull-up at prescribed normal load, landing, bump on irregular runway, rudder maximum deflection). It is based on beam theory and analytical methods for the estimation of aerodynamic loads. GUESS is also linked to a tool named CADAC and to a Weight and Balance module for the estimation of the inertia properties. The former provides a general parametric description of the geometric shape of the aircraft under design (wing, fuselage, tail). The latter provides a semi-empirical estimation of all inertia properties, as proposed by Raymer and Torenbeek [12]. The interested reader is referred to Ref. [5], where a more detailed description of the whole design environment is presented.

As soon as the airframe is sized, GUESS automatically creates the stick model for SMARTCAD in order to run the aeroelastic analysis. The airframe is actually sized without considering any static/dynamic aeroelastic effect, thus the necessity to assess the aeroelastic performances of the aircraft and eventually improve them by means of multi-disciplinary optimization procedures.

3.1 Mass distribution

The mass distribution is determined by a dedicated internal weight and balance module which computes the structural mesh inertia properties. This is particularly important when the trim condition for the free-flying aircraft is sought, since a detailed description of mass values and their location is of primary importance to correctly define inertial loads. Thus, when the beam stick model is automatically generated, all non-structural masses are correctly introduced in the structural mesh as:

- lumped non-structural masses on mesh nodes (engines, landing gears, auxiliary tanks, systems);
- non-structural densities per unit length along the beams (passengers in fuselage, fuel in wings, paint, furniture).

As introduced above, these data are either provided by statistical methods or directly by the user if available. Lumped masses are easily introduced in the model by means of rigid offsets from a reference node. Beams can also be used for this purpose, but an estimation of their stiffness is required (this may happen in the case of engine pylons). As far as distributed masses are concerned, for example fuel, the availability of an estimate of the fuel volume (see Fig. 3) available in the wing-box allows to determine the mass stored for each beam along the wing-span, and thus to estimate the mass per unit length. The same approach can be applied to any distributed mass (passengers, furniture and so on), provided their value and position is correctly estimated.

3.2 Thrust modelling

For non-linear aeroelastic applications, it may be important to include the effect of the displacement and rotation of lumped force application points. This typically highlights potential couplings and allows to check their propitious or adverse nature. Thus, SMARTCAD includes a follower-force element which can be used, for example, to include thrust effects in the aeroelastic trim solution.

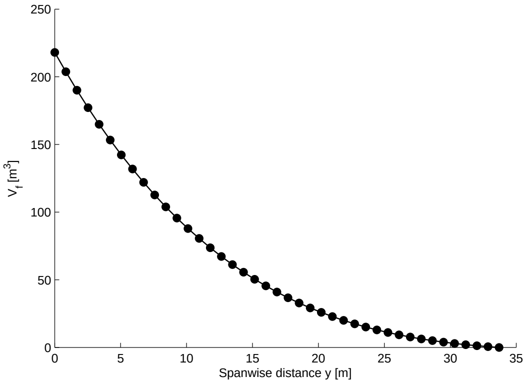


Fig. 3 Spanwise fuel volume available in wing-box.

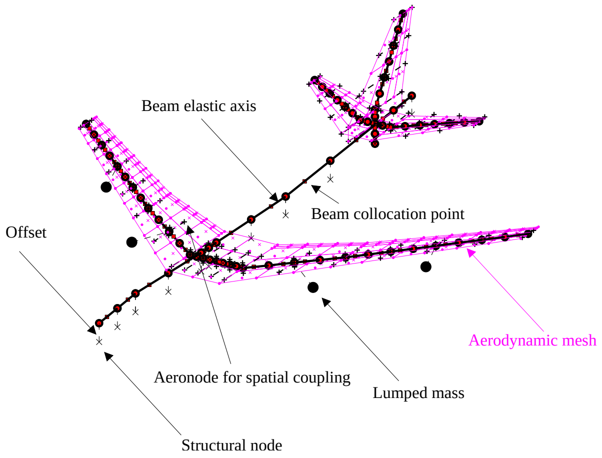
3.3 Control surfaces modelling

Control surfaces are currently represented by their aerodynamic contributions, considering the early design phase SMARTCAD is intended for. As for flutter analysis, for example, a further step would consist in the structural sizing of the control surfaces, and include a lumped static impedance to model a mechanic control-chain or to approximate the impedance of the actuators. This is currently out of the target the code is developed for and it is left to future developments.

