APPLICATION OF SHAPE OPTIMIZATION TO AIRCRAFT WING STRUCTURES

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Abstract

The demand for more efficient and lighter structural components has shifted focus to formal optimization techniques in order to change the structure's topology automatically to achieve a given objective; for example, minimizing the weight of the structure without compromising on its performance.

This paper explores the concepts for structural optimization employed in design improvement using the shape optimization technique. Shape optimization software has been successfully employed to optimize an aircraft wing structure, comprising stringers and skins to model structural parts.

The mathematical methods of structural optimization are presented. A sensitivity analysis precedes the optimization process of the stringer-skin structure. The stringers are repositioned on the skin while maintaining a smooth shape after the optimization process. It was found that the performance of the structure improved. Shape optimization can enable weight savings as well as long term cost benefits, while ensuring that the component is structurally sound and can be commercially manufactured.

1 Introduction

The enhancement of computational expertise and processing power has enabled engineers to harness technology to design more efficient structures that are optimized to fulfill the requirements for which they were built.

A structure can be optimized using different methodologies, namely, topological design, shape optimization and design of experiment (DoE). Moreover, objectives of the optimization process can also vary – ranging from optimizing the structure for performance variables such as stress, stiffness and modal frequencies, to parameters such as cost, weight and others.

Topological design enhancement is important in prototype creation and weight reduction problems. Shape optimization, on the other hand, is a technique for performance improvement by altering the geometric parameters of the component, while DoE strives to determine the number of variables that are most critical and influential on the component.

Shape optimization for structural performance improvement is the objective of this project. The technique has been applied to the shape optimization of a typical section of an aircraft wing structure. The approach outlined herein highlights that once a structure has been discretized using a Finite Element Model (FEM), a number of design variables affect the structure such that, depending on the variable under consideration, there can be a multitude of solutions to a given problem.

There is a high possibility that not all solutions will be acceptable from an engineering, manufacturability, cost, durability and efficiency standpoint. Therefore, the software used in the optimization process should be capable to sift through the valid and invalid results to ensure that the final result is a feasible, optimized and realistic option. This has been explored using optimization software, and the results are depicted in Section 4.5.

2 Mathematical Background

The objective of shape optimization is to improve the performance of an existing component through manipulating the geometry of the component. Shape change presents many challenges due to constraints that are not easily defined mathematically, for example packaging and manufacturing constraints.

The theory of the application of shape change to FE problems is well described in research and the reader is directed to a more complete procedure in Ref. [6].

2.1 Sensitivity of Parameters

The relationship between the performance of a structure based on its nodal co-ordinates can be calculated by considering the solution of the model as a result of perturbation of the nodal co-ordinates. However, by employing this methodology, the change of each nodal position in each of the degrees of freedom would require at least two analyses, resulting in a vast amount of data..

In the shape optimization software used for solving the presented problem, the sensitivities were analytically calculated for every step, which was then used to evaluate the gradients of the performance parameters under consideration. It is important to note that an irregularity in the FE geometry could have a detrimental effect on the sensitivity. To counter any unwanted influence from the mesh, nodal sensitivities are recalculated as sensitivity densities and then scaled based on the sensitive parameters of the node in the shape optimization software. The sensitivity can then be applied directly to simple structures (for a practical result) or can be used to identify the sensitive regions of a complex structure. Such information can facilitate in identifying regions in a structure with scope for performance improvement rather than investing in futile attempts to improve a design that is intrinsically bad.

2.2 Method for Calculating Sensitivities

The basis for the evaluation of sensitivities is demonstrated as outlined in Ref. [6].

