UNCERTAINTIES AT THE CONCEPTUAL STAGE: MULTILEVEL MULTIDISCIPLINARY DESIGN AND OPTIMIZATION APPROACH

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Abstract

This paper presents some uncertainties propagation methods and their use for conceptual design. Uncertainties analysis is a natural complement to the classical post optimal sensitivities analysis of a Multidisciplinary Optimization. A methodology to deal with uncertainties propagation in the case of a multilevel design process is presented, including some work currently in progress at Dassault-Aviation to deal with uncertainties in CFD simulations. An example of design under uncertainties is presented. Finally a simple method to couple uncertainties analysis and the multidisciplinary optimization of an aircraft is described.

1 Introduction

One of the primary reasons for the rising interest in uncertainties management is its application in risk-based design methods. At the conceptual design stage, the choices of global parameters aim to satisfy performance targets. The performances predicted suffer from a certain level of uncertainties which can be related to two major sources: the entire set of sizing parameters are not fully determined yet and the computational models used during this phase can be simplified and thus can only approximate the actual physical behaviour.

To better manage the risks, both technical and economical, from the earliest stage of a program one can benefit from using a probabilistic framework. At Dassault-Aviation, the conceptual design is performed using an integrated computing platform which performs the optimization of selected performance targets under the constraints of global specifications. In a classical optimization framework, the target performances are considered as design constraints. To assess the risk related to not meeting a given objective, it is necessary to replace the usual ‘fixed margin’ approach by probabilistic criteria. This allows one to relate the objective values to a probability of meeting the target. Embedding the multidisciplinary optimization process into a probabilistic framework is a required step towards robust optimization. Robust optimization can thus be simply considered as the optimization of global parameters for a not-fully-known situation.

At Dassault-Aviation a typical conceptual design is carried out in a two-level framework as shown in figure 1.

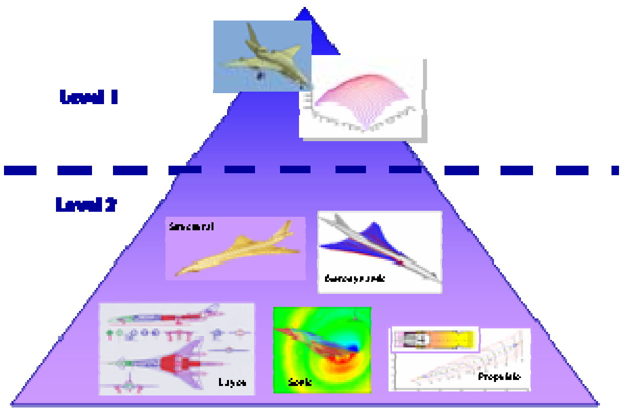

Figure 1: two-level design process

All the global optimizations, trade off studies and uncertainties management with respect to the high level specifications are carried out at level 1. This level uses low to medium fidelity tools and surrogate models which are calibrated or constructed using high fidelity analysis or optimizations performed at level 2.

The challenge is thus twofold: first, one must be able to gather all the uncertainties sources from the lower level and then, one must conceive a design that meets the specifications with a given probability.

2. Uncertainties analysis at the global level.

2.1 Uncertain design parameters

Two uncertainties propagations methods have been developed and implemented in the industrial platform that is used for level 1 analysis and design.. Along with the well known Monte Carlo method, methods derived from the structural reliability analysis have been implemented. The FORM/SORM (First and Second Order Reliability Method) rely on a local approximation of the limit state surface by a hyper plan (First Order Method) or a conic surface (Second Order Method). The risk of not satisfying a specification $g(x) > 0$ can be evaluated as a probability which reads $P\{X \in F\}$ where $F = \{x \in \Re^n : g(x) \leq 0\}$. This probability can be computed by a Monte Carlo method with a large number of random samples of X. When the computational cost prohibits a large number of samples, an approximated method such as the FORM/SORM methods allow one to estimate the desired probability. In this case, after transforming the random variables X into normalized variables U, the probability is computed using the reliability index β, which is the distance of the origin in the U space to the boundary of the limit state. The mathematical problem then reduces to a constraint minimization problem.

