APPROXIMATIONS FOR FULLY ISOTROPIC LAMINATE CONFIGURATIONS

C. B. York
Department of Aerospace Engineering, University of Glasgow, Glasgow, Scotland, G12 8QQ

Keywords: *Bending Isotropy, Lamination Parameters, Stacking Sequences*

Abstract.

Approximations for Fully Isotropic Laminate (FIL) configurations are presented as an extension to recent work on Fully uncoupled Orthotropic Laminates (FOLs) and FILs, which have recently been established as a special sub-set of FOLs. Approximations are characterized in terms of their proximity to the non-dimensional lamination parameters describing a FIL configuration. The proximity criteria applied, relates to changes in induced curvature and stiffness properties with respect to the FIL.

1. Introduction.

This article follows on from recent work detailing the definitive set of stacking sequences containing (973) π/3 and (92) π/4 Extensionally Isotropic Laminates (EILs) and (36) π/3 Fully Isotropic Laminates (FOLs), all of which are sub-sets from a definitive list of (69,506) stacking sequences for Fully Orthotropic Laminates (FOLs) with up to 21 plies. The great majority of these configurations are non-symmetric in form and many are without any sub-sequence patterns, e.g. symmetry or repeating groups, which is contrary to the assumptions on which many previous studies have been based.

Standard ply angles (±45°/0°/90° for π/4 isotropy and ±60°/0° for π/3 isotropy) were assumed in the presentation of these configuration listings, however they are otherwise generic in the sense that any orientation may be assigned to the angle-ply sub-sequence, and the (two) cross-ply orientations may be arbitrarily switched. For EIL or FIL properties of course, the choice of these arbitrary ply angles must ensure that π/3 or π/4 ply separation is maintained.

The extension presented in the current article recognizes the fact that very few laminate configurations satisfy the fully isotropic condition exactly; all are 18-ply laminates within the range of thin laminates considered.

By considering configurations that approximate the FIL the scope for broadening the number of possible configurations and/or the numbers of plies in the laminate increases substantially. However, these approximate configurations need to be carefully characterized with respect to the FIL datum so that the mechanical and/or thermal response can be accurately assessed.

There are a number of practical applications for configurations closely approximating the fully isotropic condition. These include large diameter mirrors for space-based reflector telescopes, which require low mass and low coefficient of thermal expansion mirrors, both of which are met by carbon fiber composite laminates. However, configurations for thin laminates to meet the requirements of mechanical and thermal isotropy continue to be of interest [1-4].

2. Classification of uncoupled orthotropic composite laminates.

Composite laminate materials are typically characterized in terms of their response to mechanical (and/or thermal) loading, which is generally associated with a description of the coupling behavior, unique to this type of material, i.e. coupling between in-plane (i.e. extension or membrane) and out-of-plane (i.e. bending or flexure) responses when $B_{ij} \neq 0$, coupling between in-plane shear and extension when $A_{16} = A_{26} \neq 0$, and coupling between out-of-plane bending and twisting when $D_{16} = D_{26} \neq 0$.

The well-known **ABD** relation from classical laminate plate theory, it is often expressed using compact notation hence the coupling behavior, which is dependent on the form of the elements in each of the extensional (**A**), coupling (**B**) and bending (**D**) stiffness matrices, must now described by an extended subscript notation, defined previously by the Engineering Sciences Data Unit, or ESDU [5] and subsequently augmented for the purposes of this article. Hence, balanced and symmetric stacking sequences, which generally possess bending anisotropy, give rise to coupling between out-of-plane bending and twisting and are referred to by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$, signifying that the elements of the extensional stiffness matrix (**A**) are specially orthotropic in nature, i.e. uncoupled, since

$$A_{16} = A_{26} = 0, \tag{1}$$

the bending-extension coupling matrix (**B**) is null, whilst all elements of the bending stiffness matrix (**D**) are finite, i.e. $D_{ij} \neq 0$.

Laminates possessing extensional anisotropy give rise to coupling between in-plane shear and extension only and, by the same rationale, are referred to by the designation $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$, signifying that all elements of the extensional stiffness matrix (**A**) are finite, i.e. $A_{ij} \neq 0$, the bending-extension coupling matrix (**B**) is null, and the elements of the bending stiffness matrix (**D**) are specially orthotropic in nature, i.e. uncoupled, since

$$D_{16} = D_{26} = 0 \tag{2}$$

The $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ designation is not listed as part of the ten laminate classifications described in the ESDU data item [5], but is however the subject of a recent article [6], identifying the definite list of $\mathbf{A}_F\mathbf{B}_0\mathbf{D}_S$ stacking sequences with up to 21 plies, thus complementing a new definitive list [7] of <u>F</u>ully <u>O</u>rthotropic <u>L</u>aminates or FOLs. Note that the term for FOLs is synonymous with specially orthotropic laminates, which possess none of the coupling characteristics described above and are represented by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$.

3.Development of isotropic composite laminates and their approximate configurations.

This article presents the characterization and stacking sequence configurations which approximate Fully Isotropic Laminates or FILs, with the designation $\mathbf{A}_I\mathbf{B}_0\mathbf{D}_I$. These laminates represent sub-sets of FOLs and are therefore contained within the definitive list [7], since in addition to the specially orthotropic form of each matrix, see Eq. (1), elements of the in-plane stiffness matrix simplify further such that the designation $\mathbf{A}_S$ is replaced with $\mathbf{A}_I$ to indicate that:

$$A_{11} = A_{22} \tag{3}$$

and

$$A_{66} = (A_{11} - A_{12})/2 \tag{4}$$

and elements of the out-of-plane stiffness matrix simplify further such that the designation $\mathbf{D}_S$ is replaced with $\mathbf{D}_I$ to indicate that:

$$D_{ij} = A_{ij}\, H^2/12, \tag{5}$$

where H is the laminate thickness, corresponding to the total number of plies, n, of thickness t.