Let the performance parameter be represented by p, which is a scalar function of the geometry (denoted by $\boldsymbol{x}$) and the displacement vector (denoted by $\boldsymbol{u}(x)$). In equation form, this can be written as:

$$p = p(\boldsymbol{x},\boldsymbol{u}(\boldsymbol{x})) \qquad (2)$$

The sensitivity vector (denoted by $\boldsymbol{s}$) is represented as

$$s = \left(\frac{dp}{d\mathbf{x}}\right)^T \qquad (3)$$

Hence, the derivative d$p/d\boldsymbol{x}$ is denoted by

$$\frac{dp}{d\mathbf{x}} = \frac{dp}{d\mathbf{x}} + \frac{dp}{d\mathbf{u}} \bullet \frac{d\mathbf{u}}{dx} \qquad (4)$$

From the FE equation, the matrix, d$\boldsymbol{u}/d\boldsymbol{x}$, is given by Equation 5, where $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{u}$ is the displacement vector, and $\boldsymbol{f}$ is the load vector. Hence,

$$\mathbf{K} \bullet \mathbf{u} = \mathbf{f} \qquad (5)$$

Differentiating the above equation, Equation (6) is obtained, where the number of terms on the right hand side of the equation is equal to the number of components of the vector $\boldsymbol{x}$. Consequently, in comparison to the sizing sensitivity, the number of terms is huge.

$$\mathbf{K} \bullet \frac{d\mathbf{u}}{dx} = \frac{d\mathbf{f}}{d\mathbf{x}} - \frac{d\mathbf{K}}{d\mathbf{x}} \bullet \mathbf{u} \qquad (6)$$

Due to the large number of terms, the adjoint – variable method is more appropriate for use in the case of shape optimization. According to the adjoint – variable method, the matrix $\boldsymbol{du/dx}$ can be expressed in accordance with Equation (6). Hence,

$$\frac{dp}{d\mathbf{x}} = \frac{\partial p}{\partial \mathbf{x}} + \frac{\partial p}{\partial \mathbf{u}} \bullet \mathbf{K}^{-1}\left(\frac{d\mathbf{f}}{d\mathbf{x}} - \frac{d\mathbf{K}}{d\mathbf{x}} \bullet \mathbf{u}\right) \qquad (7)$$

Substituting $\partial p/\partial \mathbf{u}.\mathbf{K}^{-1}$ by the formal parameter vector Λ^T in Equation 7, which is obtained by solving the linear system, Equation 8 is obtained.

$$\mathbf{K} \bullet \Lambda = \left(\frac{\partial p}{\partial \mathbf{u}}\right)^T \qquad (8)$$

In Equation (8), the number of terms on the right hand side is equal to the number of sensitivities, which in turn is usually equal to one. Hence, the derivatives $\partial p/\partial \mathbf{u}$ and $\partial p/\partial \mathbf{x}$ can be calculated. In the software used for the solution of the presented problem, the derivative $\mathrm{d}\mathbf{K}/\mathrm{d}\mathbf{x}$ is also calculated analytically for commonly used elements.

2.3 Application of Shape Optimization in Software

The relationship between a generalized coordinate system and the nodal geometry, $\mathrm{d}\mathbf{x}/\mathrm{d}\mathbf{q}$, is the basis on which a FEM can be optimized for shape. This relationship is often called the *influence matrix*.

In the software the influence matrix is first used to determine the sensitivity in design variables by considering the nodal sensitivities, and then the results are fed back to realign the FEM from the change in design variables.

Recalling the sensitivity of the performance parameters with respect to a change in the shape of the FE model:

$$\mathrm{d}\boldsymbol{p}/\mathrm{d}\boldsymbol{x} \qquad (9)$$

These ‘raw’ sensitivities can already be used for simple structures, such as trusses, as well as to establish sensitive areas of a model which could assist in optimizing the structure for maximum returns. The raw sensitivities cannot be applied directly to change the shape of complex surface or solid-based models since the original smoothness of the FE model cannot be maintained.

To address this issue of FE smoothness, a mesh-geometry associativity parameter is introduced into the shape optimization process. The mesh-geometry associativity factor is a set of generalized coordinates, and can be represented by Equation (10).

$$\mathbf{x} = \mathbf{x}(\mathbf{q}) \qquad (10)$$

As such the generalized coordinates could encompass:

- Modal vectors (where q denotes the individual modal vectors), or
- Displacement vectors (where q is the scaling factor of the applied virtual load), or
- Splines (wherein q denotes represents the spline parameters), or
- Influence functions (in which q describes user-defined points for generalized co-ordinates).