In practical applications, the boundary of the limit state is not a known function and needs to be approximated. We use the DACE [1] Matlab toolbox to generate the approximation with a Kriging method.

As a first example we evaluate the probability of exceeding a maximum takeoff weight assuming that the design parameters are random variables. Using the FORM method the solution is obtained in 16 iterations of the global sizing model. The FORM/SORM methods have been compared to the Monte Carlo method. In this case, we used 10,000 samples to estimate the probability with the Monte Carlo method. Since each evaluation of a sampling data requires the solution of the global sizing problem, the Monte-Carlo simulation is much more CPU intensive than the reliability analysis.

In figure 2, we present the limit state frontier as determined by the FORM method (red curve), the SORM method as the Kriging model iterates (black) and Monte-Carlo (blue). We can notice that around the critical point (point that minimizes the distance of the origin to the limit state frontier) the approximate methods and Monte-Carlo are in very good agreement.

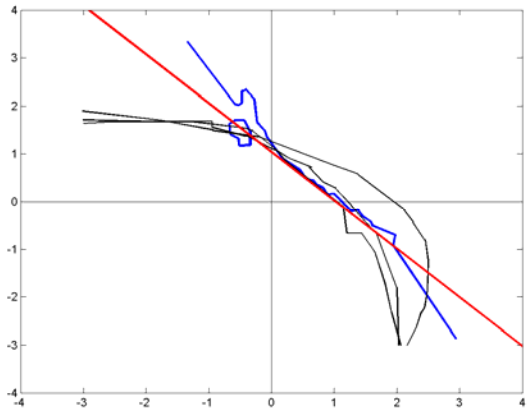

Figure 2: Limit state frontier

Figure 3 presents the population generated by the Monte-Carlo simulation; the points in black meet the target (MTOW less than specified MTOW) whilst the purple points exceed the requirements.

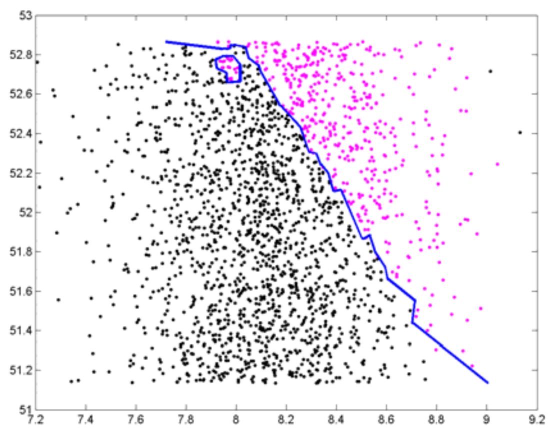

Figure 3: MC simulation population

Finally, the probabilities are compared for the different methods: Figure 4 shows the probability computed by Monte-Carlo as the simulation progresses (black curve) and the probability computed with the FORM method (red curve).

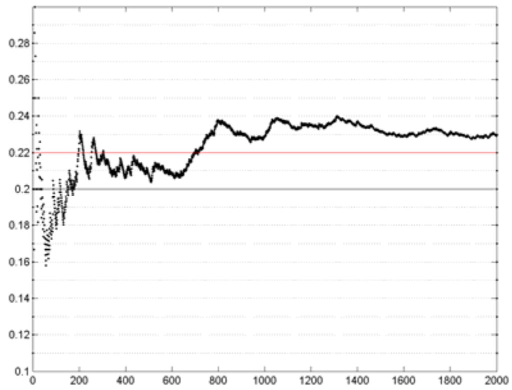

Figure 4: probability (red FORM, black MC)

The reliability method gives a probability with a margin of error of 10^{-2} compared to Monte Carlo. This margin of error is truly negligible for our case of interest and allows supports the hypothesis that the reliability method can be systematically used for robust design problems.