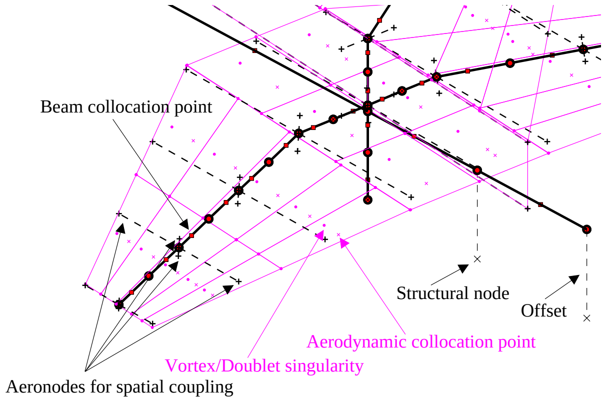
When static aeroelastic trim is sought, the user can specify arbitrary constraints among the control surfaces, with different gains. For example, antisymmetric ailerons deflection, symmetric elevators deflection, or wing flaps deflection can be imposed as needed.

4 Spatial coupling methods

SMARTCAD adopts a staggered approach, where different structural and aerodynamic independent codes, each one optimal for its purpose, are used for each field. In order for the codes to interact, a spatial coupling scheme is required. The adoption of a partitioned approach [10] requires the definition of an interface scheme to exchange displacements, velocities and loads between the structural grid and the CFD boundary surfaces. The two models are typically discretized in very different, often incompatible ways. Structural models usually present complex geometries, including many discontinuities. They are often based on schematic models, which are of common use in the aerospace industry, using elements with very different topologies, like beams and plates, which usually hide the real structural geometry up to the point where the aircraft external shape partially or completely disappears. Despite the computational power available nowadays, these simplified models are still used, and will probably be used in the future in aerospace industry because of their efficiency and effectiveness, especially in the early design stages SMARTCAD is supposed to be adopted for. As a consequence, it is extremely important to be able to cope with them. On the contrary, aerodynamic meshes typically require a more ac-



(a) Overview of the stick model and nomenclature.



(b) Detailed overview for tail planes.

Fig. 4 Stick model for Boeing 747 aircraft.

Type	$\Phi(\delta)$, $\Phi(r)$
Volume Spl.	r
Thin Plate Spl.	$r\log(r)$
Gaussian	e^{-r}
Euclid Hat	$\pi\cdot((1/12r^3)-(r^2_{\max}\cdot r)$ $+(4/3r^3_{\max}))$
Wendland C^0	$(1-\delta)^2$
Wendland C^2	$(1-\delta)^4\cdot$ $(4\delta+1)$
Wendland C^4	$(1-\delta)^6\cdot$ $(35/3\delta^2+18/3\delta+1)$
Wendland C^6	$(1-\delta)^8\cdot$ $(32\delta^3+25\delta^2+8\delta+1)$

Table 1 Weight functions used as RBF.

curate description of boundary surfaces (especially when Computation Fluid Dynamics solvers are used to solve for Euler or Navier-Stokes equations). As a consequence, two radically different representations of the same aircraft geometry must be made compatible in order to transfer information between them. This is a well known problem, deeply investigated in the literature; for further reference, see [21; 3].

An innovative scheme, based on a 'meshfree' Moving Least Square (MLS) method [18], is used to cope with incompatible situations, when the two meshes do not share a common surface. A second approach is represented by the Radial Basis Function (RBF) method [4; 20]. Both methods ensure the conservation energy transfer between the fluid and the structure and they are suitable for the treatment of complex configurations. To guarantee the conservation between the two models, the correct strategy consists in enforcing the coupling conditions in a weak sense, through the use of simple variational principles. Using the Virtual Works Principle (VWP) the energy exchange can be investigated as reported in [16].