The fundamentals of smoothing are based on the premise of creating the best fit for nodal changes, $\Delta\boldsymbol{x}$. This can be explained by considering the effect of smoothing on the raw sensitivities, which would be proportional to the function $dp/d\mathbf{x}$. For the change to be smooth, the nodal sensitivities need to be calculated as the sensitivities of generalized coordinates, $dp/d\boldsymbol{q}$. Hence, if the influence matrix is denoted by $d\boldsymbol{x}/d\boldsymbol{q}$, then it follows that:

$$\frac{dp}{d\mathbf{q}} = \frac{dp}{d\mathbf{x}} \bullet \frac{d\mathbf{x}}{d\mathbf{q}} \qquad (11)$$

The change in the design variables, represented as $\Delta\boldsymbol{q}$, using the method of steepest descent is therefore given:

$$\Delta\mathbf{q} = \alpha \bullet \left(\frac{dp}{d\mathbf{q}}\right)^T \bullet \left\|\frac{dp}{d\mathbf{q}}\right\|^{-1} \qquad (12)$$

In Equation (12), $\|dp/d\boldsymbol{q}\|$ is the normalized vector, $dp/d\boldsymbol{q}$, and α is the user-defined iteration step. Consequently, the mesh change, $\Delta\boldsymbol{q}$, defined as a function of the influence matrix is given by:

$$\Delta\mathbf{x} = \left(\frac{d\mathbf{x}}{d\mathbf{q}}\right) \bullet \Delta\mathbf{q} \qquad (13)$$

The technique transforms nodal sensitivities to generalized co-ordinate sensitivities and is then repeats in reverse from generalized co-ordinate sensitivities to nodal sensitivities in the process of re-meshing.

This method is valid only for regular meshes and regular generalized co-ordinate systems. In reality, meshes are rarely regular and neither are generalized co-ordinates. In order to fix this discrepancy, it is important to impose the same best fit on the nodal changes as that on the generalized co-ordinates using

$$\Delta \mathbf{x}^T \bullet \Delta \mathbf{x} = \Delta \mathbf{q}^T \bullet \Delta \mathbf{q} \tag{14}$$

Rewriting Equation (14) by substituting values from Equation (13), the best fit criteria can be defined as

$$\Delta \mathbf{q}^T \bullet \left(\frac{d\mathbf{x}}{d\mathbf{q}}\right)^T \bullet \left(\frac{d\mathbf{x}}{d\mathbf{q}}\right) \bullet \Delta \mathbf{q} = \Delta \mathbf{q}^T \bullet \Delta \mathbf{q} \tag{15}$$

As is evident, the above equation can only be satisfied if the matrix ***dx/dq*** is ortho-normal. That is to say

$$\left(\frac{d\mathbf{x}}{d\mathbf{q}}\right)^T \bullet \left(\frac{d\mathbf{x}}{d\mathbf{q}}\right) = \mathbf{I} \tag{16}$$

Such ortho-normalization can be achieved by a number of established methods including the Gram-Schmidt ortho-normalization algorithm.

4 Example Application

The steps involved in optimizing a stringer-skin structure using the shape optimization software have been explained in Ref. [1], and are briefly described hereafter:

- **Analysis**

The *Analysis* procedure is non-iterative and offers linear Finite Element (FE) analysis using elements used in the FEM. The FE capability employed for the problem discussed in this paper was a linear elastic stress-displacement solution (SOL 101 in MSC NASTRAN©) (Ref. [3]).

- **Sensitivity**

The *Sensitivity* procedure calculates the sensitive regions of an FE model with reference to the target of the sensitivity study. The sensitivity is calculated based on the nodes only – there is no influence from the mesh generated in the FEM.

