# TRAJECTORY CONTROL IMPROVEMENT BASED ON A NONLINEAR DYNAMIC INVERSION APPROACH

**L Höcht, F Holzapfel, G Sachs**
**Institute of Flight System Dynamics**
**Technische Universität München, Boltzmannstr. 15, 85748 Garching, Germany**
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## Abstract

*In recent years many research efforts focused on the improvement of higher level supervisory logics for flight control systems pursuing the capability to autonomously accomplish complete missions. The main interface between those higher level logics and the flight control system core is the navigation loop which enables the aircraft to fly along defined trajectories. The improvement of the trajectory performance by enhancement of the robustness of the inner loops in case of failure cases without any mode switching can facilitate the task of autonomous mission accomplishment by decreasing the number of decisions that have to be made.*

*The paper presents a physically motivated approach of resolving control objective conflicts in the path loop in case of saturated controls. The control concept is based on a nonlinear dynamic inversion approach with reference models as command filters.*

*The control objective conflict resolution is carried out on the one hand by a nonlinear extension of the total energy control system (TECS) leading to prioritization between flight path angle and acceleration and on the other hand by generalization of TECS with could be named total force control system (TFCS). Here a consideration of the total transverse force results in a prioritization between the curvature of the trajectory in the horizontal and vertical plane. This approach is especially interesting for aircraft with the necessity of high maneuverability and full flight envelope exploitation.*

## 1 Introduction

Control of the flight path namely the airspeed as well as the inclination and course angle serve as basis for the trajectory control loop which may be intended to form an interface to high level flight management systems (FMS) conducting complete missions autonomously. While recently many efforts focus on the improvement of such FMS, an optimized performance of the inner loops of the flight control system (FCS) can significantly increase the capabilities of those FMS.

The aspects discussed in the paper at hand are related to performance optimizations in the path loop especially at the edge of the flight envelope. Feedback linearization, or often referred to as nonlinear dynamic inversion [1] is the chosen control concept. In order to fully exploit the flight envelope reference is made to a physical interpretation of the linear momentum equations of motion representing the path dynamics. While somewhere inside the flight envelope the path variables can be controlled decoupled from each other, as soon as one more controls run into limits the decoupling can no longer be held up. Without accounting for those interdependencies in the path commands, the aircraft would react in a non deterministic way. The resolution of those control objective conflicts is thus indispensable. The novel approach presented utilizes nonlinear cross-feed limits between the path loop reference models resolving the control objective conflicts when controls run into saturations. There are two limiting factors concerning the path dynamics.

The first one is the limited energy flow provided by the engine, which can either be distributed into kinetic or potential energy. The concept for resolution is a nonlinear extension of the total energy control system (TECS) [2] [3] [4] [5] [6] [7]. The considerations lead to a trade-off between flight path angle and acceleration. Depending on the prioritization of the supervisory FMS either of the objectives is sacrificed in order to achieve the other prioritized one. In case of low and over speed of course the speed must be prioritized not to leave the flight envelope boundaries. An implementation of the speed envelope protection based on the energy consideration is presented in detail, too.

The second main limitation in the path dynamics is the transverse force which is either limited by the maximum lift at low speeds or by the maximum load factor at high speeds. For this issue the TECS philosophy is generalized to a total force control system (TFCS). The available transverse force may either be applied in the horizontal or vertical plane. For resolving this conflict, again a prioritization has to be made where either the turn or the flight path rate is sacrificed in order achieve the other one.

The new approach has proven its effectiveness and performance in simulations of various types of vehicles including piloted aircraft as well as unmanned aerial vehicles.

## 2 Basic Control Concept

### 2.1 Nonlinear Dynamic Inversion

This section gives a short overview for the basic control strategy, while the following section details the actual implementation of the path dynamics controller.

The applied control strategy is nonlinear dynamic inversion [1] [7]. Due to the application of the full non linear equations of motions for the controller design the FCS can control highly dynamic maneuvers at high bandwidth.

Further developments added an adaptive term [8] [9], e.g. by neural networks, to cancel the inversion error between the assumed dynamic for inversion and the real plant.

The control approach presented here is reference model based. The reference models play a significant role in the developed concept, since the control objective conflict resolutions, prioritizations and protections are implemented by means of dynamic limiters at different positions inside the reference model.

Further extensions to account for saturated controls developed in the past which slow down the reference model in order to hide saturation effects from the error dynamic between plant and reference model and to avoid unbounded weights in adaptive control terms [10] [11]. Pseudo-Control hedging (PCH) is the approach established for that purpose in flight control [12]. A detailed description of the dynamic inversion approach can be found in [1] and is thus dropped at this point. Fig. 1 illustrates the complete dynamic inversion controller structure of a single cascade with reference models and PCH.

### 2.2 Physical Interpretation of Path Dynamics

The variables controlled in the path loop are the airspeed $V_A$, respectively the kinematic velocity $V_K$, the kinematic flight path angle $\gamma_K$ and the kinematic course angle $\chi_K$. Basis for their dynamics are the point mass equations of motion which describe the dynamics of the aircraft center of gravity with respect to the earth surface. Forces applied to the aircraft by aerodynamic and propulsion system serve as controls. While the propulsion force mainly acts into the direction of the velocity controlling the linear acceleration, lift, the main aerodynamic force, effects an acceleration perpendicular to the direction of motion and thus a curvature in the horizontal and vertical direction.