2.2 Uncertainties of models

The global performances of the aircraft depends on the elementary performances of the individual disciplines involved in the design. For instance, if we consider the Breguet range formula:

$$R = \frac{V}{gC_s}\frac{C_z}{C_x}\ln\left(\frac{W_b}{W_e}\right)$$

where V is the cruise speed, C_s the fuel specific consumption, C_z the lift coefficient, C_x, the drag coefficient and W_b and W_e respectively the weight of the aircraft at the beginning and at the end of the cruise. If one wants to evaluate the uncertainties of the range using this formula, the uncertainties on each of the individual terms are necessary. One of the most sensitive parameters for an aircraft performance is the drag coefficient. To derive uncertainties on the predicted drag coefficient, a multilevel approach is needed. Even at the conceptual stage the evaluation of performances critical to the design may require high fidelity analysis. We present below how the uncertainties for the aerodynamic coefficients can be obtained using high fidelity CFD analysis.

3. Uncertainties calculations using CFD.

Uncertainties is relatively new in the CFD community and can probably be explained by the relatively new use of CFD in design (especially compared to structural design) and its computational cost. In the context of CFD, the uncertainties are usually separated into two sources, uncertainties due to the physical model and uncertainties due to the boundary conditions, the operating conditions or more generally the input of the computation.

For the first type, the source is usually a lack of knowledge of the actual physical behaviour. In CFD, the model with the greatest amount of uncertainty is most probably the turbulence model because turbulence is not a fully understood phenomenon leading to models which are a drastic simplification of the physics. In this case, we are technically dealing with “errors” since these uncertainties originate in either an acknowledged deliberate simplification or an acknowledged lack of understanding. However, since the exact solution is unknown, the errors cannot be corrected and we have no alternative but to treat them in a non-deterministic manner as uncertainties. A valid model can also introduce uncertainties through the value of its parameters which can be either unknown or known only in a probabilistic manner.

Some inputs of the computation can be viewed as true aleatoric variables. Among the possible variations we can list the actual shape of the geometry, the atmospheric conditions (temperature, pressure, density, wind,…) and the operating conditions. To deal with this kind of uncertain parameters we use two methods: the method of moments and Monte-Carlo simulation using surrogate models.

3.1 Method of moments

The method of moments for uncertainties propagation has been used for a long time in the risk management community. This method is very attractive for CFD applications since it only needs one deterministic nonlinear computation, which is the CPU intensive part, to estimate the mean and the standard deviation of the output. The major impediment in its practical use was that it requires the derivatives (Jacobian or Hessian matrix) of the output with respect to the fluctuating parameters. The advances in Automatic Differentiation enable one to easily calculate the needed derivatives and the moments method is becoming feasible for CFD applications (see references [2], [3] and [4] for example).

The method of moments is based on a Taylor expansion of the response around the mean of the input parameters. Let $y = f(x)$ denote the random output of a *deterministic* process f with *random* inputs $x = \{x_i\}_{i=1,\dots,n}$. The Taylor expansion can be either a first order expansion (First order method) or a second order expansion (Second order method). A second order Taylor expansion of the output y around the expected value of x reads:

$$\begin{aligned} y = f(E(x)) &+ \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(x_i - E(x_i)) \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j}(x_i - E(x_i))(x_j - E(x_j)) \\ &+ O(\|x - E(x)\|^3) \end{aligned}$$

This Taylor expansion is used to compute the expected value and variance of the random variable y.

For the first order method the expected value of y is the value given by the deterministic computation $E(y) = f(E(x))$ and its variance reads:

$$\mathrm{var}(y) = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}\mathrm{cov}(x_i, x_j).$$

The covariance of the random variables x_i and x_j is defined as

$$\mathrm{cov}(x_i, x_j) = E(x_i x_j - E(x_i)E(x_j)).$$

The second order method uses the curvature of f to correct the expected value and the variance of y which read respectively:

$$E(y) = f(E(x)) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j}\mathrm{cov}(x_i, x_j)$$

$$\begin{aligned} \mathrm{var}(y) = &\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}\mathrm{cov}(x_i, x_j) \\ &+ \frac{1}{4}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial^2 f}{\partial x_i \partial x_j}\mathrm{cov}(x_i, x_j)\right)^2 \end{aligned}$$

The second order approximation is much richer. For instance, if x derives from an optimization problem and is a free optimum, the first order method does not give any information about the variance since $\frac{\partial f}{\partial x_i} = 0$.