- **Improvement**

Once the sensitive regions of the structure have been identified, the software's *Improvement* procedure amends the shape of the FEM iteratively, using the *method of steepest descent* to achieve the most optimized result subject to the initial constraints of the problem. The modification process ensures that the mesh topology remains smooth (and hence can be manufactured) during each iterative process.

4.1 Improvement Process

An existing FE model can be enhanced to achieve the following outcomes using the shape optimization technique to improve performance. For example, to:

- Minimize the weight of the overall structure, or in local regions.
- Maximize the stiffness of the component, either locally or on the whole.
- Decrease the stress acting on the body or a particular region of the body
- Modify the modal response / frequency of vibration in the FEM, to improve the response of the real-life component
- Increase the buckling resistance of the part (Ref. [1]).

Based on the factors that need to be improved, seven different improvement processes can be employed to optimize the FE model for shape (Ref. [1]). As mentioned in Section 3.1, most improvement processes commence by identifying the sensitive areas and/or sensitive parameters that would result in maximum improvement with minimal changes.

For this exercise of improving the placement of stringers on an aircraft skin in a typical aircraft configuration, Process Geometry has been used for this problem from the software's analyses suite. The theory behind the analysis technique has been explained in the following sections, while the approach taken as part of this problem has been further elaborated sequentially in Section 5.

4.2 Selection of the Generalized Coordinates

Due to the regularity of the structure of the presented problem, the generalized coordinate to be used for the shape change is selected as parametric surfaces-in the software used, bi-cubic spline surfaces are available as a generalized coordinate. Excerpts of the selection process, as outlined in Ref. [6] have been explained in the following paragraphs.

The cubic spline parameters are expressed as $\boldsymbol{q}$, and so for a one-dimensional spline using the relationship between the geometry and the generalized coordinate can be written as

$$x(r) = [q_1, q_2, q_3, q_4] \bullet [f_1(r), f_2(r, f_3(r), f_4(r)]^T \quad (17)$$

The co-ordinates of the end points of the spline are represented by q_1 and q_2, and q_3 and q_4 the slopes dq_1/dr, dq_2/dr in the respective end nodes, and the variable r denotes the parametric co-ordinate of the respective nodes. The quantities f_j (where $j = 1 \ldots 4$) are the polynomials in r such that

$$f_1 = 1 - 3r^2 + 2r^3,$$
$$f_2 = 3r^2 - 2r$$
$$f_3 = r - 2r^2 + r^3, \text{ and}$$
$$f_4 = - r^2 + r^3 \quad (18)$$

Subsequently, the values of the influence matrix, $d\boldsymbol{x}/d\boldsymbol{q}$ can be evaluated as

$$(d\mathbf{x}/d\mathbf{q})_{i,j} = f_j(r_j) \quad (19)$$

for $j = 1,\ldots,4$ and $i = 1,\ldots,3*Number\ of\ nodes$

It is important to note that in this specific case the geometry is processed via the best fit to the changed nodal positions in accordance with the raw sensitivities rather than the influence matrix.

4.3 Analysis Approach

The objective of the stringer-skin application is to minimize the stresses in the structure. One method to achieve this objective was by maximizing the stiffness of the material by appropriately placing the stiffeners along the skin, thereby stiffening the material and subsequently reducing the stress.

Figure 1 represents a simplified flowchart depicting the major milestones in improving a typical aircraft stringer-skin model using shape optimization software.

