ADVANCED METHODOLOGIES FOR AVERAGE PROBABILITY CALCULATION FOR AEROSPACE SYSTEMS

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Abstract

Airplane manufacturers must certify the airplane systems to be safe for flight. One means of safety certification is by analytic demonstration that the probability of failure in a typical flight is bounded. The probability bound requirement for a system is based on the criticality of system failure. Therefore, probability of failure calculations for these systems is necessary for certification. The probability of failure for a system which can have latent faults is non-constant across flights. So we are led into the concept of "Average Probability of Failure per Flight" or "Average Probability of Failure per Flight Hour".

In this paper we will discuss new methodologies and equations. We will also review and compile previous efforts (many of which are our own) to develop efficient methods and tools to compute the "Average Probability of Failure per Flight" for aerospace systems.

1 Introduction

Fault-tolerance in commercial aircraft applications is typically achieved by redundancy. Dual or triple redundancy is common, and higher redundancy aircraft systems exist. This is also true for the Military, Defense and Space vehicles. In many cases the backups are provided for safety, and are used only to provide functionality when the primary fails. In such systems the primary component is checked before the start of a flight to see if it operates correctly. The aircraft will not take off unless the primary component is functioning.

Usually backup components are checked (inspected) at intervals that span multiple flights. This leads to flights where the system although functional could have potential latent failures in the backup components. The probability of failure in such cases varies from flight to flight due to the different exposure times for components in the system. In avionics systems, redundancy management techniques are used to detect, isolate and reconfigure the systems. Automatic reconfiguration is a standard redundancy management technique.

Another scenario is the inspection of aging parts typically associated with the airframe structure. An example will be bolts or rivets on a wing that can potentially expose a path to lightning due to degradation with age. Aging components are typically modeled by the Weibull lifetime distribution. Depending on the inspection interval the risk per flight hour changes. For very small intervals the cost of inspections can become exorbitant.

Airplane manufacturers must certify the airplane systems to be safe for flight. One means of safety certification is by analytic demonstration that the probability of failure in a typical flight is bounded. The probability bound requirement for a system is based on the safety criticality level (consequences) of system failure. Typical criticality levels are Minor, Major, Hazardous and Catastrophic in increasing order of severity. Therefore, probability of failure calculations for these systems is necessary for certification. As mentioned in the previous paragraph, the probability of failure for a system which can have latent failures is non-constant across flights. So we are led into the concept of the metric "Average Probability of Failure per Flight" or "Average Probability of Failure per Flight Hour".

In the following sections we define the metric and discuss methodologies and equations to compute the metric.

1.1 Definition of Average Probability

Assume that an airplane is operated for a total of *M* flights in its life time with each flight being of a constant duration of *t* flight hours. This is an idealization of reality and *t* represents the average flight length of a typical airplane of a specific type such as the Boeing 737. An example could be 3000 flights per year of average duration of 1.5 hours and an airplane life of 20 years. This results in $M = 60000$ with $t = 1.5$ hours. Airplane life can also be expressed in flight hours as *T*, which in this example would be 90000 hours.

Assume that the system under analysis comprising one or more redundant components operates and provides a function for the full flight of duration *t*. If the system operates for only a portion of the flight, the analysis methods can be used by adjusting the flight length to system operation time.

If P_i is the probability of system failure in the *i*-th flight then we define the Average Probability per Flight as

$$P_{avg} = \frac{\sum_{i=1}^{M} P_i}{M} \qquad (1)$$

Note that P_i is the probability the system fails anywhere in the typical flight of duration t given that it was working at the start of a flight. If the system fails there is a loss of function that is noticeable or indicated and results in the repair or replacement of the system on the ground before the next flight. The implication is that a system can fail only once in a given flight and hence the definition of average probability of failure as in Eq. (1) makes sense.

Another way of describing the metric in (1) is

$$P_{avg} = \frac{E[N(T)]}{M} \qquad (2)$$

where *E[N(T)]* is the expected number of system failures (repairs/replacements) in the life *T* of the airplane. Based on the assumption of repairs/replacements only on ground *N(T)* is less than or equal to *M*, the measure above makes sense and is equivalent to Eq. (1).