So the primary controls of path dynamics are the aerodynamic angle of attack $\alpha_A$, which controls the absolute lift, the kinematic bank angle $\mu_K$ to rotate the direction of lift into a desired plane and the thrust lever state $\delta_T$ which controls the amount of thrust. Secondary controls are the angle of sideslip $\beta_A$ to produce a side force perpendicular to the lift which allows quick changes in the plane of curvature compared to a change in the bank angle, which is a comparatively slow control, the position of high lift devices $\delta_F$ which allows to produce extra lift at limited amount and bandwidth, air brakes to produce drag in order to decelerate and the thrust azimuth $\kappa$ and elevation angle $\sigma$, if thrust vectoring is available. Although there are more controls than variables to be controlled, which would make room for various optimizations in control allocation algorithms, the consideration in the following is restricted to the primary controls since they are sufficient to control the path variables independently and a detailed discussion of control allocation algorithms at this point would go beyond the scope of the paper. Specified in the kinematic (path-axis) frame the required forces for a desired trajectory are:

$$(X_{DES})_K = m \cdot \dot{V}_K + m \cdot g \cdot \sin\gamma_K \quad (9)$$

$$(Y_{DES})_K = m \cdot V_K \cdot \dot{\chi}_{K,DES} \cdot \cos\gamma_K \quad (10)$$

$$(Z_{DES})_K = -m \cdot V_K \cdot \dot{\gamma}_{K,DES} - m \cdot g \cdot \cos\gamma_K \quad (11)$$

Concerning envelope protections, the airspeed $V_A$ is to be controlled instead of the kinematic velocity $V_K$ appearing in equations (9)-(11) whereas for the calculation of the desired forces the kinematic velocity is the one to take. The relationship between airspeed and kinematic speed relative to the ground results from the wind speed $\vec{\mathbf{V}}_W$

$$\vec{\mathbf{V}}_K = \vec{\mathbf{V}}_A + \vec{\mathbf{V}}_W \quad (12)$$

Since wind acceleration cannot be measured anyway it is feasible to assume constant wind for the controller design and thus the aerodynamic acceleration equals to the kinematic acceleration $\dot{V}_A = \dot{V}_K$. While the kinematic velocity is utilized for the computation of the desired forces, the error feedback of the velocity reference model has to be calculated from the airspeed.

The actual forces acting onto the aircraft are:

$$(\vec{\mathbf{F}}_{ACT})_K = \begin{pmatrix} X_{ACT} \\ Y_{ACT} \\ Z_{ACT} \end{pmatrix}_K = \mathbf{M}_{K\bar{K}}(\mu_K)\mathbf{M}_{\bar{K}B}(\alpha_K, \beta_K) \cdot \left\{ \mathbf{M}_{BA}(\alpha_A, \beta_A) \cdot \left[ \bar{q}S \begin{pmatrix} -C_D \\ C_Q \\ -C_L \end{pmatrix}_A - \begin{pmatrix} T_I \\ 0 \\ 0 \end{pmatrix}_A \right] + \begin{pmatrix} \cos\sigma\cos\kappa \\ \cos\sigma\sin\kappa \\ \sin\kappa \end{pmatrix}_B T_O \right\} \quad (13)$$

Here $\mathbf{M}_{\bar{K}K}$ denotes the rotation matrix about the x-axis of the kinematic frame by the kinematic bank angle $\mu_K$. $\mathbf{M}_{\bar{K}B}$ is the transformation matrix from the body fixed to the kinematic frame without the rotation about the kinematic bank angle and $\mathbf{M}_{BA}$ is the transformation matrix from the aerodynamic to the body frame. $\bar{q}$ and $S$ denote the dynamic pressure and the wing reference area respectively. $C_L$, $C_D$ and $C_Q$ denote the aerodynamic lift, drag and side force coefficients specified in the aerodynamic frame.

There are two types of influences to the forces. First there are flight condition quantities like the Mach number and the air density. Those cannot be used to control the path dynamic. The second type of influence is formed by the variables that can be used for control. There are the aerodynamic angle of attack $\alpha_A$, controlling the magnitude of lift and the aerodynamic angle of side slip $\beta_A$. But as stated above the latter is not used for control since it is considered a secondary control. The flight path bank angle, although it has no influence on the aerodynamic coefficients, is used to turn the lift vector into the desired direction. Finally the thrust is made up by two components, an inlet impulse $T_I$ and an outlet impulse $T_O$. While the inlet impulse acts into the direction of the aerodynamic x-axis, the outlet impulse is fixed relative to the body-fixed frame. In case of conventional propulsion system the relative direction of the outlet impulse is fixed by the thrust elevation angle $\sigma$ and the thrust azimuth angle $\kappa$. In case of thrust vectoring these angles could be used for control, too. The magnitude of inlet- and outlet impulse are both influenced by the thrust lever state $\delta_T$. Of course they are also dependent on the Mach number and the air density. To sum it up, the primary controls for path dynamics are:

| | |
|---|---|
| Aerodynamic angle of attack: | $\alpha_A$ |
| Flight path bank angle: | $\mu_K$ |
| Thrust lever state: | $\delta_T$ |

Looking at equation (13) it is clear that it cannot be analytically solved for the control variables. In order to find a remedy the global inversion is suspended by an incremental inversion. Thus the required force increment can be calculated.