Fully orthotropic laminates, from which all stacking sequences presented in this article are derived, minimize distortion during manufacturing and maximize compression buckling strength [8], particularly in comparison to balanced and symmetric laminates, which are commonly adopted in aircraft and spacecraft construction, despite the fact that such laminates generally possess bending anisotropy. Valot and Vannucci provide recent examples of laminate stacking sequences for FOLs with anti-symmetric sequences [9], following a previous article [10] on FILs. These two articles are part of, and provide reference to, a growing number of related articles by a community of co-workers addressing the development of laminate stacking sequences exhibiting a range of physical characteristics, all of which refer back to an original article by Caprino and Crivelli-Visconti [11], identifying the specially orthotropic angle-ply laminate with eight plies. Much of this related work has been focused on laminate stacking sequences for Quasi-Isotropic or Extensionally Isotropic Laminates (EILs), Fully Isotropic Laminates (FILs), and laminates with material homogeneity. By contrast, original work by Bartholomew [12,13], which forms the basis of the Engineering Sciences Data Unit (ESDU) publication [14] for the so called definitive list of fully orthotropic angle-ply laminates with designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_S$, precedes the findings of Caprino and Crivelli-Visconti [11], but appears to have been completely overlooked in the literature described above. This published list of stacking sequences contains 75 symmetric sequences for laminates with up to 21 plies, 653 anti-symmetric sequences for laminates with up to 20 plies and 49 additional non-symmetric (asymmetric) sequences, which were derived by combining the symmetric and anti-symmetric sequences. Further inspection reveals that there are no angle-ply laminates possessing specially orthotropic properties with fewer than 7 plies. Indeed, there is only one generic 7-layer angle-ply anti-symmetric stacking sequence. This number increases to 233 generic anti-symmetric sequences with 20 ply layers. There are no symmetric stacking sequences with less than 12 layers, and only 25 combinations with 20 layers. These twenty-five generic stacking sequences possess balanced and symmetric combinations of angle plies, together with cross plies, which may be 0 and/or 90°, symmetrically disposed about the laminate mid-plane; all possess angle-ply layers on the outer surfaces of the laminate.

For compatibility with the previously published data, similar symbols have been adopted for defining all stacking sequences that follow. Additional symbols and parameters are necessarily included to differentiate between cross plies (0° and 90°), given that symmetry about the laminate mid-plane is no longer assumed.

The resulting sequences are characterized by sub-sequence symmetries using a double prefix notation, the first character of which relates to the form of the angle-ply sub-sequence and the second character to the cross-ply sub-sequence. The double prefix contains combinations of the following characters: *A* to indicate Anti-symmetric form; *N* for Non-symmetric; and *S* for Symmetric. Additionally, for cross-ply sub-sequence only, *C* is used to indicate Cross-symmetric form.

To avoid the trivial solution of a stacking sequence with cross plies only, all sequences have an angle-ply (+) on one outer surface of the laminate. As a result, the other outer surface may have an angle-ply of equal (+) or opposite (–) orientation or a cross ply (○), which may be either 0 or 90°. A subscript notation, using these three symbols, is employed to deferential between similar forms of sequence.

The form (and number) of FOL stacking sequences contained in the definitive list [7] for up to 21 plies can be summarized as: *AC* (210), *AN* (14,532), *AS* (21,906), *SC* (12), *SN* (192), *SS* (1,029), ${}_{+}NS_{+}$ (220), ${}_{+}NS_{-}$ (296), ${}_{+}NN_{+}$ (5,498), ${}_{+}NN_{-}$ (15,188) and ${}_{+}NN_{\circ}$ = ${}_{+}NN_{\bullet}$ (10,041).

The 18 FILs, with π/3 isotropy, identified as a sub-set of FOLs, are of the form (and number) ${}_{+}NN_{+}$ (2), ${}_{+}NN_{-}$ (8) and ${}_{+}NN_{\circ}$ (8).

Wu and Avery [15] obtained exact FIL configurations by varying, or shuffling, the stacking sequences of fully uncoupled **EIL**s in order to produce bending isotropy. Eighty-nine stacking sequences were presented as the symmetric halves of 36-ply laminates with fully isotropic properties, deemed to be the minimum number of plies for π/3 FILs. Limited details of 54-ply sequences for FILs with π/3 isotropy are also provided, together with π/4, π/5 and π/6 isotropy for 48-, 50- and 72-ply sequences, respectively. The symmetry assumption, on which this work is based, precluded the possibility that the listings would contain non-symmetric 18-ply FILs. However, inspection of this list reveals that nine of these stacking sequences represent 18-ply non-symmetric FILs.

Paradies [1] demonstrated that bending isotropy could be closely approximated without necessarily satisfying in-plane isotropy, which is contrary to the procedure for obtaining exact

FIL configurations. Other examples of approximate laminate configurations were presented by Fukunaga [16] and Grediac [17]. Fukunaga obtained exact solutions for a 40-ply laminate but for 20-ply configurations, based on unusual symmetric combinations of four angle-ply pairs, all six solutions presented were approximate, whilst Grediac, who adopted an optimization strategy, obtained close approximations to a FIL with 12 plies but with seemingly random combinations of angle-ply sequences.

4. Non-dimensional parameters.

For IM7/8552 carbon-fiber/epoxy material with Young's moduli E_1 = 161.0GPa and E_2 = 11.38GPa, shear modulus G_{12} = 5.17GPa and Poison ratio $\nu_{12} = 0.38$, lamina thickness t = 0.1397mm and stacking sequence *NN* 1071: $[+/-_2/\bigcirc_3/+_2/\bigcirc/-/+_2/-_3/\bigcirc_2/+]_T$, the non-dimensional parameters are verified by the calculations presented in Table 1, where the first two columns provide the ply number and orientation, respectively. Subsequent columns illustrate the summations, for each ply orientation, of $(z_k - z_{k-1})$, $(z_k^2 - z_{k-1}^2)$ and $(z_k^3 - z_{k-1}^3)$, relating to the **A**, **B** and **D** matrices, respectively. The distance from the laminate mid-plane, z, is expressed in term of ply thickness t, which is set to unit value in the non-dimensional expressions.

Table 1 – Derivation of non-dimensional stiffness parameters for $[+/-_2/\bigcirc_3/+_2/\bigcirc/-/+_2/-_3/\bigcirc_2/+]_T$.