A near equivalent continuous analogue of the above metric can be defined for ultra-high reliable systems with not much loss in accuracy if we remove the restriction of replacement/repair only on ground. Considering that for safety critical systems the probability of system failure is very small in a flight (of the order of 1E-06 to 1E-09 per flight) and that the flight length *t* is much smaller than life of airplane *T* the following definition is nearly equivalent to those in Eqs. (1) and (2).

$$P_{avg} = \frac{t}{T}\int_0^T m(x)dx = t * \frac{E[N(T)]}{T} \qquad (3)$$

The integrand *m(x)* is also called the instantaneous system renewal rate. P_{avg}/t is the "Average Probability of Failure per Flight Hour". It is equivalent to the average system renewal rate. Renewal theory methods [5, 7] can be used to obtain the renewal rate as we will show in the following sections.

Many other definitions [2, 6] have been previously proposed such as "Average System Unavailability", "Average System Failure Rate" etc with the intent to calculate the same metrics as in Eqs. (1)-(3) but the definitions here are precise and unambiguous.

2 Average Probability Calculation Methods

2.1 Closed-Form Solutions for Simple Systems

For systems that have a small number of redundant components such as dual and triple redundant systems it is possible to obtain closed-form solutions for the average per flight failure probability.

Dual Redundant System

Consider a dual redundant system of two identical units where the failure of a single component is not indicated in a flight but the system failure (loss of both units) is identified because of the resulting loss of function. Periodically both units are inspected and failed units are repaired at a fixed interval T. If both units fail between inspection intervals, the system will be fixed within a flight (nearly instantaneously) after its failure. Suppose the failure rates λ (assumed constant with time) are the same for both units and we consider the impact of an inspection/scheduled maintenance interval for the system. The system has the possibility of failing and being renewed multiple times during the inspection interval, depending on the length of the interval. If the interval is too long one encounters the risk of multiple system failures. On the other hand if the interval is too short then the cost of schedule maintenance goes up. In this scenario we are interested in the average probability of failure over one inspection interval.

The system renewal rate *m(x)* can be obtained from a transient analysis of a Markov chain [5] for the system. The Markov chain for the dual redundant system with system repair on failure is shown in Figure 1.

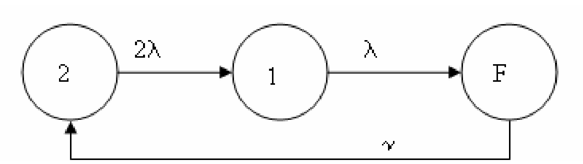

Figure 1: Markov Chain of Dual Redundant System

State 2 corresponds to a system that is fully functional (both components working), 1 to a system where one component has failed, and F represents system failure. The flow into the failure state is P_1*λ where P_1 is the state probability of state 1. If repair rate γ is taken to be infinite, i.e. the repair is instantaneous, failure state F can be mapped to the starting state 2, yielding the simpler Markov chain in Figure 2. If flight length is much smaller than the inspection interval, instantaneous repair is a good assumption.

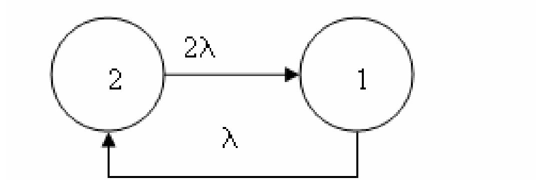

Figure 2: Simplified Markov Chain of Dual Redundant System

The state probability equations [5] for this Markov chain are

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + 2\lambda p_2(t)$$

Using the conservation equation $p_1 + p_2 = 1.0$ we have $p_2 = 1 - p_1$ and substituting for p2 in the differential equation above we obtain

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + 2\lambda(1 - p_1(t)) = -3\lambda p_1(t) + 2\lambda$$

The solution to this one variable simple Ordinary Differential Equation (ODE) can be obtained easily as

$$p_1(t) = \frac{2}{3}(1 - e^{-3\lambda t})$$

The instantaneous renewal rate is

$$m(t) = \lambda * p_1(t) = \frac{2\lambda}{3}(1 - e^{-3\lambda t})$$

Note that $m(\infty) = \frac{2\lambda}{3}$ which is the inverse of the MTBF (Mean Time Between Failures) of the dual redundant system. Using Eq. (3) we obtain the average probability of failure per flight as

$$P_{avg} = \frac{t}{T}\int_0^x m(x)dx = \frac{2\lambda t}{3}\left[1 - \frac{1 - e^{-3\lambda T}}{3\lambda T}\right] \quad (4)$$

Note that T in equation above is the inspection interval and t is the flight length. For large T the average probability expression is close to $2\lambda t/3$ which equals t/MTBF for the system.