$$\Delta\left(\vec{\mathbf{F}}\right)_K = \left(\vec{\mathbf{F}}_{DES}\right)_K - \left(\vec{\mathbf{F}}_{ACT}\right)_K \tag{14}$$

Provided that the path commands are compliant with the plant capabilities it is reasonable to assume that the reference model error and thus the required force increment is quite small. Therefore it is legitimate to linearize equation (13) by building a local Jacobian with respect to the primary controls. Before the linearization is carried out the assumption “no wind” is applied to equation (13).

$$\left(\vec{\mathbf{F}}\right)_K = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}_K =$$

$$\mathbf{M}_{K\bar{K}}(\mu)\cdot \left\{\left[\bar{q}S\begin{pmatrix} -C_D \\ C_Q \\ -C_L \end{pmatrix}_A - \begin{pmatrix} T_I \\ 0 \\ 0 \end{pmatrix}_A\right] + \mathbf{M}_{\bar{K}B}(\alpha,\beta)\begin{pmatrix} \cos\sigma\cos\kappa \\ \cos\sigma\sin\kappa \\ \sin\kappa \end{pmatrix}_B T_O\right\} \tag{15}$$

This assumption is appropriate since the effect of the wind onto the force increment is canceled out since the wind is applied to the actual **and** the desired force where the increment is build from the difference between them.

The differentiation with respect to the primary controls is done analytically as far as possible. Therefore the gradients of the aerodynamic coefficients with respect to the controls such as $C_{L\alpha}$ and $C_{D\alpha}$ have to be known, as well as the gradients of the inlet and outlet impulse with respect to the controls $\partial T_O / \partial \delta_T$ .

$$\mathbf{B} = \begin{bmatrix} \frac{\partial X}{\partial \alpha} & \frac{\partial X}{\partial \mu} & \frac{\partial X}{\partial \delta_T} \\ \frac{\partial Y}{\partial \alpha} & \frac{\partial Y}{\partial \mu} & \frac{\partial Y}{\partial \delta_T} \\ \frac{\partial Z}{\partial \alpha} & \frac{\partial Z}{\partial \mu} & \frac{\partial Z}{\partial \delta_T} \end{bmatrix} \tag{16}$$

The result is a local quadratic control allocation matrix which describes the relationship between the required force increment and the control vector increment $\Delta\mathbf{u}_C = \left(\Delta\alpha_C \quad \Delta\mu_C \quad \Delta\delta_{T,C}\right)^T$ .

$$\Delta\left(\vec{\mathbf{F}}\right)_K = \mathbf{B}\cdot\Delta\mathbf{u}_C \tag{17}$$

Thus the required control increment is computed by simply solving the system of linear equations (17). The absolute control commands are obtained by adding the desired command increments to the actual control states, which can be either be measured or estimated by an actuator model $\hat{\mathbf{G}}_A$ .

## 3 Reference Model Limitation

The dynamic inversion approach allows independent control of the path variables as long as no limitations come into effect. In case of saturated controls the reference model commands for airspeed $V_A$ , flight path angle $\gamma_K$ and course angle $\chi_K$ may no longer be independent. In order to make the behavior in case of limited controls determinate, the reference models are equipped with several limiters for the reference model command signals as well as for the first time derivative of the reference model state and the reference model state itself. These limiters are filled with the values derived by the energy and force considerations in the following chapter. Fig. 2 illustrates the limiter structure of the energy related prioritizations and Fig. 3 presents the force related cross-feed limits in the flight path and course angle reference models.

**Command limiter**

The command value limiter limits the raw commands into a valid range. Furthermore in case of a direct command to the path loop, i.e. cutting the trajectory loop commands, the upper and lower limit can be faded to the same, the desired, value. This, contrary to switches, avoids steps in the commands with any further effort.

**Reference model rate command limiter**

Just as the command limiter, the rate limiter cuts the reference model rate at first into a legitimate range (namely the linear acceleration, the flight path angle rate and the turn rate). Besides it can also be used for specifying a desired rate by fading the upper and lower limit to the same value.

**Prioritization cross-feed limiter**

The cross-feed limiters implement the prioritization between conflicting variables in case of saturated controls. Generally the limiters are placed downstream in the signal flow relative to the command limiters, since they shall override the commands because of the higher relevance for the aircraft safety. The energy principle discussed in the next chapter leads to a trade-off between linear acceleration and the flight path angle. Therefore one cross-feed limiter is placed after the acceleration command limiter in the velocity reference model and the other is placed after command limiter of the flight path angle reference model.

The force principle discussed in the following leads to a trade of between the flight path angle rate and the turn rate. Thus two cross-feed limiters are placed as rate limitation in the flight path angle and the course angle reference model each after the reference model rate command limiter.

**Protection limiters**

The protection limiters are placed further downstream relative to the cross-feed limiters since they have the highest priority related to the aircraft safety and thus should override the other limiters. The limits are set by physical deliberations as described in detail later. In case of the linear acceleration the maximum available thrust is the limiting factor. For the speed envelope protection a phase-plane approach is chosen in order to avoid dynamic overshoots beyond the limit. The flight path angle and turn rate are restricted by the maximum available load-factor protecting the aircraft against stall.

**Integrator state limit**

Finally the integrator state itself is limited despite the command value is limited, which would avoid the integrator state to exceed the limits since it's a first order model. Nevertheless due to the pseudo-control hedging signal which is added to the reference model rate after all the rate limiters, the integrator state could leave the valid domain when not limited.