			A				**B**				**D**		
Ply	θ	$(z_k - z_{k-1})$	$^{A}\Sigma_{\bigcirc}$ / 6	$^{A}\Sigma_{-}$ / 6	$^{A}\Sigma_{+}$ / 6	$(z_k^2 - z_{k-1}^2)$	$^{B}\Sigma_{\bigcirc}$ / 0	$^{B}\Sigma_{-}$ / 0	$^{B}\Sigma_{+}$ / 0	$(z_k^3 - z_{k-1}^3)$	$^{D}\Sigma_{\bigcirc}$ / 486	$^{D}\Sigma_{-}$ / 486	$^{D}\Sigma_{+}$ / 486
1	+	1			⇢ 1	-17			⇢ -17	217			⇢ 217
2	–	1		⇢ 1		-15		⇢ -15		169		⇢ 169	
3	–	1		⇢ 1		-13		⇢ -13		127		⇢ 127	
4	○	1	⇢ 1			-11	⇢ -11			91	⇢ 91		
5	○	1	⇢ 1			-9	⇢ -9			61	⇢ 61		
6	○	1	⇢ 1			-7	⇢ -7			37	⇢ 37		
7	+	1			⇢ 1	-5			⇢ -5	19			⇢ 19
8	+	1			⇢ 1	-3			⇢ -3	7			⇢ 7
9	○	1	⇢ 1			-1	⇢ -1			1	⇢ 1		
10	–	1		⇢ 1		1		⇢ 1		1		⇢ 1	
11	+	1			⇢ 1	3			⇢ 3	7			⇢ 7
12	+	1			⇢ 1	5			⇢ 5	19			⇢ 19
13	–	1		⇢ 1		7		⇢ 7		37		⇢ 37	
14	–	1		⇢ 1		9		⇢ 9		61		⇢ 61	
15	–	1		⇢ 1		11		⇢ 11		91		⇢ 91	
16	○	1	⇢ 1			13	⇢ 13			127	⇢ 127		
17	○	1	⇢ 1			15	⇢ 15			169	⇢ 169		
18	+	1			⇢ 1	17			⇢ 17	217			⇢ 217

Table 2 – Transformed reduced stiffnesses (N/mm^2) for IM7/8552 carbon-fiber/epoxy.

θ	Q'_{11}	Q'_{12}	Q'_{16}	Q'_{22}	Q'_{26}	Q'_{66}
-60	22,149	31,508	-17,059	97,731	-48,396	32,309
60	22,149	31,508	17,059	97,731	48,396	32,309
0	162,660	4,369	0	11,497	0	5,170

The non-dimensional parameters arising from the summations of Table 1 are: n_+ (= $_A\Sigma_+$) = 6, n_- = 6 and $n_{\circ}$ = 6, where $n_\pm$ = 12, and; ζ_+ (= 4 × $_D\Sigma_+$) = 1944, ζ_- = 1944 and $\zeta_{\circ}$ = 1944, where $n^3 = 18^3 = \zeta = \zeta_+ + \zeta_- + \zeta_{\circ} = 5832$ and $\zeta_\pm$ = 3888. The **B** matrix summations confirm that $B_{ij} = 0$ for this laminate.

The calculation procedure for the elements (A_{ij} and D_{ij}) of the extensional (**A**) and bending (**D**) stiffness matrices using the dimensionless parameters are as follows:

$$A_{ij} = \{n_\pm/2 \times Q'_{ij+} + n_\pm/2 \times Q'_{ij-} + n_{\circ}Q'_{ij\circ} + n_{\bullet}Q'_{ij\bullet}\} \times t \tag{6}$$

$$D_{ij} = \{\zeta_\pm/2 \times Q'_{ij+} + \zeta_\pm/2 \times Q'_{ij-} + \zeta_{\circ}Q'_{ij\circ} + \zeta_{\bullet}Q'_{ij\bullet}\} \times t^3/12 \tag{7}$$

The form of Eqs. (6) and (7) was chosen because they are readily modified to account for laminates with extensional and bending anisotropy by replacing $n_\pm/2 \times Q'_{ij+}$ with $n_\pm(n_+/n_\pm)Q'_{ij+}$ and $n_\pm/2 \times Q'_{ij-}$ with $n_\pm(1 - n_+/n_\pm)Q'_{ij-}$, and $\zeta_\pm/2 \times Q'_{ij+}$ with $\zeta_\pm(\zeta_+/\zeta_\pm) \times Q'_{ij+}$ and $\zeta_\pm/2 \times Q'_{ij-}$ with $\zeta_\pm(1 - \zeta_+/\zeta_\pm) \times Q'_{ij-}$. The use of these modified equation requires the calculation of an additional stiffness parameter, n_+ and ζ_+, relating to the extensional and bending stiffness contribution of positive (θ) angle plies, respectively.

The transformed reduced stiffness terms in Eqs. (6) and (7) are given by:

$$\begin{aligned}
Q'_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\sin^4\theta \\
Q'_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\cos^2\theta\sin^2\theta + Q_{12}(\cos^4\theta + \sin^4\theta) \\
Q'_{16} &= \{(Q_{11} - Q_{12} - 2Q_{66})\cos^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^2\theta\}\cos\theta\sin\theta \\
Q'_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\cos^2\theta\sin^2\theta + Q_{22}\cos^4\theta \\
Q'_{26} &= Q'_{62} = \{(Q_{11} - Q_{12} - 2Q_{66})\sin^2\theta + (Q_{12} - Q_{22} + 2Q_{66})\cos^2\theta\}\cos\theta\sin\theta \\
Q'_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta + \sin^4\theta)
\end{aligned} \tag{8}$$

and the reduced stiffness terms by:

$$\begin{aligned}
Q_{11} &= E_1/(1 - \nu_{12}\nu_{21}) \\
Q_{12} &= \nu_{12}E_2/(1 - \nu_{12}\nu_{21}) \\
Q_{22} &= E_2/(1 - \nu_{12}\nu_{21}) \\
Q_{66} &= G_{12}
\end{aligned} \tag{9}$$

For fiber angles θ = ±60° and 0° in place of symbols ± and O respectively, the transformed reduced stiffnesses are given in Table 2, which are readily calculated using Eqs. (8), and through Eqs (6) and (7), the final stiffness matrices are derived for the laminate:

$$[\mathbf{A}] = \begin{bmatrix} 173{,}473 & 56{,}482 & 0 \\ & 173{,}473 & 0 \\ \text{Sym.} & & 58{,}496 \end{bmatrix} \text{N/mm}$$

$$[\mathbf{D}] = \begin{bmatrix} 91{,}409 & 29{,}762 & 0 \\ & 91{,}409 & 0 \\ \text{Sym.} & & 30{,}823 \end{bmatrix} \text{N.mm}$$

given that:

$$A_{16} = \{n_+Q'_{16+} + n_-Q'_{16-} + n_{\circ}Q'_{16\circ}\} \times t$$

$$A_{16} = A_{26} = \{6 \times 17{,}059 + 6 \times -17{,}059 + 6 \times 0\} \times 0.1397 = 0 \text{ N/mm}$$

$$D_{12} = \{\zeta_\pm/2 \times Q'_{12+} + \zeta_\pm/2 \times Q'_{12-} + \zeta_{\circ}Q'_{12\circ}\} \times t^3/12$$

$$D_{12} = D_{21} = \{1944 \times 17{,}059 + 1944 \times -17{,}059 + 1944 \times 0\} \times 0.1397^3/12 = 0 \text{ N.mm}$$