The practical implementation of the method of moments in Dassault-Aviation Reynolds Averaged Navier-Stokes solver is presented in details in [5].

The major benefit of the method of moments arises from its moderate computat-ional overhead. The real limitation of this approach is that it only estimates the moments which may not provide enough information to compute a probability or estimate a distribution, should the output distribution depart from a normal distribution.

3.2 Monte Carlo using surrogate models

A Monte Carlo simulation would provide, in theory at least, an estimate of the probability density function of the outputs. The major drawback of Monte Carlo is the computational effort required. In the CFD context, each sample requires the solution of a *non linear* problem and the number of samples can be high since the convergence rate of the Monte Carlo method scales only as the reciprocal of the square root of the number of samples.

Surrogate models appear to be an elegant manner to overcome the computing time associated with Monte-Carlo simulations. The idea is extremely simple and consists of using an approximation of the expansive non linear problem for the Monte Carlo simulation. The computational advantage is evident since an evaluation of the surrogate model is performed at a negligible cost, the CPU intensive part being the construction of the model which is done once and for all.

Surrogate models can be constructed using a wide variety of techniques. Here we investigate three different surrogate models that can be separated into local and global models.

Local surrogate models are based on a Taylor expansion. The derivatives computed for the method of moments can be used to directly compute the terms of the Taylor expansion of the output functions. A *local* surrogate model is thus a *direct* byproduct of the method of moments.

The construction of a global model can be computationally expansive for a large number of parameters. Radial Basis Functions, thanks to their ability to incorporate derivatives information to improve the accuracy of the model (see reference [6]).are attractive to construct surrogate models. Kriging methods are an alternate approach for the construction of global surrogate models. By definition, Kriging methods can provide an estimate of the surrogate model error [7] which can be used both during the construction of the model and later in its exploitation phase.

3.3 Comparison of the methods

The methods described above are applied to the evaluation of the impact on the drag coefficient of a geometrical deformation close to the leading edge on the suction side of an airfoil. A parametric CAD definition of the airfoil is used to generate the new geometries. In Fig. 4 we present the reference airfoil in red and the extremes of the deformation in blue and green.. The airfoil considered here is a RAE2822 at a transonic Mach number. The calculations are performed using a $k-\varepsilon$ turbulence model. The drag coefficient for the reference point is C_x=0.0131.

The uncertainties propagation methods used here are the first order moments method, the second order moments method, Monte Carlo simulation using a first order Taylor expansion, Monte Carlo simulation using a second order expansion, Monte Carlo simulation using the RBF model and finally Monte Carlo simulation using the Kriging model. All the Monte Carlo simulations are performed using 10,000 samples. The geometrical parameter is assumed to follow a normal distribution with a standard deviation of 0.02.

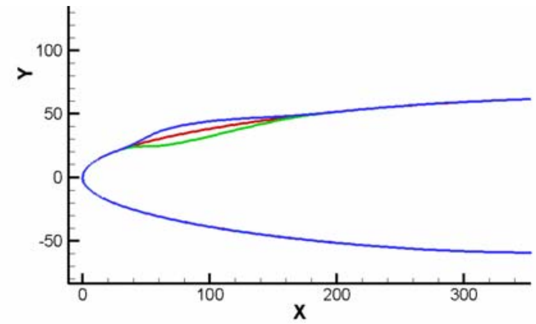

Figure 5: Geometrical deformation (green reference)

The table below summarizes the results for the predicted means and variances.