The design of the reference model time constants the feedback gains is automated by considerations of the step response of a system model with uncertainties and an assumed linear actuator dynamic representing the next inner loop. However a detailed description of the design algorithm would go beyond the scope of the paper.

## 4 Energy Considerations

This chapter describes the energy based prioritization and protection mechanisms in detail. The total energy of the aircraft, consisting of kinetic and potential energy is:

$$E_T = E_{kin} + E_{pot} = \frac{1}{2} m V_K^2 + mgh \qquad (18)$$

Derived once to the time yields the total energy flow

$$\dot{E}_T = m V_K \dot{V}_K + mg\dot{h} = m V_K \left(\dot{V}_K + g \sin \gamma_K\right) \qquad (19)$$

which equals to the energy provided by the engine less the power dissipated by the aerodynamic drag.

$$\frac{\dot{E}_T}{m V_K} = \frac{T - D}{m} = \dot{V}_K + g \sin \gamma_K \qquad (20)$$

Equation (20) makes clear that the specific excess power is either used in order to gain kinetic or potential energy. As both, acceleration and flight path angle can be measured, the total energy equivalent acceleration can be calculated

$$\dot{V}_{TE} = \dot{V}_K + g \sin \gamma_K \qquad (21)$$

The same holds for the energy equivalent flight path angle

$$\gamma_{TE} = \arcsin\left(\frac{\dot{V}_K}{g} + \sin \gamma_K\right) \qquad (22)$$

### 4.1 Energy Based Prioritizations

Knowing the total energy equivalent acceleration, a flight path angle can be computed that is achieved with the current energy flow provided by the thrust.

$$\gamma_X = \arcsin\left(\frac{\dot{V}_{TE} - \dot{V}_{K,DES}}{g}\right) \qquad (23)$$

The desired acceleration is taken from the reference model pseudo command $\nu_{V,RM}$ and the resulting $\gamma_X$ is applied to the flight path reference model cross-feed command limiter (Fig. 2). Thus the flight path angle command is sacrificed in order to achieve the desired acceleration.

Analogous a flight path angle results for a desired acceleration.

$$\dot{V}_X = \dot{V}_{TE} - g \sin \gamma_{K,DES} \tag{24}$$

The desired flight path angle is taken from the reference model state $\gamma_{K,RM}$ and the resulting acceleration $\dot{V}_X$ is applied to the cross-feed rate limiter of the airspeed reference model.

The two cross-feed limitations are not applied simultaneously but exclusively depending whether the prioritization is set to speed or flight path angle priority by an external moding logic.

A further important fact is that the energy flow is computed for the actual thrust lever state. The cross-feed limiters are engaged as soon as the commanded thrust lever state $\delta_{T,C}$ runs into the saturation. At this moment the actual thrust lever state has not reached the limit yet due to the engine dynamics. So the calculated energy flow is too low and thus the cross-feed limits are too conservative. So the actual energy flow must be corrected by the estimated thrust reserves to full thrust and idle.

$$\dot{E}_{T,MAX} = m \cdot V_K \cdot \dot{V}_K + m \cdot g \cdot \dot{h} + \Delta T_{MAX} \cdot V_K \tag{25}$$

$$\dot{E}_{T,MIN} = m \cdot V_K \cdot \dot{V}_K + m \cdot g \cdot \dot{h} + \Delta T_{MIN} \cdot V_K \tag{26}$$

So the thrust-corrected cross-feed values for the flight path angle are:

$$\gamma_{X,T_{MAX}} = \arcsin\left( \sin \gamma_K - \frac{1}{g} \cdot \left( \dot{V}_{K,DES} - \dot{V}_K \right) + \frac{\Delta T_{MAX}}{mg} \right) \tag{27}$$

$$\gamma_{X,T_{INX}} = \arcsin\left( \sin \gamma_K - \frac{1}{g} \cdot \left( \dot{V}_{K,DES} - \dot{V}_K \right) + \frac{\Delta T_{MIN}}{mg} \right) \tag{28}$$

Analogous the thrust corrected acceleration cross-feed values are:

$$\dot{V}_{X,T_{MAX}} = \dot{V}_K - g\left( \sin \gamma_{K,DES} - \sin \gamma \right) + \frac{\Delta T_{MAX}}{mg} \tag{29}$$

$$\dot{V}_{K,T_{MIN}} = \dot{V}_K - g\left( \sin \gamma_{K,DES} - \sin \gamma \right) + \frac{\Delta T_{MIN}}{mg} \tag{30}$$

## 4.2 Energy Based Protections

Derived from the total energy philosophy, a speed envelope protection can be implemented into the reference model limit structure. Therefore a phase-plane limit is calculated for the protection limiter of the velocity reference model when the dynamic pressure $\bar{q}$ approaches the upper of lower boundary (Fig. 5).

$$V_{\bar{q}_{MIN}} = k\left( \bar{q}_{MIN} - \bar{q} \right) \qquad V_{\bar{q}_{MAX}} = k\left( \bar{q}_{MAX} - \bar{q} \right) \tag{31}$$

In case of flight path angle priority, $\gamma_K$ has to be reduced when the aircraft is approaching the low speed limit in order to achieve the necessary phase-plane protection acceleration. The same holds for the upper speed limit, where the flight path angle must be increased to increase the deceleration capability. In order to avoid step inputs the flight path angle reference model command limit is blended to the total energy coupled value derived in equation (17) in order to achieve the phase plane acceleration.