5 Aerodynamic methods available

Two low-fidelity aerodynamic methods are available in SMARTCAD, depending on whether steady or unsteady analysis is carried out:

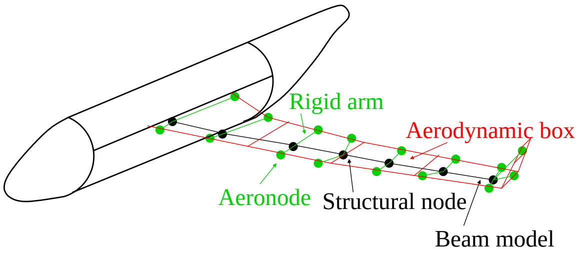


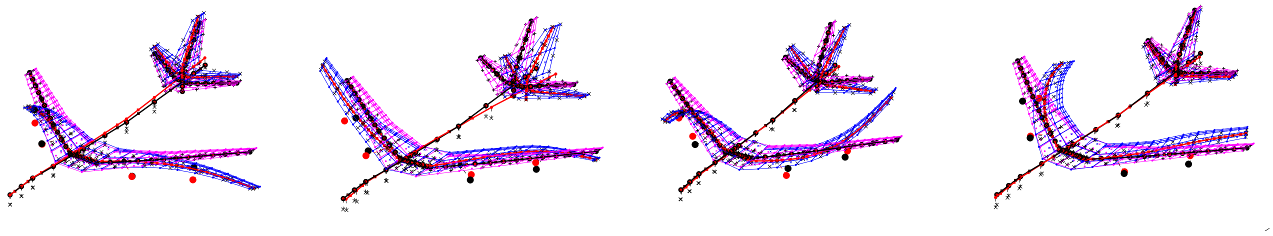
Fig. 6 Aeronodes for aeroelastic interpolation

- Vortex Lattice Method (VLM) with camber contribution on normalwash once the airfoil description is provided;
- Doublet Lattice Method (DLM) for the prediction of the generalized forces and flutter analysis in the subsonic regime.

Both methods are based on the same geometric discretization of the aerodynamic surfaces as a flat lifting surface with singularities and collocation points located respectively at $1/4$ and $3/4$ of each panel chord. Thus the same mesh generator can be used, with the only exception that the VLM allows camber contributions to be included and requires trailing vortexes for wake modelling.

5.1 Steady aerodynamics

The available VLM is derived from Tornado [17], which has been enhanced with new functionalities to support aeroelastic analysis. Since the adopted beam model allows to model large displacements and rotations, the aerodynamic mesh is physically deformed to accurately follow the deformation of the structural shape. The same occurs when control surfaces are deflected. Whenever the VLM mesh is deformed, the new position of its panel nodes, collocation and singularities points and trailing wake are updated using the techniques introduced in Section 4. The two spatial coupling methods determine an influence matrix **H** that maps field data from the structural to the aerodynamic mesh. To overcome the problem of mapping the one-dimensional beam domain to a two-dimensional or three-dimensional lifting surfaces and CFD domain, an extra set of nodes, named *aeronodes* as sketched in Fig. 6,



(a) 1st bending - 0.60 Hz (b) 1st torsional - 0.93 Hz (c) 2nd bending - 1.61 Hz (d) 1st inplane - 1.83 Hz

Fig. 5 Boeing 747 vibration modes for maximum take-off configuration.

are introduced in the model; their displacements are directly recovered from the real structural nodes under the hypothesis of rigid beam section. Finally, a new flow solution is determined on the deformed lattice, and the corresponding forces are transferred to the structural model by means of the same spatial coupling matrix **H** for a new structural solution. These steps are repeated until the displacement field converges under the assumption that the system is not approaching or exceeding static aeroelastic divergence conditions.

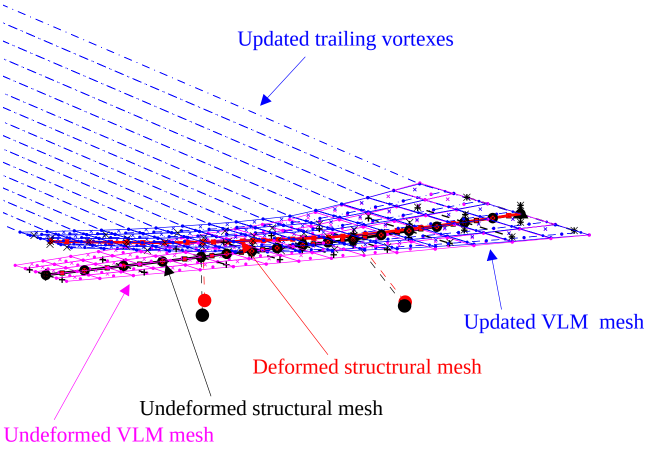


Fig. 7 Deformed configuration for Boeing 747 clamped wing, M_∞=0.8, α=6deg, z=5000m.