For optimum design of angle ply laminates, lamination parameters are often preferred, since these allow the stiffness terms to be expressed as linear variables. The optimized lamination parameters may then be matched against a corresponding set of laminate stacking sequences. In the context of the FILs presented in the current article, the necessary four lamination parameters are related through the following expressions:

$$\xi_1^A = \xi_1 = \{n_\pm \mathrm{Cos}(2\theta_\pm) + n_\circ \mathrm{Cos}(2\theta_\circ) + n_\bullet \mathrm{Cos}(2\theta_\bullet)\}/n$$
$$\xi_2^A = \xi_2 = \{n_\pm \mathrm{Cos}(4\theta_\pm) + n_\circ \mathrm{Cos}(4\theta_\circ) + n_\bullet \mathrm{Cos}(4\theta_\bullet)\}/n \quad (10)$$

$$\xi_1^D = \xi_9 = \{\zeta_\pm \mathrm{Cos}(2\theta_\pm) + \zeta_\circ \mathrm{Cos}(2\theta_\circ) + \zeta_\bullet \mathrm{Cos}(2\theta_\bullet)\}/\zeta$$
$$\xi_2^D = \xi_{10} = \{\zeta_\pm \mathrm{Cos}(4\theta_\pm) + \zeta_\circ \mathrm{Cos}(4\theta_\circ) + \zeta_\bullet \mathrm{Cos}(4\theta_\bullet)\}/\zeta \quad (11)$$

Elements of the extensional stiffness matrix (**A**) are related to the lamination parameters by:

$$A_{11} = U_1 + \{\xi_1 \times H\}U_2 + \{\xi_2\}U_3$$
$$A_{22} = U_1 + \{-\xi_1 \times H\}U_2 + \{\xi_2\}U_3$$
$$A_{12} = A_{21} = \{-\xi_2\}U_3 + U_4 \quad (12)$$
$$A_{66} = \{-\xi_2\}U_3 + U_5$$

and the flexural stiffness matrix (**D**) by:

$$D_{11} = U_1 + \{\xi_9 \times H^3/12\}U_2 + \{\xi_{10}\}U_3$$
$$D_{22} = U_1 - \{\xi_9 \times H^3/12\}U_2 + \{\xi_{10}\}U_3$$
$$D_{12} = U_4 - \{\xi_{10}\}U_3 \quad (13)$$
$$D_{66} = U_1 - \{\xi_9 \times H^3/12\}U_2 + \{\xi_{10}\}U_3$$

where the laminate invariants are given in terms of the reduced stiffnesses of Eq. (9) by:

$$U_1 = \{3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66}\}/8$$
$$U_2 = \{Q_{11} - Q_{22}\}/2$$
$$U_3 = \{Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66}\}/8 \quad (14)$$
$$U_4 = \{Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66}\}/8$$
$$U_5 = \{Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66}\}/8$$

The extensional lamination parameters (ξ_1, ξ_2) are calculated from Eqs. (10):

$$\xi_1^A = \{n_\pm \mathrm{Cos}(2\theta_\pm) + n_\circ \mathrm{Cos}(2\theta_\circ)\}/n$$

$$\xi_1^A = \{12 \times \mathrm{Cos}(90°) + 8 \times \mathrm{Cos}(0°)\}/18 = 0.0$$

$$\xi_2^A = \{n_\pm \mathrm{Cos}(4\theta_\pm) + n_\circ \mathrm{Cos}(4\theta_\circ)\}/n$$

$$\xi_2^A = \{12 \times \mathrm{Cos}(180°) + 6 \times \mathrm{Cos}(0°)\}/18 = 0.0$$

and the bending lamination parameters from Eqs. (11):

$$\xi_9 = \{\zeta_\pm \mathrm{Cos}(2\theta_\pm) + \zeta_\circ \mathrm{Cos}(2\theta_\circ)\}/\zeta$$

$$\xi_9 = \{3888 \times \mathrm{Cos}(90°) + 1944 \times \mathrm{Cos}(0°)\}/5832 = 0.0$$

$$\xi_{10} = \{\zeta_\pm \mathrm{Cos}(4\theta_\pm) + \zeta_\circ \mathrm{Cos}(4\theta_\circ)\}/\zeta$$

$$\xi_{10} = \{3888 \times \mathrm{Cos}(180°) + 1944 \times \mathrm{Cos}(0°)\}/5832 = 0.0$$

5. Approximation for FILs.

In the context of the work presented herein FILs are readily found by inspection of the feasible domains of bending lamination parameters for FOLs, from which FILs, exact or approximate, are sub-sets: the degree of approximation depends on the proximity to the point (ξ_1, ξ_2) = (ξ_9, ξ_{10}) = (0, 0), described in the next section. The minimum number of plies giving rise to a FIL has been shown to be 18. Normalization of the results for other ply numbers therefore requires extrapolation of the equivalent FIL properties. This is can be achieved exactly from:

$$E = (Q_{11} + 2Q_{12} + Q_{22})U_5/U_1 \quad (15)$$

and

$$\nu = U_4/U_1 \quad (16)$$

The Young's modulus, E, and Poisson ratio, ν, are the equivalent isotropic material properties of a composite laminate of thickness, *H*, corresponding to the total number of plies, *n*, of uniform thickness *t*. The shear modulus follows in the usual form:

$$G = E/2(1 + \nu) \quad (17)$$

Together these material properties provide the equivalent isotropic stiffness properties for laminates with any number of plies:

$$A = EH/(1 - \nu^2)$$
$$B = EH^2/4(1 - \nu^2) = 0 \qquad (18)$$
$$D = EH^3/12(1 - \nu^2)$$

6. Approximation criteria and Results.

Curvature criterion.