$$\gamma_{X;MAX}^{(\gamma-\mathrm{Prio})} = \underset{\left(\gamma_X, \frac{\pi}{2}\right)}{sat}\left( \gamma_X + \frac{\pi/2 - \gamma_X}{\dot{V}_{Trans,MIN} - \dot{V}_{\bar{q},MIN}} \cdot \left( \dot{V}_K - V_{\bar{q},MIN} \right) \right) \tag{32}$$

$$\gamma_{X,MIN}^{(\gamma-\mathrm{Prio})} = \underset{\left(-\frac{\pi}{2}, \gamma_X\right)}{sat}\left( \gamma_X + \frac{\gamma_X - (-\pi/2)}{\dot{V}_{\bar{q},MAX} - \dot{V}_{Trans,MAX}} \cdot \left( \dot{V}_K - V_{Trans,MAX} \right) \right) \tag{33}$$

Fig. 4 shows the limiting characteristic, applied to the cross-feed limiter. The fading begins at a transition acceleration $\dot{V}_{Trans}$, which is also defined in the phase-plane as illustrated in Fig. 4. In case of speed priority, the flight path angle is limited to the thrust corrected values $\gamma_{X,T,MAX}$ and $\gamma_{X,T,MIN}$ anyway. Nevertheless when the aircraft is approaching the dynamic pressure limits, the saturation value is reduced to the non thrust corrected cross-feed value $\gamma_X$ since it is relevant for safety of the aircraft. The fading is done in an analogous way to equations (32) and (33) as illustrated in Fig. 4.

In addition to the reduction of the flight path angle limits, the allowed thrust region is limited too. When approaching the lower speed limit the minimum thrust limit is faded to the maximum thrust, so that the aircraft uses its full capabilities to leave the dangerous region, at first by reducing the flight path angle and second by increasing the thrust. An analogous strategy is pursued at the upper speed limit. When approaching the maximum dynamic pressure, the flight path angle is increased and the thrust is reduced

## 5 Force Considerations

Analogous to the energy considerations, an abstraction of the philosophy to the transverse force leads to a trade-off between the curvature of the trajectory in the horizontal and in the vertical plane.

The forces in y- and z-axis direction of the path-axis frame, e.g. perpendicular to the direction of movement, effect the compensation of gravity and a flight path angle and turn rate as shown in equations (10) and (11). The maximum load-factor, which is a measure for the transverse force is limited by the maximum lift coefficient at low speeds and by structural limits at high speeds.

$$n_{Z,MAX} = \min\left(n_{Z,MAX,Aero}, n_{Z,MAX,Struct}\right) \tag{34}$$

At any time the force perpendicular to the trajectory must not exceed the load-factor limit.

$$n_{ACT} = \frac{\sqrt{(Y_{ACT})_K^2 + (Z_{ACT})_K^2}}{mg} = \\ = \sqrt{\left(\frac{V_K}{g}\cos\gamma_K \dot{\chi}_K\right)^2 + \left(\cos\gamma_K + \frac{V_K}{g}\dot{\gamma}_K\right)} \geq n_{Z,MAX} \tag{35}$$

The load-factor commanded by the flight path and course angle reference models analogously is:

$$n_C = \sqrt{\left(\frac{V_K}{g}\cos\gamma_K \cdot \nu_\chi\right)^2 + \left(\cos\gamma_K + \frac{V_K}{g}\nu_\gamma\right)} \tag{36}$$

From these equations the cross-feed limits for each, path-angle and turn rate, can be calculated in order to achieve the other one. In case for pitch priority:

$$\dot{\chi}_{X,MAX} = \frac{g}{V_K\cos\gamma_K}\sqrt{n_{Z,MAX}^2 - \left(\cos\gamma_K + \frac{V_K}{g}\nu_{\gamma,RM}\right)} \tag{37}$$

$$\dot{\chi}_{X,MIN} = -\frac{g}{V_K\cos\gamma_K}\sqrt{n_{Z,MAX}^2 - \left(\cos\gamma_K + \frac{V_K}{g}\nu_{\gamma,RM}\right)} \tag{38}$$

and in case for turn priority

$$\dot{\gamma}_{X,MAX} = \frac{g}{V_K}\sqrt{n_{Z,MAX}^2 - \left(\frac{V_K}{g}\cos\gamma_K\nu_{\chi,RM}\right)} - \frac{g}{V_K}\cos\gamma_K \tag{39}$$

$$\dot{\gamma}_{X,MIN} = -\frac{g}{V_K}\sqrt{n_{Z,MAX}^2 - \left(\frac{V_K}{g}\cos\gamma_K\nu_{\chi,RM}\right)} - \frac{g}{V_K}\cos\gamma_K \tag{40}$$

These values are applied to the cross-feed rate limiters of the flight path and course angle reference model, dependent on the prioritization that is chosen from the higher level moding logic. Considering equations (37) and (38) the turn-rate limit only has a real valued solution when the expression under the square root is bigger than zero. The pitch rate for which the square root becomes zero is, physically interpreted, the maximum pitch rate that can be commanded at that maximum load-factor. Thus there is no capacity left for maneuvers in the horizontal plane.

$$\dot{\gamma}_{PROT,MAX} = \frac{g}{V_K}\left(-n_{Z,MAX} - \cos\gamma_K\right) \tag{41}$$

$$\dot{\gamma}_{PROT,MIN} = \frac{g}{V_K}\left(n_{Z,MAX} - \cos\gamma_K\right) \tag{42}$$

Analogously the square root in equations (39) and (40), set to zero, yields the maximum turn rate capacity.

$$\dot{\chi}_{PROT,MAX} = \frac{g}{V_K\cos\gamma_K}n_{Z,MAX} \tag{43}$$

$$\dot{\chi}_{PROT,MIN} = -\frac{g}{V_K\cos\gamma_K}n_{Z,MAX} \tag{44}$$

The protection values of equations (41) – (44) are applied to the protection rate limiters of the flight path and course angle reference models, further downstream relative to the cross-feed rate limiters, as illustrated in Fig. 3.