	Mean	Std deviation
First order	130.94e-4	1.55e-4
Second order	134.00e-4	4.60e-4
MC Taylor 1	130.95e-4	1.55e-4
MC Taylor 2	134.02e-4	4.52e-4
MC RBF	133.97e-4	4.81e-4
MC Kriging	134.04e-4	4.95e-4

The results clearly show two types of behavior. On the one hand the linear methods, first order method and Monte Carlo using a first order Taylor expansion, and the non linear method on the other hand. The two linear methods give, as expected, the same results. It is worth noting that in this example, the deterministic value of the drag coefficient does not correspond to the expected value of the drag coefficient considered as a random variable; there is a difference greater than 3 drag counts between the linear methods and the non linear methods. The non linear methods give the same results for the mean value, the predictions of the standard deviation are more scattered.

4. Sizing under uncertainties

To assess the probability of success is a nice feature to have but is clearly not sufficient. The next step forward is to generate a design that meets the requirements in a probabilistic sense. In the case considered here, we are interested in designing a Supersonic Business

Jet such that the aircraft will have a 95% probability to meet the required range. The uncertain parameters are in this case the drag coefficient, the fuel specific consumption and the aircraft empty weight. A two level approach is used: the aircraft is sized using detailed models, then a surrogate model is calibrated and used to determine the probability of meeting the specification. The probability is computed using Monte Carlo with the surrogate model. Figure 6 presents a sketch of the process.

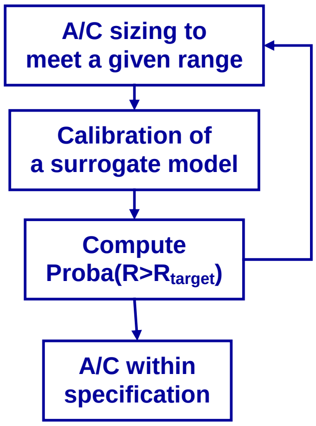

Figure 6: sizing under uncertainties process

The histogram of the range distribution for the selected aircraft is presented in figure 7 below.

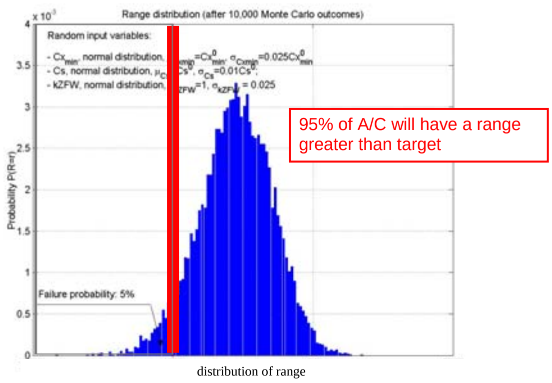

Figure 7: range distribution

6. Robust optimization.

Our multi-level design process is actually a multi-level MDO process and the objective is to take into account uncertainties in the MDO procedure. We want to be able to treat constraints described in terms of probability, for instance that the final design must have a 90% probability to exceed a target range.

The method relies only on deterministic optimizations, the target value of the constraints are adjusted in order to meet the required probability. Figure 8 presents the overall process: first a standard deterministic MDO problem is solved, then using Monte Carlo simulation around the optimal solution we compute the probabilities, if the probabilities are met we have a final robust optimal design, otherwise the constraints of the optimization problem are shifted and a new optimization cycle is performed.