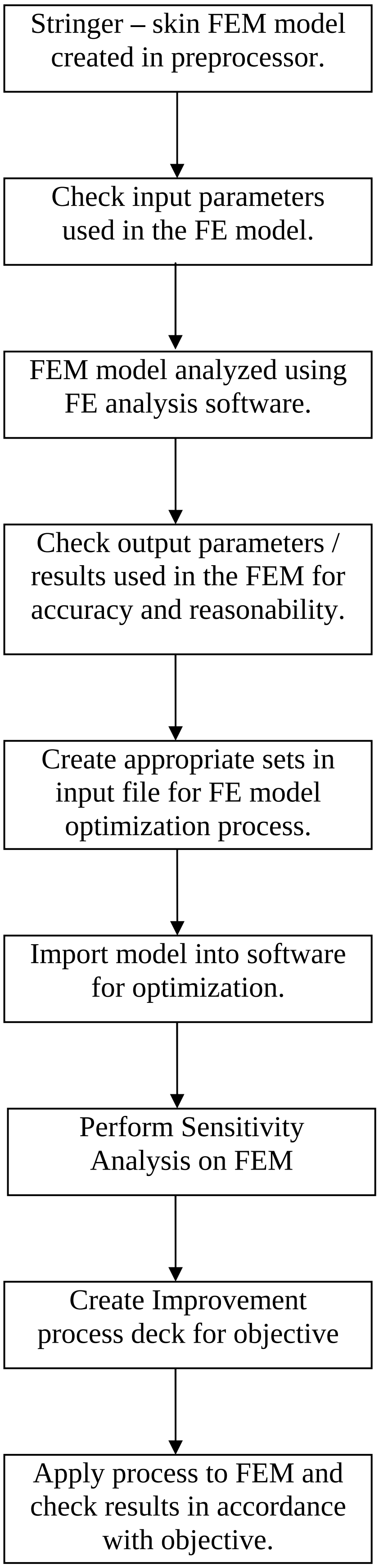

Fig. 1. Flowchart Depicting Methodology

4.4 Finite Element Model

The first step of the optimization process requires the creation of the Finite Element Model. The FEM was constructed using the geometry parameters of a typical aircraft. Representative aircraft loads and boundary conditions were applied to a stringer-skin model constructed in MSC PATRAN©.

The FE model was constructed in 2-D using shell elements for the stringer webs and skin, while the interfaces between the stringer and skin were modeled as common nodes. Fig. 2 shows the isometric view of the stringer elements as they attach to the skin. The material properties applied were that of aircraft-grade aluminum, together with appropriate constraints to model real-life conditions for the aircraft Ref. [5]. Subsequently, the resultant stringer-skin model comprised hundreds of nodes and elements, resulting in a multitude of equations and several unknowns.

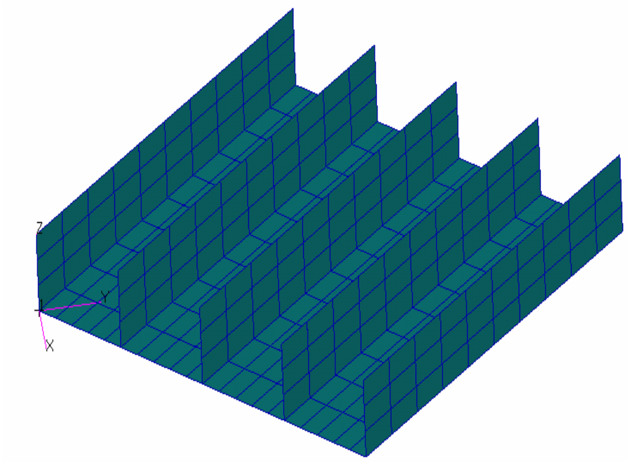

Fig. 2. Stringer-skin FE model

The model shown in Fig. 2 was analyzed for linear static analysis using MSC Nastran© as the solver. The results for the corresponding stringer-skin configuration have been depicted in Fig. 3, and are the von mises stress tensor results at the element centroids. The exact calibration of the original fringe plot cannot be depicted due to confidentiality reasons. However, for the purpose of comparison with the optimized results, an equivalent, but approximate scale has been incorporated into the fringe plots to highlight the reduction in the maximum stress values post the shape optimization process.