The protection values computed in equations (43) and (44) give full priority to the horizontal plane to an extent that not even the flight path angle could be maintained due to gravity, which can be seen in equations (39) and (40) when the square root is set to zero. Thus an alternative limit for the course rate could be obtained, when $\gamma_{X,MAX}$ and $\gamma_{X,MIN}$ in equations is (39) and (40) is set to zero.

$$\dot{\chi}_{PROT,MAX} = \frac{g}{V_K}\sqrt{\left(\frac{n_{Z,MAX}^2}{\cos\gamma_K}\right)^2 - 1} \tag{45}$$

$$\dot{\chi}_{PROT,MAX} = -\frac{g}{V_K}\sqrt{\left(\frac{n_{Z,MAX}^2}{\cos\gamma_K}\right)^2 - 1} \qquad (46)$$

Here in case of turn priority the maximum turn rate limits are restricted, so that at least the gravity can be compensated and thus the flight path angle in maintained.

## 6 Conclusions

The paper presented a physical interpretation of the path dynamics which forms the basis for a reference model based dynamic inversion controller. Several limitations in the reference models are used to resolve the control objective conflicts in case of saturated controls. Two limiting factor were identified. The total energy flow, which is limited by the engine power, leads to a trade-off between flight path angle and acceleration. The derived equations are leaned against the total energy control system (TECS) [4] [5].

A second limitation that was identified is the transverse force, produced by lift. A generalization of the total energy philosophy to the maximum load-factor leads to a trade-off between horizontal and vertical maneuvering capability.

The derived equations are based on physical consideration of the path dynamic and allow a stable determinate behavior in case of saturated inputs dependent on the prioritization chosen by a higher level moding logic. Nevertheless at normal flight conditions a decoupled control of the three path variables is possible.

Further research will focus on the extension of the principles. Until now the prioritization is only made in full favor of one variable. In case of the force principle a prioritized plane of curvature could be defined, which e.g. allows an optimal performance in a terrain following flight, when the plane of curvature is specified to be perpendicular to the terrain.

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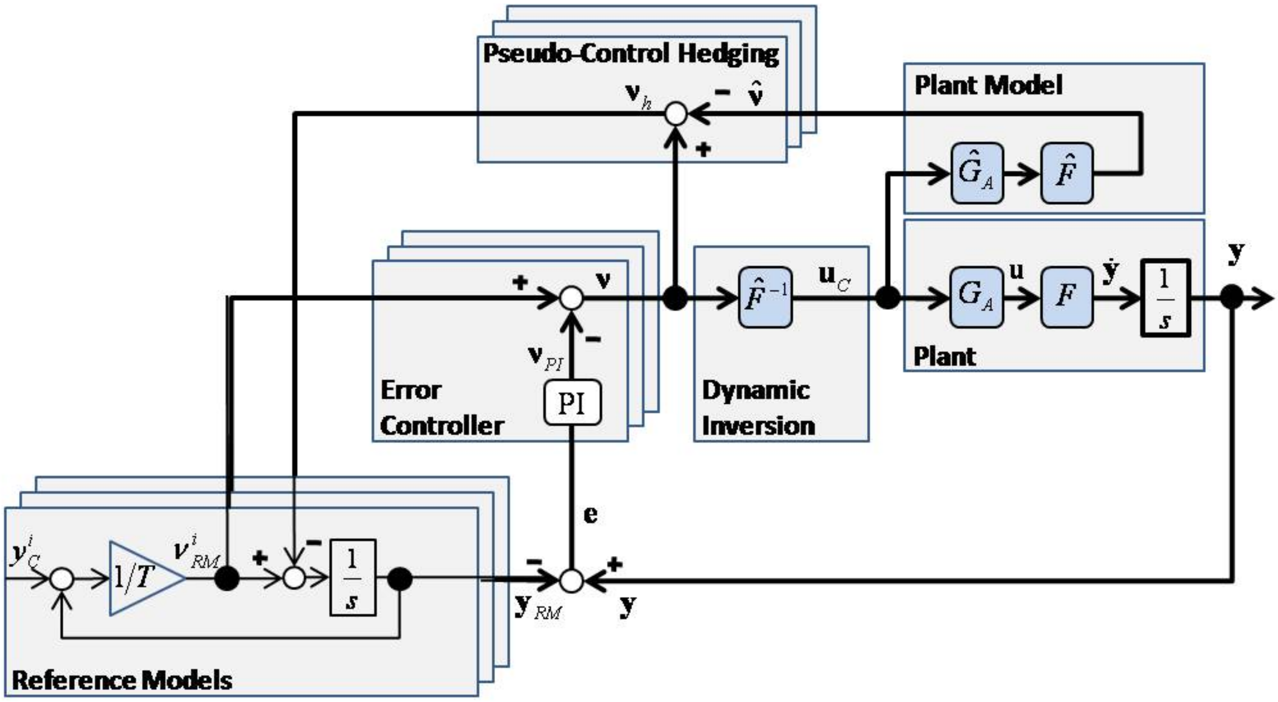