5.2 Harmonic aerodynamics

SMARTCAD adopts a built-in DLM [1] to compute the aerodynamic transfer matrix $\mathbf{H}_{am}(jk,M_\infty)$ for the generalized forces in the reduced frequency domain jk and Mach number M_∞. Two different approximations are available for doublet distributions across the span of the box bound vortex: parabolic, which restricts the box aspect ratio to about 3, and quartic, which relaxes the limitation on box aspect ratio and the number of spanwise divisions required in high-frequency analyses [19]. Particular care was dedicated to test the solver for nearly coplanar wing/tail configurations as illustrated in [13].

Once the transfer matrix is available, flutter instabilities in the whole flight envelope are determined using the classic *p*-*k* method, solving the equation

$$\left(s^2\mathbf{M}+s\mathbf{C}+\mathbf{K}-q_\infty\mathbf{H}_{am}(jk,M_\infty)\right)\mathbf{q}=\mathbf{0} \quad (1)$$

where **q**, **M**, **C**, **K** are respectively modal amplitudes, mass, damping, and stiffness matrices, and q_∞ is the dynamic pressure. The solution of Eq.(1) determines flutter velocity, frequency and modes. A well-known three-dimensional standard aeroelastic configuration, the AGARD 445.6 weakened wing [22], is considered to validate the whole procedure. It was originally tested in the Transonic Dynamics Tunnel (TDT) at NASA Langley. The wing semispan model is made of laminated mahogany, with the NACA 65A004 airfoil, a quarter-chord sweep angle of 45 deg, an aspect ratio of 1.65 and a taper ratio of 0.66. The structural model consists in the first four normal modes. Two flight conditions are investigated by means of SMARTCAD; results are successfully compared with both experimental and NASTRAN results, as summarized in Table 2. The first flight condition at a Mach number $M_\infty = 0.678$ is subsonic, while the sec-

Mach	$V_{F_{Exp}}$ (m/s)	$V_{F_{NAS}}$ (m/s)	$V_{F_{SMA}}$ (m/s)	ω_{Exp} (Hz)	ω_{NAS} (Hz)	ω_{SMA} (Hz)
0.678	231.37	237.94	235.86	13.89	14.82	14.30
0.960	309.00	337.07	324.23	17.98	20.53	20.06

Table 2 Comparison of flutter velocity V_F and frequency ω_F for the AGARD 445.6 aeroelastic benchmark

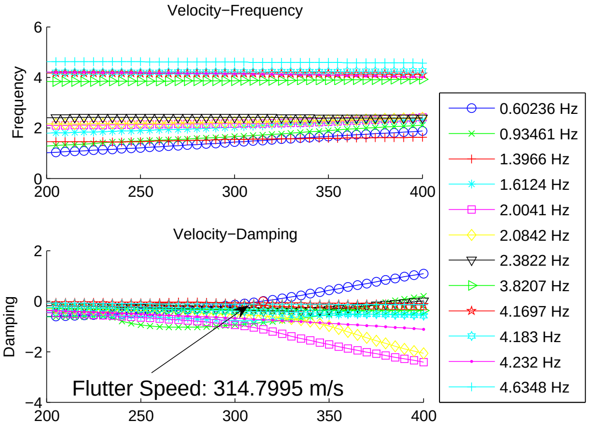


Fig. 8 Flutter diagrams for Boeing 747 in free flight, M_∞=0.9, z=0m.

ond one at a Mach number $M_\infty = 0.960$ is transonic. This flight condition is investigated despite the linearized potential theories are known to overestimate the flutter velocity in this regime.

Fig. 8 shows flutter results through the DLM for the Boeing 747 in free flight at M_∞=0.9 and zero altitude. All rigid body and deformable normal modes are used for this purpose due to the lack of an adequate frequency separation margin. As it can be seen, a flutter instability associated to the fist bending mode showed in Fig. 5(a) is detected and a second instability for the first torsional mode showed in Fig. 5(b) occurs at higher velocity.