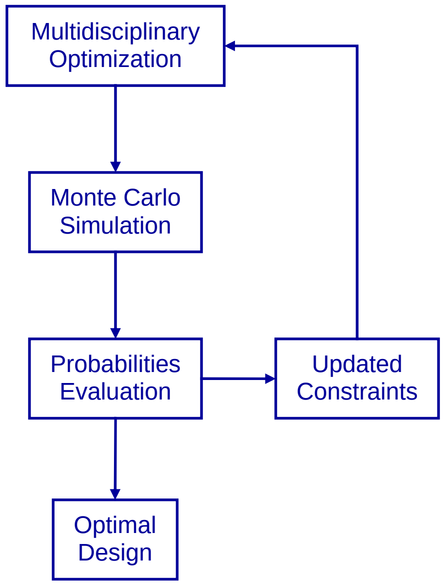

Figure 8: robust optimization process

The way the constraints are shifted is presented first for the special case of a single constraint. Let assume that the constraint reads:

$$\Pr(X \leq P_T) \geq 0.9$$

The Monte Carlo simulation performed after the optimization step allows one to construct the empirical cumulative distribution of the random variable X presented in figure 9. From the cumulative distribution we can compute P_{90} such that $\Pr(X \leq P_{90}) = 0.9$. If $P_{90} \leq P_T$ the constraint is satisfied, otherwise we determine a new target for the optimizer based on a simple shift between P_T and P_{90}. If the distribution is not to far from a normal distribution, defining the new target as $P_T^N = P_T - \alpha(P_{90} - P_T)$ with a relaxation parameter $\alpha \approx 0.7 - 0.8$ works well.

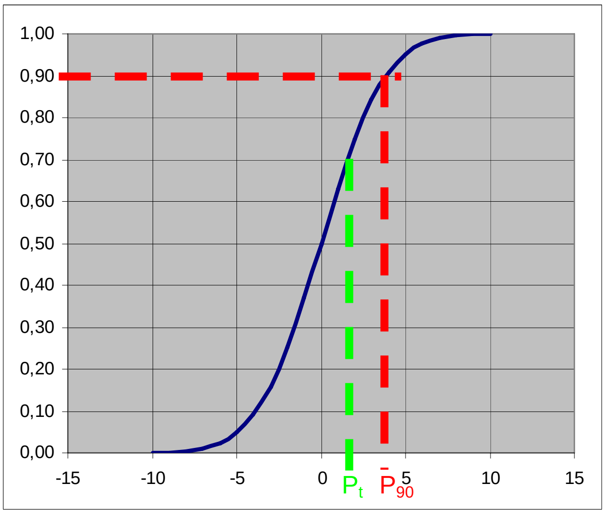

Figure 9: CdF of performance

The case with more than one constraint is more complex since it involves determining a conditional probability. The method that can be used for any number of constraints, is detailed for the case with two constraints for the sake of clarity. In this case the problem reads:

$$\Pr(X_1 \geq X_1^T \ and \ X_2 \geq X_2^T) \geq 0.9$$

We first transform the random variables X_1 and X_2 to their reduced centred form :

$$\tilde{X}_i = \frac{X_i - E(X_i)}{\sigma_{X_i}}$$

We now work in the reduced space in which the results of the Monte-Carlo simulation are plotted (see figure 10).

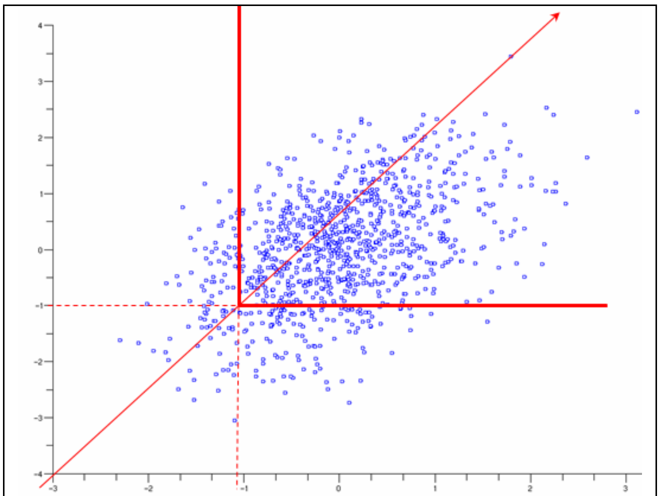

Figure 10: target selection in reduced space

As can be seen in figure 10 we define a box that is moved along the diagonal of the reduced axis until it captures 90% of the sampled points. The intercepts of the box edges with the axis defines the values of $\tilde{X}_i^{90}$ such that

$$\Pr(\tilde{X}_1 \geq \tilde{X}_1^{90} \ and \ \tilde{X}_2 \geq \tilde{X}_2^{90}) = 0.9$$

The values of X_i^{90} are then computed and using the cumulative distribution functions for X_i we apply the same type of shift as the one constraint case for both of the constraints.