**Fig. 1** **Dynamic Inversion Controller with Reference Models and Pseudo-Control Hedging**

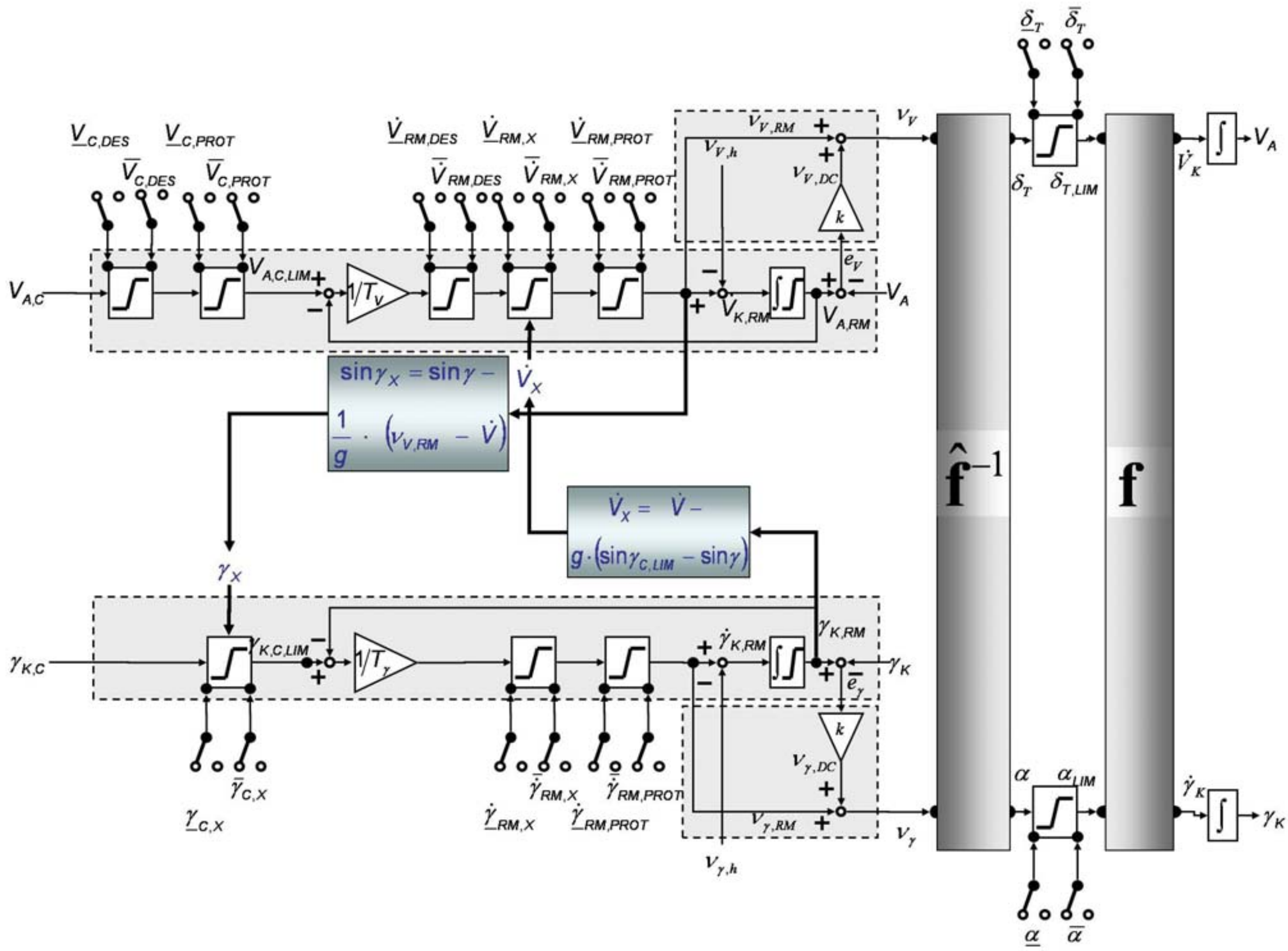

**Fig. 2** **Cross-Feed Limitations between Airspeed and Flight path Angle Reference Models**

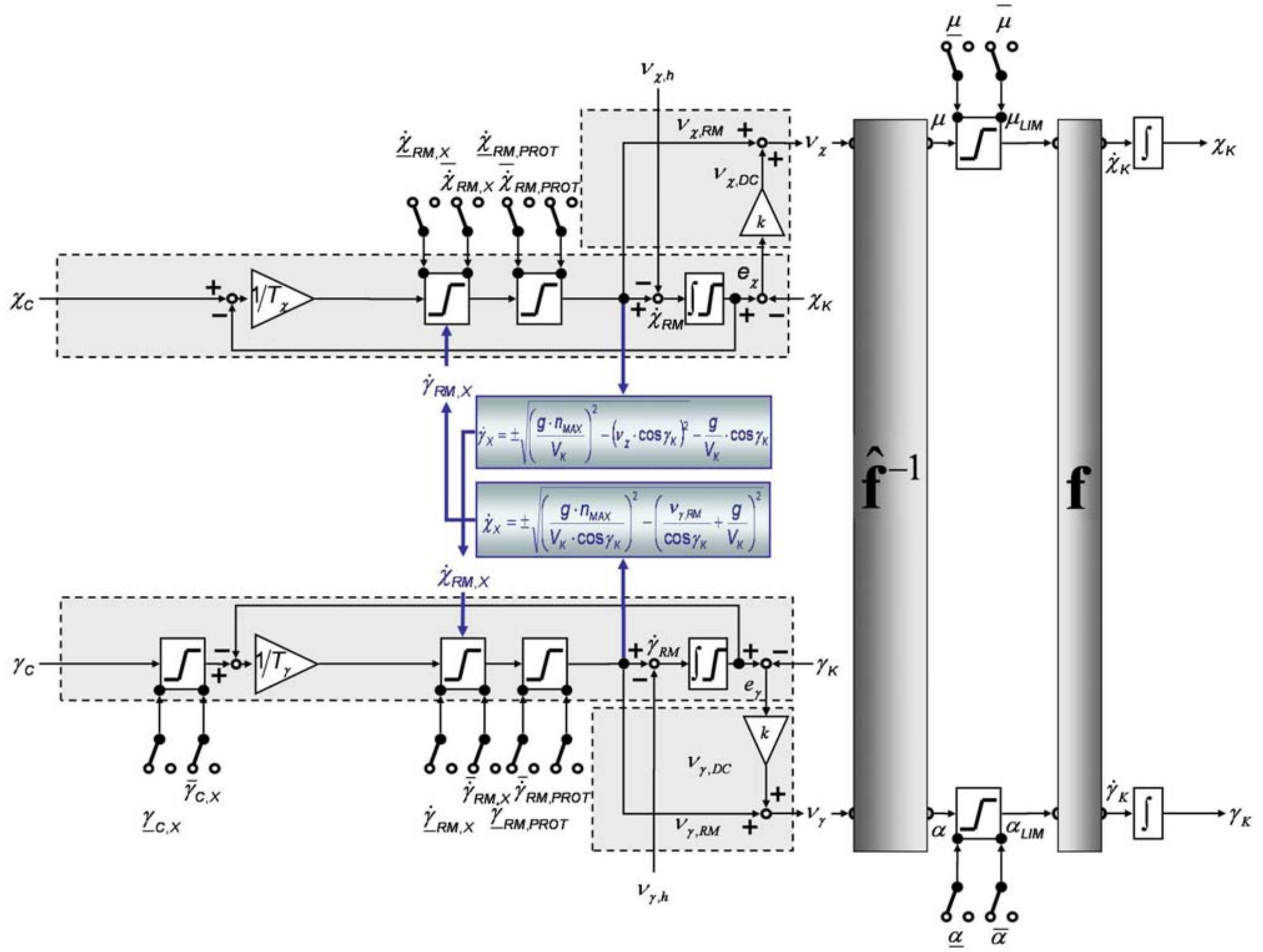