5.3 High-fidelity aerodynamics

Complex aeroelastic phenomena may appear in the transonic speed range, where moving shock waves arise in the flow field caused by unsteady motions of the aircraft structure. The presence of shock waves may cause a significant drop in flutter velocity: the so-called *transonic dip* effect. In these cases the flutter velocity is often overpredicted by classical linear velocity potential methods used to determine unsteady aerodynamic loads. More complex fluid dynamics models need to be adopted, like those based on Euler or Navier-Stokes equations. Following the same approach presented in [6], SMARTCAD can export vibration modes to allow the determination of the aerodynamics transfer matrix by means of the Edge flow-solver [9]. The results for the AGARD wing with the inviscid flow model are summarized in Fig. 9, where the transonic dip is correctly captured in the transonic Mach region. To highlight the good quality of the results, the same pictures also show the ones presented in [6; 15], obtained using classical methods like the DLM and the Harmonic Gradient Method (HGM, [8]: ZONA51) and Navier-Stokes equations.

6 Trim solution for the free-flying aircraft

SMARTCAD allows to determine the trimmed solution for the rigid/deformable free-flying aircraft in steady maneuver, characterized by steady aerodynamic and inertial forces in body-axes. The approach is the same presented in [7], with the exception that the full model is used instead of a reduced modal one by component mode synthesis. The governing idea consists in determining the trim condition in a staggered manner when the flight conditions and some parameters regarding the attitude of the aircraft are imposed. This allows, for example, to determine the angle of attack and the controls deflections that make the aircraft fly with an imposed linear and angular velocity in the inertial reference frame.

A matrix-free Jacobian Free Newton-Krylov Method (JFNK) [14] or Newton-Raphson method is applied only to the flight mechanics equilibrium equations. This approach is equivalent to solving the coupled problem given by the flight mechanics and structural equilibrium equations through a block Gauss-Seidel method. Aerodynamic loads are subsequently transferred

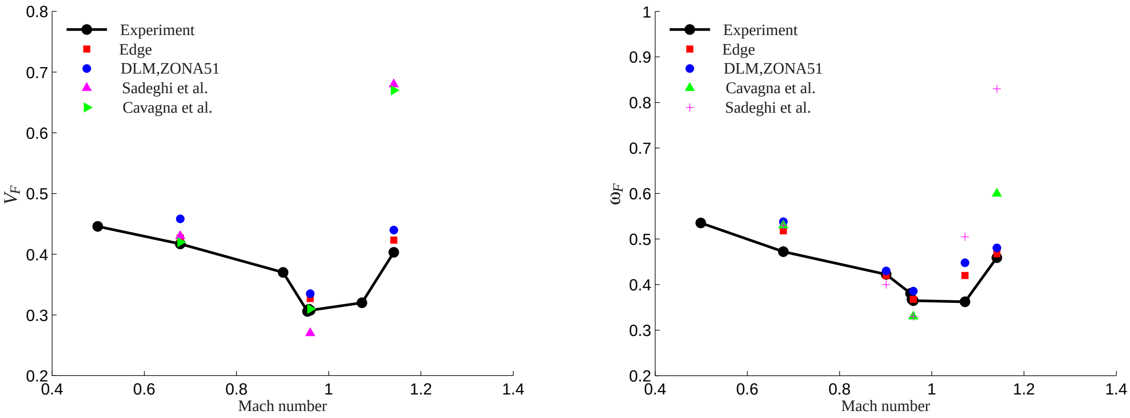
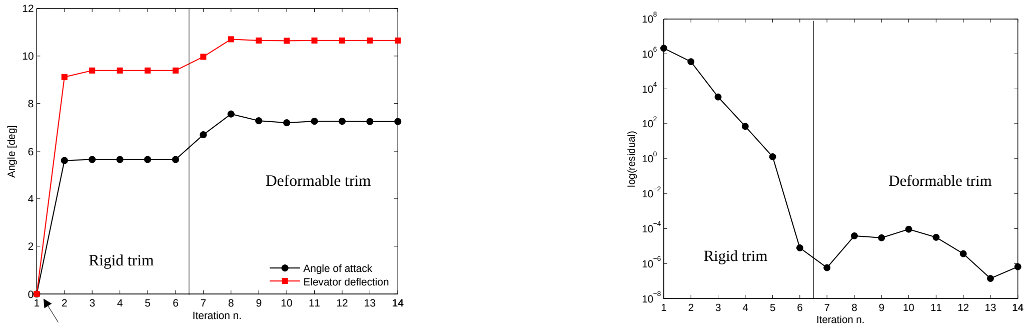
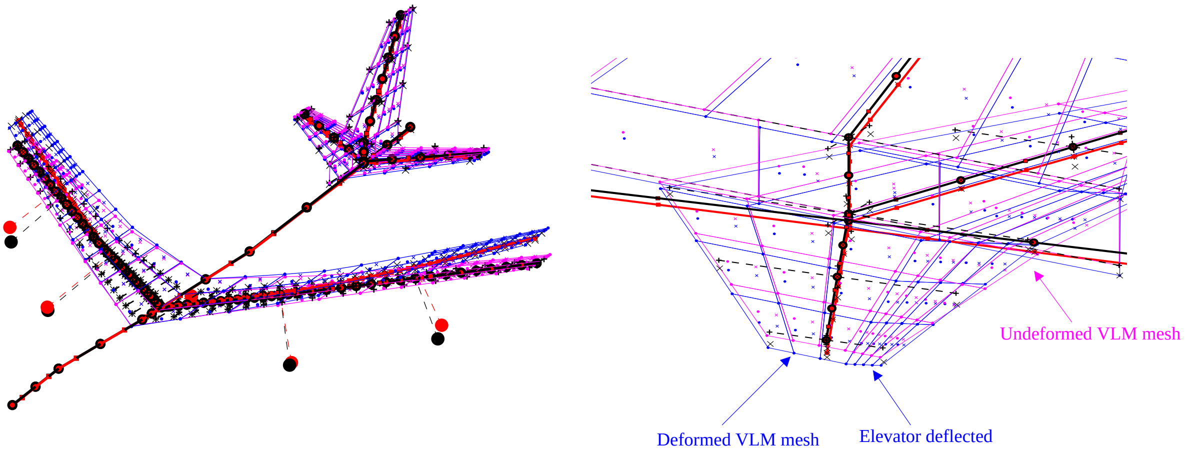