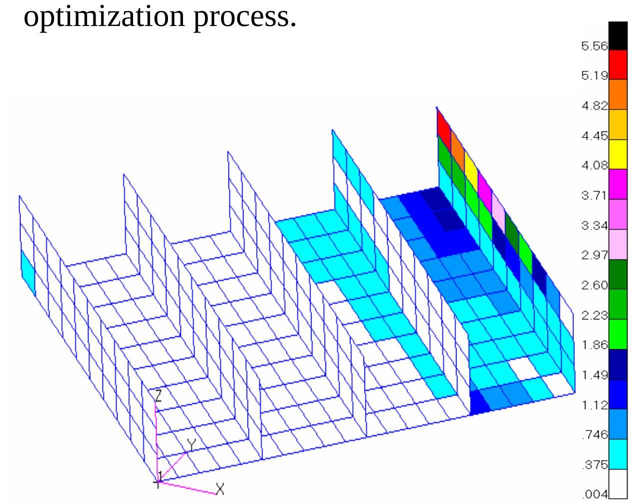

Fig. 3. Stress Tensor Results for Linear Analysis

4.5 Shape Optimization

The FE model shown in Fig. 2 was optimized for minimum stress so that the load applied is reacted by the structure, without having to change the material for higher stiffness. Process Geometry was applied to the FEM. Due to the confidential nature of the results, the actual numbers cannot be mentioned in this paper. Nevertheless, typical results have been shown to give the readers an appreciation of the methodology used, as well as to highlight the capabilities of the software in improving design to accommodate the objective of minimum stress of the structure without compromising on material selection and stiffness.

To reduce the stress acting on the structure, the geometry was modified to maximize the stiffness of the structure by repositioning the stiffeners. The fringe plot for the initial stresses on the sample stringer-skin model is shown in Fig. 4.

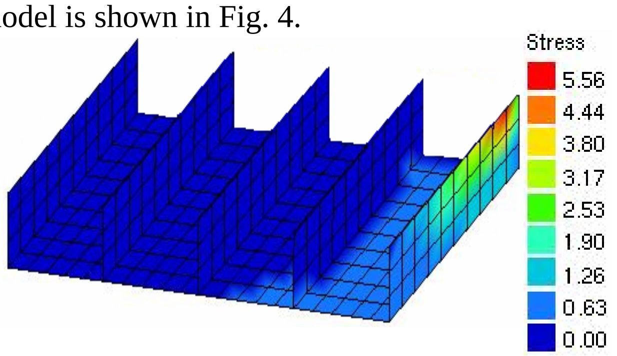

Fig. 4. Initial Stress Results

As seen from Fig. 4, the stiffeners are originally located at equal distances from each other. If the structure would have been optimized for minimum stress by merely 're-shaping' the topology of the stiffeners and without repositioning the stiffeners, then the resultant 'optimized' structure would have been as shown in Fig. 5. As seen from the results depicted in Fig. 5, the stiffeners are warped and not quite easy to manufacture, even though the maximum stresses have been reduced by modifying the shape of the stiffeners.

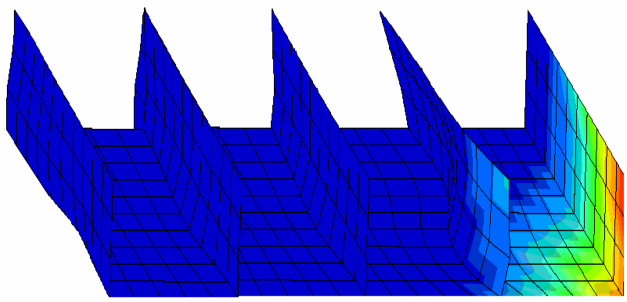

Fig. 5. Improved Stress Results without Repositioning of Stiffeners

Consequently, appropriate commands and side conditions were introduced in the shape optimization command file for the optimization of the stringer-skin configuration, to ensure that the stiffeners remain linear, vertical and planar after the optimization process.

The subsequent iterative shape improvement process was completed in 20 steps. The intermediate shape changes at various steps have been shown from Fig. 6 through to Fig. 10.