**Fig. 3 Cross-Feed Limitations between Flight path Angle and Course Angle Reference Models**

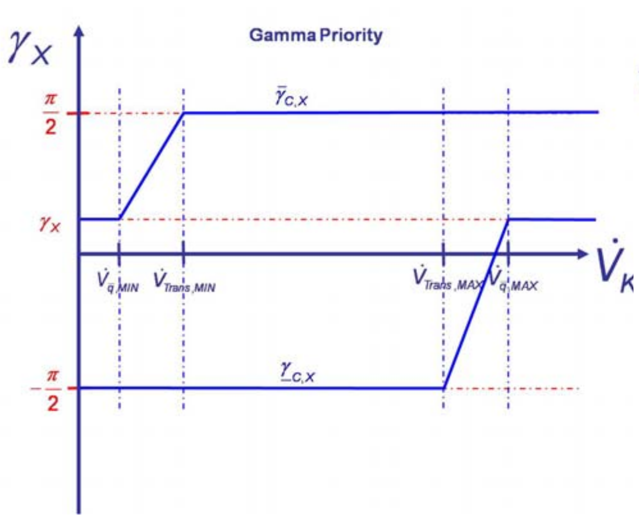


**Fig. 4 Flight path Limit for Speed Protection Flight path Priority**

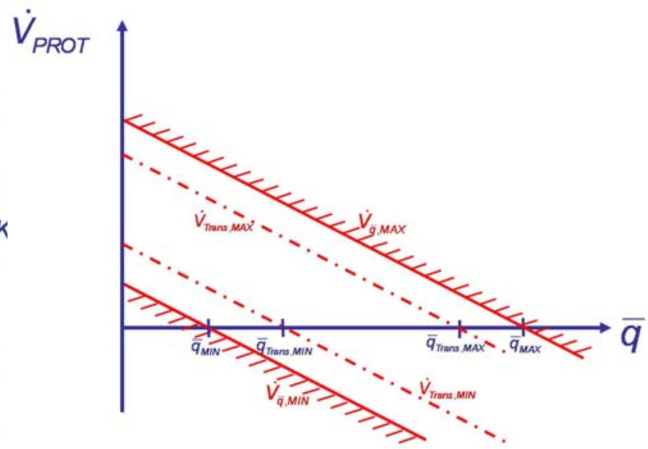

**Fig. 5 Phase-Plane Limits for Speed Protection**