Fig. 9 Flutter speed index V_F and frequency ω_F for the AGARD 445.6 wing by Edge solver



(a) Trim variables history (b) Residual norm for flight mechanics equations



(c) Deformed trimmed solution (d) Horizontal tail detail

Fig. 10 Trim solution for the free flying deformable aircraft, M_∞=0.6, z=5000m.

to the structural model to calculate the new deformed shape. The latter is then used to determine the new trimmed solution. The process is repeated until convergence on structural displacements and rigid-body equilibrium is reached (see Fig. 10(b)). The same process can be applied when the trim condition is sought for both the rigid and deformable aircraft. For this last case, the rigid trim is always determined to obtain a good initial guess solution and improve the convergence. This is also important to highlight the contribution of structural deformability on the trimmed solution.

Fig. 10 shows the results obtained by applying the process illustrated above. In this simple case, a steady levelled symmetric cruise condition is sought. For this simple case, the user specifies null angular velocities, the deflection of the horizontal tail, the components of the linear velocity in the inertial frame and requires a symmetric flight by enforcing null sideslip angle. The process then determines the angle of attack, the elevator deflection (see Fig. 10(a)) and the structural displacements (Fig. 10(c)). The remaining parameters such as aileron/rudder deflections and bank angle are null. Fig. 10(d) shows the horizontal tail bent down as expected, along with the deflected elevator in order to produce the required negative lift.

7 Conclusions and future developments

The paper outlined the different tools developed in order to enhance the conceptual design phase with aeroelastic analysis. Particular care is dedicated to provide hierarchical tools from low to high fidelity able to:

- be easily coupled with other codes, as multidisciplinary analyses are required;
- be relatively accurate for the conceptual phase and give correct trend data to let the design progress in the correct direction; ;
- be computationally efficient, as several configurations need to be examined;
- give the capability to trade accuracy for speed to the designer who is left free to decide the level of discretization and to rule accuracy of modelling by the adoption of different solvers available in the toolbox;
- require minimal time for model preparation and modification; the development of automatic procedures and the exploitation of geometry parametrization that can be used as design variables, guarantees to easily reflect the changes of a design variable in all the numerical models;
- provide sensitivity derivatives of the design variables.

Future developments will allow to provide aeroelastic corrections to flight stability derivatives to be used in flight dynamic applications. Furthermore, SMARTCAD will be used within an MDO environment to eventually improve the first guess structural solution in order to satisfy aeroelastic constraints like flutter speed, divergence, static aeroelastic deflections and control effectiveness.

Acknowledgments

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Copyright Statement

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