The curvature criterion employed involves normalizing the curvature κ_x from the approximate configurations with that of the FILs, expressed as a percentage:

$$(\kappa_x - \kappa_{x,FIL})/\kappa_{x,FIL} \qquad (19)$$

where curvature κ_x is calculated using elements from the inverse of the bending stiffness matrix, **D**, i.e.:

$$\kappa_x = d_{11}N_x + d_{12}N_y \qquad (20)$$

and

$$d_{11} = D_{22}D_{66}/|D|$$
$$d_{12} = -D_{12}D_{66}/|D|$$
$$d_{22} = D_{11}D_{66}/|D| \qquad (21$$
$$d_{66} = (D_{11}D_{22} - D_{12}^2)/|D|$$
$$|D| = D_{11}D_{22}D_{66} - D_{12}^2D_{66}$$

Noting that the 3rd and 4th equations allow comparison of κ_y and κ_{xy}:

$$\kappa_y = d_{12}N_x + d_{22}N_y$$
$$\kappa_{xy} = d_{66}N_{xy} \qquad (22)$$

Stiffness criterion

The stiffness criterion employed involves normalizing the stiffness A_{ij} and D_{ij} from the approximate configurations with that of the FILs, expressed as a percentage:

$$(A_{ij} - A_{ij,FIL})/A_{ij,FIL}$$
$$(D_{ij} - D_{ij,FIL})/D_{ij,FIL} \qquad (23)$$

The stiffness terms are calculated from the using lamination parameters lying on the perimeter of the proximity regions. These are circular regions around the FIL condition, representing 1%, 2% and 5% of the lamination parameter bounds, i.e. $-1 \leq \xi_{1,2}^{A,D} \leq 1$. The 1% proximity region therefore lies between $-0.01 \leq \xi_{1,2}^{A,D} \leq 0.01$, see Figs 1 – 4.

These circular proximity regions were assessed against the curvature and stiffness criteria, revealing that for a 1% approximation, curvatures $\kappa_x \neq \kappa_y$ varied by up to a maximum of -1.41% and 1.43%, whilst stiffness elements $A_{ij} = D_{ij}$ varied by up to a maximum of ±1.04%. Increasing the proximity region to 2% exactly doubles the curvature and stiffness variations with respect to the FIL.

Approximations can be derived for ξ^D only, or ξ^A and ξ^D, to the desired level of accuracy by looping through integer ply angles in the range $0 \leq \theta \leq 90°$ and noting the number of sequences satisfying the proximity criteria, see Figs 3 and 4. The range over which curvatures and stiffnesses can be approximated to within ±1.4% and ±1%, respectively, are found to correspond to $39 \leq \theta \leq 60°$, see Figs . Note that for the non-symmetric (angle-ply and cross-ply) laminates chosen for this study, the approximation applies equally to both ξ^A and ξ^D only when $\theta = 60°$; only 19 of the 84 sequences. It is therefore apparent that laminate configurations with bending stiffness closely approximating the isotropic conditions are readily obtained using this method. However the number of configurations with extensional and bending stiffness is limited.

Table 3 presents stacking sequences for $_{+}NN_0$ laminates which represent FOLs within 1% of the FIL, based on the curvature and stiffness criteria, with [±/○/●] = [±60°/0°/90°]. They are presented together with their FOL reference numbers and non-dimensional parameters, including lamination parameters. Note that the flexural lamination parameters are omitted since all fall below $\xi_{1,2}^D = 0.00$.

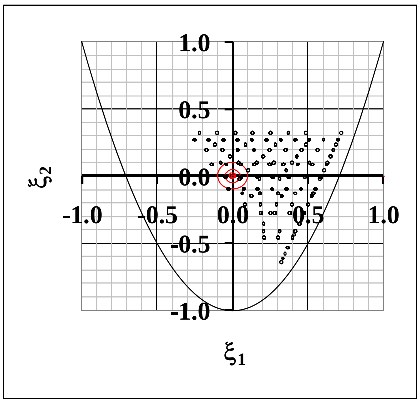

Fig. 1 – Extensional lamination parameters for (10,041) $_{+}NN_{0}$ stacking sequences with ply angles [±/○/●] = [±39°/0°/90°].

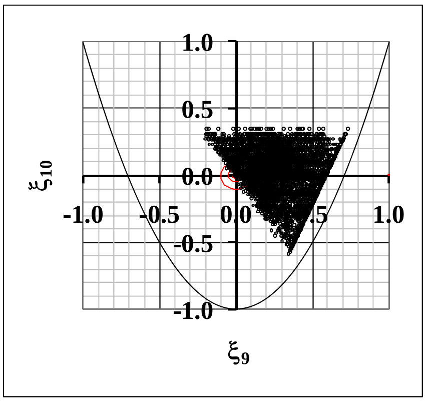

Fig. 2 – Flexural lamination parameters for (10,041) $_{+}NN_{0}$ stacking sequences with ply angles [±/○/●] = [±39°/0°/90°].

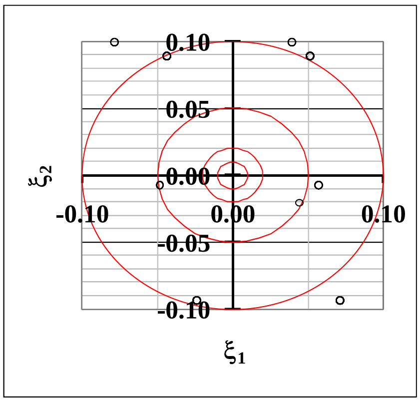

Fig. 3 – Enlargement of Fig. 1, revealing extensional lamination parameters within 1%, 2%, 5% and 10% proximity region.

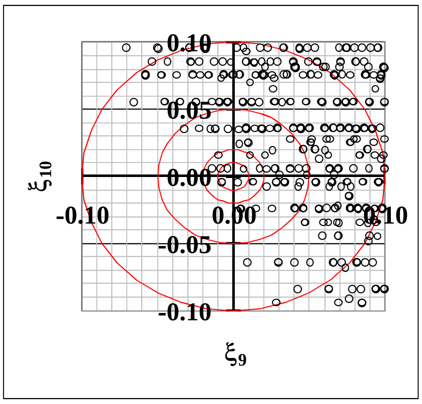

Fig. 4 – Enlargement of Fig. 2, revealing flexural lamination parameters within 1%, 2%, 5% and 10% proximity region.