It is worth noting that in the case of independent random variables X_1 and X_2 this method leads to $\Pr(\tilde{X}_i \geq \tilde{X}_i^{90}) = \sqrt{0.9}$. Since in the case of independent variables

$$\Pr(\tilde{X}_1 \geq \tilde{X}_1^{90} \ and \ \tilde{X}_2 \geq \tilde{X}_2^{90}) =$$
$$\Pr(\tilde{X}_1 \geq \tilde{X}_1^{90}) \Pr(\tilde{X}_2 \geq \tilde{X}_2^{90})$$

the two constraints share equally the "burden" of meeting the global specification.

The correlation between the random variables X_1 and X_2 are taken into account. Figure 10 presents the case of two variables with a positive correlation, whilst figure 11 shows the case of two variables with a negative correlation.

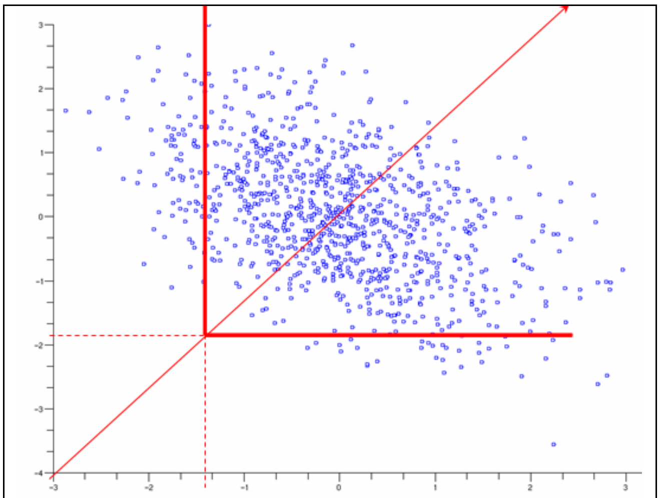

Figure 11: impact of negative correlations

The negative correlation indicates that the two constraints cannot be reached at the same time and this shows up clearly on the width of the box that contains 90% of the samples which is much larger in this case.

7. Conclusion

Uncertainties propagation needs to be considered in the multi-level paradigm to fully tackle the source of uncertainties and build a hierarchy with respect to their impact on the probability of success of the global specification. The estimation of uncertainties in crucial disciplines such as aerodynamics are thus of paramount importance. The uncertainties propagation through a CFD simulation is still in its infancy and results available today show the advantage of considering a second order method and justify the extra work that is involved for the computation of the second order derivatives. It is, once again, important to stress that the method of moment is one amongst a large panel and that it has some shortcomings. For instance the information about the first two moments does not suffice to determine a confidence interval if the output distribution departs too much from a normal distribution, whilst the empirical distribution obtained with Monte-Carlo will allow one to estimate the confidence interval. In this case it will then be necessary to assess the statistical error between the actual model (Navier-Stokes) and the surrogate model. It is also important to keep in mind that the difficulty and the computing cost of building a surrogate model grows exponentially with the number of parameters. It is thus necessary to build an "uncertainties" toolbox for aerodynamics and the methods presented in this paper are the first candidates.

Uncertainties propagation at the global aircraft level is a necessary post processing after a multidisciplinary optimization and complements the usual post optimal sensitivities analysis; it can even be integrated within an optimization loop with minor modifications. Some of the necessary ingredients for a robust multi-level multi-disciplinary optimization are now available and are being integrated into our design process to eventually allow to perform Optimization Under Uncertainties which is the actual goal.

8.References

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