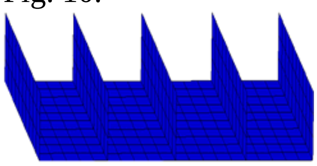

Fig. 6. Initial Shape

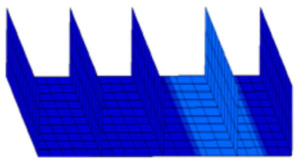

Fig. 7. Step 5 Shape

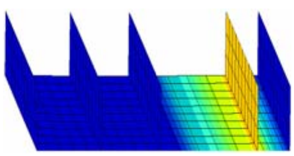

Fig. 8. Step 10 Shape

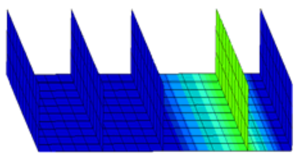

Fig. 9. Step 15 Shape

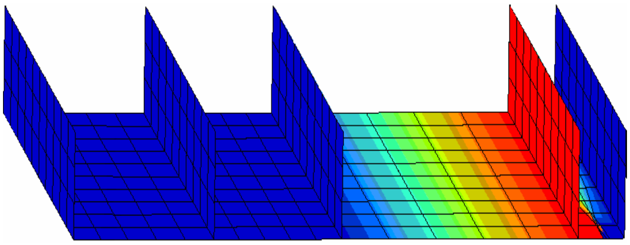

Fig. 10. Final Shape at Step 20

From the above figures, it is seen that the stringer-skin configuration is 'reshaped' step-by-step using shape optimization methodologies. The re-shaping process involved repositioning the stiffeners with the objective of minimizing the stress on the structure.

Once the stiffeners were repositioned, a linear static analysis was conducted to confirm whether or not the stresses had reduced in the structure. The results of the analysis are shown in Fig. 11.

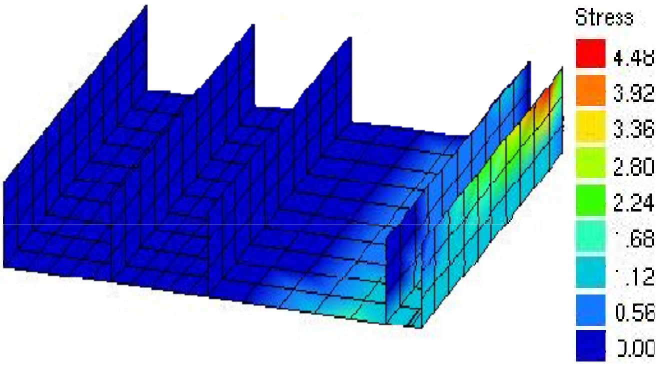

Fig. 11. Stress Results for Optimized Stiffeners

The results indicate that by repositioning the stringers, it is possible to minimize the stress of the stringer-skin model. In this instance, the stress was minimized without warping the shape of the stringers, hence the structure can be manufactured. As seen in Fig. 11, the maximum stresses have reduced by almost 20% of the original, while maintaining the integrity of the structure. This has been possible due to shape optimization software.

5 Conclusion

The concepts of shape optimization for FE models have been discussed in this paper. The optimization process was extended to include a stringer-skin configuration for a typical aircraft wing structure. From the simulations done using the shape optimization software, it was concluded that a sensitivity analysis can be preformed to determine the sensitive parameters and locations of the FE model. Typical aircraft structural components were optimized using the software, which in turn also improved the performance of the structure by minimizing stresses and maximizing the stiffness of the original parent material without compromising on material selection.

It was seen that the structure maintains a smooth shape, enabling ease of manufacture. Hence, instead of employing an alternative (stiffer) material system and maintaining the stringer spacing as that of the original material to achieve the objective of minimizing stresses, by harnessing the optimization capabilities of available software and repositioning the stiffeners on the wing skin, an overall and long term cost and weight savings was also achieved.

In future, the shape optimization process using shall further be extended to more complex aircraft shapes such as a complete aircraft wing. Furthermore, additional research will be conducted to determine methods wherein input parameters and additional results could be obtained by specifying values in the software.

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