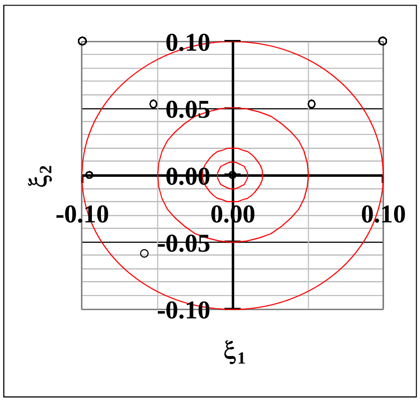

Fig. 5 – Extensional lamination parameters for (10,041) $_{+}NN_{0}$ stacking sequences with ply angles [±/○/●] = [±60°/0°/90°].

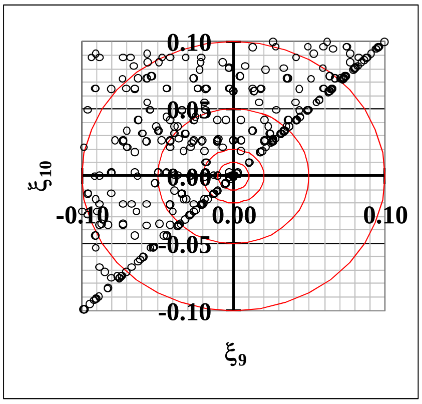

Fig. 6 – Flexural lamination parameters for (10,041) $_{+}NN_{0}$ stacking sequences with ply angles [±/○/●] = [±60°/0°/90°].

6. Conclusions.

A method has been demonstrated for deriving uncoupled laminate configurations, which approximate the fully isotropic laminate or FIL in either bending stiffness alone, or in bending and extensional stiffness.

Lamination parameters were used to assess the proximity of a given fully uncoupled orthotropic laminate to the fully isotropic condition. It was found that a 0.01 or 1% change in lamination parameter approximately translated to a 1% change in the extensional and bending stiffnesses and a 1.4% change in the induced bending curvature.

Laminate configurations with bending stiffness closely approximating the isotropic condition are readily obtained. However the number of configurations with both extensional and bending stiffness within the same tolerance is limited for the non-symmetric laminate group investigated.

7. Additional remarks and suggestions for futher work.

The application of this work to large diameter mirrors for space-based reflector telescopes was mentioned briefly but not developed. Such mirrors are typically formed by the bonding of a thin metallic surface layer to a composite laminate substrate. It should be noted however that this additional metallic layer has a significant effect on the behavioral properties: A fully uncoupled isotropic laminate, of the sort presented herein, with an isotropic metallic layer bonded to one outer surface possesses combined anticlastic bending-extension and shear-twist couplings. The design of metallic surface coated carbon fibre substrate FILs therefore require the consideration of coupled laminate configurations, in order to counteract the induced coupling effect of the metallic surface.

References

[1] Paradies R. Designing quasi-isotropic laminates with respect to bending. *Composites Science and Technology*, Vol. 56, pp 461-472, 1996.

[2] Paradies R and Hertwig M. Shape control of adaptive composite reflectors. *Composites: Part B*, Vol. 30, pp 65-78, 1999.

[3] Kim K-P and Hale D. Design of directionally uniform extensional and flexural stiffness in quasi-isotropic laminates. *Proc 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, Schaumburg, Illinois, AIAA-2008-1749, 2008.

[4] Kim K-P and Hale D. Analysis of surface deformation due to thermal load on circular quasi-isotropic laminate mirrors. *Proc 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, Schaumburg, Illinois, AIAA-2008-2185, 2008.

[5] Engineering Sciences Data Unit, “Stiffnesses of laminated plates”, ESDU Item No. 94003, 1994.

[6] York CB. On composite laminates with extensional-anisotropy *Proc 49th AIAA/ASME/ ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, Schaumburg, Illinois, AIAA-2008-1752, 2008.

[7] York, C. B., “Characterization of non-symmetric forms of fully orthotropic laminates.” *AIAA Journal of Aircraft*, Accepted for publication. See also, York, C. B. “Characterization and ply mixing rules for non-symmetric forms of fully orthotropic laminates,” *Proc. 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conf.*, AIAA-2007-2083, Honolulu, Hawaii, 2007.

[8] Fukunaga, H., Sekine, H., Sato, M. and Iino, A., “Buckling design of symmetrically laminated plates using lamination parameters” *Computers and Structures*, Vol. 57, No. 4, 1995, pp. 643-649.

[9] Valot, E. and Vannucci, P., “Some exact solutions for fully orthotropic laminates,” *Composite Structures*, Vol. 69, 2005, pp. 157-66.

[10] Vannucci, P. and Verchery, G., “A new method for generating fully isotropic laminates,” *Composite Structures*, Vol. 58, 2002, pp. 75-82.

[11] Caprino, G. and Crivelli-Visconti, I., “A note on specially orthotropic laminates,” *Journal of Composite Materials*, Vol. 16, No. 5, 1982, 395-399.

[12] Bartholomew, P., “Ply stacking sequences for laminated plates having in-plane and bending orthotropy,” Royal Aircraft Establishment Technical Report No. 76003, 1976.

[13] Bartholomew, P., “Ply stacking sequences for laminated plates having in-plane and bending orthotropy,” *Fibre Science and Technology*, Vol. 10, No. 4, 1977, pp. 239-253.

[14] Engineering Sciences Data Unit, “Laminate stacking sequences for special orthotropy (Application to fibre reinforced composites)”, ESDU Item No. 82013, 1982.

[15] Wu, K., M. and Avery, B. L., “Fully isotropic laminates and quasi-homogeneous anisotropic laminates”, *Journal of Composite Materials*, Vol. 26, No. 14, 1992, pp. 2107-2117.

[16] Fukunaga H. On isotropic laminate configurations. *Journal of Composite Materials*, Vol. 24, pp 519-535, 1990.

[17] Grediac M. A procedure for designing laminated plates with required stiffness properties. Application to thin quas-isotropic quasi-homogeneous uncoupled laminates. *Journal of Composite Materials*, Vol. 33, No. 20, pp 1939-1956, 1998.

Table 3 – ${}_{+}NN_{0}$ laminates represent those within 1% of the FIL with [±/○/●] = [±60°/0°/90°], based on the curvature and stiffness criteria.

Ref.	Sequence	n	$n_{\pm}$	$n_{○}$	$\zeta_{\pm}$	$\zeta_{○}$	ξ_1	ξ_2
NN 63	+ − ○ ○ − + − + + − ○ − + + − ○	16	12	4	2736	1360	-0.13	-0.13
NN 64	+ − ○ ○ + − − − + + ○ + − + − ○	16	12	4	2736	1360	-0.13	-0.13
NN 553	+ − ○ ○ + − − ○ ● ○ + ○ + − + − ○	17	10	6	3274	1638	0.00	0.12
NN 554	+ ○ − − + ○ ○ − ● ○ + ○ + − + − ○	17	10	6	3274	1638	0.00	0.12
NN 557	+ − ○ ○ + − − ○ ○ ○ + ○ + − + − ○	17	10	7	3274	1639	0.12	0.12
NN 558	+ ○ − − + ○ ○ − ○ ○ + ○ + − + − ○	17	10	7	3274	1639	0.12	0.12
NN 581	+ − ○ − ○ + ○ + − − + + ○ − − + ○	17	12	5	3276	1637	-0.06	-0.06
NN 582	+ − ○ − ○ + ○ + − + − − ○ + + − ○	17	12	5	3276	1637	-0.06	-0.06
NN 1069	+ ○ − − ○ + − + ● ● + − ○ − + + − ○	18	12	4	3888	1936	-0.22	0.00
NN 1070	+ ○ − + ○ − − − ● ● + + ○ + + − − ○	18	12	4	3888	1936	-0.22	0.00
NN 1081	+ − − ○ ○ + ○ ○ + + − − − + ○ − + ○	18	12	6	3888	1944	0.00	0.00
NN 1082	+ − ○ ○ − + − ○ + + ○ − ○ + − − + ○	18	12	6	3888	1944	0.00	0.00
NN 1083	+ − ○ − ○ + ○ + − ○ − + + ○ − − + ○	18	12	6	3888	1944	0.00	0.00
NN 1084	+ − ○ − ○ + ○ + − ○ + − − ○ + + − ○	18	12	6	3888	1944	0.00	0.00
NN 1085	+ ○ − − ○ + − + ○ ○ + − ○ − + + − ○	18	12	6	3888	1944	0.00	0.00
NN 1086	+ − ○ ○ + − − ○ + − ○ + ○ + − + − ○	18	12	6	3888	1944	0.00	0.00
NN 1087	+ ○ − − + ○ ○ − + − ○ + ○ + − + − ○	18	12	6	3888	1944	0.00	0.00
NN 1088	+ ○ − + ○ − − − ○ ○ + + ○ + + − − ○	18	12	6	3888	1944	0.00	0.00
NN 3952	+ − ○ − ○ + ○ + − ● ○ − + + ○ − − + ○	19	12	6	4572	2286	-0.05	0.05
NN 3953	+ − ○ − ○ + ○ + − ● ○ + − − ○ + + − ○	19	12	6	4572	2286	-0.05	0.05
NN 3954	+ − ○ ○ + − − ○ + ● − ○ + ○ + − + − ○	19	12	6	4572	2286	-0.05	0.05
NN 3955	+ ○ − − + ○ ○ − + ● − ○ + ○ + − + − ○	19	12	6	4572	2286	-0.05	0.05
NN 3960	+ − ○ − ○ + ○ + − ○ ○ − + + ○ − − + ○	19	12	7	4572	2287	0.05	0.05
NN 3961	+ − ○ − ○ + ○ + − ○ ○ + − − ○ + + − ○	19	12	7	4572	2287	0.05	0.05
NN 3962	+ − ○ − ○ + ○ + ○ − − + ○ ○ − + + − ○	19	12	7	4572	2287	0.05	0.05
NN 3963	+ − ○ ○ + − − ○ + ○ − ○ + ○ + − + − ○	19	12	7	4572	2287	0.05	0.05
NN 3964	+ ○ − − + ○ ○ − + ○ − ○ + ○ + − + − ○	19	12	7	4572	2287	0.05	0.05
NN 7140	+ − ○ − ○ + ○ + − ● ● ○ − + + ○ − − + ○	20	12	6	5328	2664	-0.10	0.10
NN 7141	+ − ○ − ○ + ○ + − ● ● ○ + − − ○ + + − ○	20	12	6	5328	2664	-0.10	0.10
NN 7142	+ − ○ ○ + − − ○ + ● ● − ○ + ○ + − + − ○	20	12	6	5328	2664	-0.10	0.10
NN 7143	+ ○ − − + ○ ○ − + ● ● − ○ + ○ + − + − ○	20	12	6	5328	2664	-0.10	0.10
NN 7166	+ − − ○ ○ + ○ + ○ ○ − ○ − + + − ○ − + ○	20	12	8	5328	2672	0.10	0.10
NN 7167	+ − ○ − ○ + ○ + − ○ ○ ○ − + + ○ − − + ○	20	12	8	5328	2672	0.10	0.10
NN 7168	+ − ○ − ○ + ○ + − ○ ○ ○ + − − ○ + + − ○	20	12	8	5328	2672	0.10	0.10
NN 7169	+ − ○ − + ○ ○ ○ − + ○ + ○ − − ○ + + − ○	20	12	8	5328	2672	0.10	0.10
NN 7170	+ − ○ ○ + − − ○ + ○ ○ − ○ + ○ + − + − ○	20	12	8	5328	2672	0.10	0.10
NN 7171	+ ○ − − + ○ ○ − + ○ ○ − ○ + ○ + − + − ○	20	12	8	5328	2672	0.10	0.10
NN 7172	+ − ○ + ○ − − ○ ○ ○ + − ○ + + ○ − + − ○	20	12	8	5328	2672	0.10	0.10

Continued.

Continued.

NN 33088	+ – ○ – ○ + ○ + – ●	●	● ○ – + + ○ – – + ○	21	12	6	6156	3078	-0.14	0.14
NN 33089	+ – ○ – ○ + ○ + – ●	●	● ○ + – – ○ + + – ○	21	12	6	6156	3078	-0.14	0.14
NN 33090	+ – ○ ○ + – – ○ + ●	●	● – ○ + ○ + – + – ○	21	12	6	6156	3078	-0.14	0.14
NN 33091	+ ○ – – + ○ ○ – + ●	●	● – ○ + ○ + – + – ○	21	12	6	6156	3078	-0.14	0.14
NN 33098	+ – ○ – ○ + ○ + – ●	○	● ○ – + + ○ – – + ○	21	12	7	6156	3079	-0.05	0.14
NN 33099	+ – ○ – ○ + ○ + – ●	○	● ○ + – – ○ + + – ○	21	12	7	6156	3079	-0.05	0.14
NN 33100	+ – ○ ○ + – – ○ + ●	○	● – ○ + ○ + – + – ○	21	12	7	6156	3079	-0.05	0.14
NN 33101	+ ○ – – + ○ ○ – + ●	○	● – ○ + ○ + – + – ○	21	12	7	6156	3079	-0.05	0.14
NN 33108	+ – – + ○ ○ ○ ○ ○ –	●	+ ○ + + – – – ○ + ○	21	12	8	6156	3104	0.05	0.14
NN 33109	+ – ○ – ○ + ○ + – ○	●	○ ○ – + + ○ – – + ○	21	12	8	6156	3104	0.05	0.14
NN 33110	+ – – ○ ○ + ○ + ○ ○	●	– – + ○ – ○ + + – ○	21	12	8	6156	3104	0.05	0.14
NN 33111	+ – ○ – ○ + ○ + – ○	●	○ ○ + – – ○ + + – ○	21	12	8	6156	3104	0.05	0.14
NN 33112	+ – – ○ + ○ ○ ○ + ○	●	– – ○ + – + ○ + – ○	21	12	8	6156	3104	0.05	0.14
NN 33113	+ – ○ ○ + – – ○ + ○	●	○ – ○ + ○ + – + – ○	21	12	8	6156	3104	0.05	0.14
NN 33114	+ ○ – – + ○ ○ – + ○	●	○ – ○ + ○ + – + – ○	21	12	8	6156	3104	0.05	0.14
NN 33131	+ – – + ○ ○ ○ ○ ○ –	○	+ ○ + + – – – ○ + ○	21	12	9	6156	3105	0.14	0.14
NN 33132	+ – ○ – ○ + ○ + – ○	○	○ ○ – + + ○ – – + ○	21	12	9	6156	3105	0.14	0.14
NN 33133	+ – – ○ ○ + ○ + ○ ○	○	– – + ○ – ○ + + – ○	21	12	9	6156	3105	0.14	0.14
NN 33134	+ – ○ – ○ + ○ + – ○	○	○ ○ + – – ○ + + – ○	21	12	9	6156	3105	0.14	0.14
NN 33135	+ – – ○ + ○ ○ ○ + ○	○	– – ○ + – + ○ + – ○	21	12	9	6156	3105	0.14	0.14
NN 33136	+ – ○ ○ + – – ○ + ○	○	○ – ○ + ○ + – + – ○	21	12	9	6156	3105	0.14	0.14
NN 33137	+ ○ – – + ○ ○ – + ○	○	○ – ○ + ○ + – + – ○	21	12	9	6156	3105	0.14	0.14
NN 33247	+ – – ○ + ○ ○ + ○ –	●	+ – – – + + + ○ – ○	21	14	6	6158	3102	-0.10	0.00
NN 33256	+ – ○ + – – ○ ○ ○ ○	+	+ – + + – – – ○ + ○	21	14	7	6158	3103	0.00	0.00
NN 33257	+ – ○ ○ – – + + ○ ○	+	+ ○ – – – ○ + – + ○	21	14	7	6158	3103	0.00	0.00
NN 33258	+ – – ○ ○ + ○ + ○ –	+	– ○ – + + – ○ – + ○	21	14	7	6158	3103	0.00	0.00
NN 33259	+ – ○ – ○ + ○ + – ○	–	+ + – ○ ○ + – – + ○	21	14	7	6158	3103	0.00	0.00
NN 33260	+ – ○ ○ – + – + ○ ○	+	– ○ – + + ○ – – + ○	21	14	7	6158	3103	0.00	0.00
NN 33261	+ – ○ ○ – + – + ○ ○	+	– ○ + – – ○ + + – ○	21	14	7	6158	3103	0.00	0.00
NN 33262	+ – ○ – ○ + ○ + – ○	+	– – + ○ ○ – + + – ○	21	14	7	6158	3103	0.00	0.00
NN 33263	+ – ○ + ○ – – ○ ○ +	–	+ – ○ + + ○ – + – ○	21	14	7	6158	3103	0.00	0.00
NN 33264	+ – – ○ + ○ ○ + ○ –	○	+ – – – + + + ○ – ○	21	14	7	6158	3103	0.00	0.00
NN 33265	+ – ○ ○ + – ○ + – –	–	+ ○ ○ ○ + + + – – ○	21	14	7	6158	3103	0.00	0.00
NN 33270	+ – ○ – ○ + – ○ + +	+	– – + – – + ○ – + ○	21	16	5	6160	3101	-0.14	-0.14
NN 33271	+ – ○ – ○ + + ○ – +	–	– – + + + – ○ + – ○	21	16	5	6160	3101	-0.14	-0.14
NN 33272	+ – ○ + ○ – – ○ – +	+	+ – – + + – ○ + – ○	21	16	5	6160	3101	-0.14	-0.14
NN 33273	+ – ○ + ○ – – ○ + –	–	+ + + – + – ○ + – ○	21	16	5	6160	3101	-0.14	-0.14
NN 33350	+ ○ – – + ○ – ○ ○ +	●	○ + ○ – ○ + – – + ○	21	12	8	6204	3056	0.05	0.14
NN 33353	+ ○ – – + ○ – ○ ○ +	○	○ + ○ – ○ + – – + ○	21	12	9	6204	3057	0.14	0.14
NN 33374	+ – ○ – ○ + + – ○ ○	●	+ – – + – + ○ + – ○	21	14	6	6206	3054	-0.10	0.00
NN 33377	+ – ○ – ○ + + – ○ ○	○	+ – – + – + ○ + – ○	21	14	7	6206	3055	0.00	0.00
NN 33405	+ ○ – – ○ – + + + ○	+	– – – + – ○ + + – ○	21	16	5	6208	3053	-0.14	-0.14
NN 33406	+ – ○ – + ○ ○ – + +	–	+ – – + – + ○ + – ○	21	16	5	6208	3053	-0.14	-0.14
NN 33407	+ – ○ – + ○ ○ + – –	+	– + + – – + ○ + – ○	21	16	5	6208	3053	-0.14	-0.14
NN 33408	+ ○ + – ○ – – – – ○	+	+ + + + – ○ – + – ○	21	16	5	6208	3053	-0.14	-0.14
NN 33409	+ ○ – + ○ – – + – ○	–	+ – + + + ○ + – – ○	21	16	5	6208	3053	-0.14	-0.14