In Table 1 we compare the result in Eq (4) with a computer package called HIMAP [1] that computes P_{avg} numerically from Eq. (1), using Markov chain analysis method described in detail in [3] and briefly in Section 2.2.

Inspection interval T is set at 10000 hours and flight length t at 1.0 hour. The correlation is very good and shows that removing the restriction of repair/replacement on the ground only as assumed in Eq. (4) does not affect the result much except for large component failure rates.

λ	λ*T	2*λ/3	Eq. (4)	Numerical Markov Solver (Eq. (1))
1E-01	1000	6.6666E-02	6.6644E-02	6.4495E-02
1E-02	100	6.6666E-03	6.6444E-03	6.6224E-03
1E-03	10	6.6666E-04	6.4444E-04	6.4424E-04
1E-04	1	6.6666E-05	4.5508E-05	4.5549E-05
1E-05	0.1	6.6666E-06	9.0707E-07	9.0707E-07
1E-06	0.01	6.6666E-07	9.9007E-09	9.9007E-09

Table 1: Equation (4) vs. Eq. (1)

Suppose one of the units is considered the primary unit and the other unit is the backup. Assume the primary is checked to make sure it is functional before every flight and the backup is checked at T hours. The probability that both the primary and backup fail in the same flight leading to a system failure is much smaller than the probability that the backup fails latent in an earlier flight and the primary fails in a subsequent flight, leading to system failure. Although both scenarios can be modeled, we consider only the main failure path as represented by the Markov chain in Figure 3.

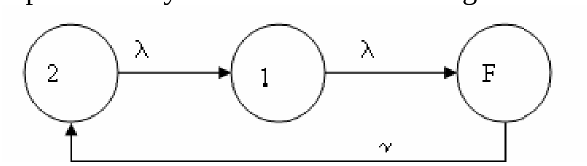

Figure 3: Markov Chain of Dual Redundant System Failure Path with Backup Failing First Followed by Primary Failure

λ	λ*T	Eq. (5)	Numerical Markov Solver (Eq. (1))
1.00E-01	1.00E+03	4.9975E-02	4.9936E-02
1.00E-02	1.00E+02	4.9750E-03	4.9752E-03
1.00E-03	1.00E+01	4.7500E-04	4.7503E-04
1.00E-04	1.00E+00	2.8383E-05	2.8386E-05
1.00E-05	1.00E-01	4.6827E-07	4.6831E-07
1.00E-06	1.00E-02	4.9668E-09	4.9673E-09

Table 2: Equation (5) vs. Eq. (1)

Using the methodology shown for the no-inspection of both components case described earlier we can obtain the average probability as

$$P_{avg} = \frac{t}{T}\int_{0}^{x} m(x)dx = \frac{\lambda t}{2}\left[1-\frac{1-e^{-2\lambda T}}{2\lambda T}\right] \quad (5)$$

Inspection interval T is set at 10000 hours and flight length t at 1.0 hour. The correlation in Table 2 is very good and shows that removing the restriction of repair/replacement on the ground only as assumed in Eq. (5) and also ignoring the replacement probability due to primary and backup failure in the same flight with primary failing first does not affect the result much except for large component failure rates.

Triple Redundant System

Consider a triple redundant system with identical components where none of the components are inspected. Assume a constant failure rate of λ for the 3 components. Assume that the loss of function due to system failure will be detected which will trigger a system renewal where all 3 components are restored to full working condition almost instantaneously within a flight. The loss of one or two components is latent and is not detected.

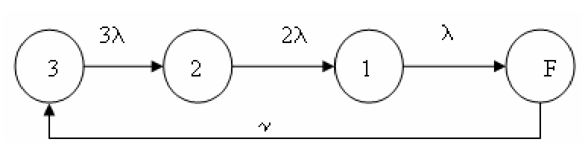

Figure 4: Markov Chain of Triple Redundant System

The Markov chain for this system in shown in Figure 4. The flow into the failure state is P_1*λ. If repair rate γ is taken to be infinite, i.e. the repair is instantaneous, failure state F can be mapped to the starting state 3, yielding the simpler Markov chain shown in Figure 5.

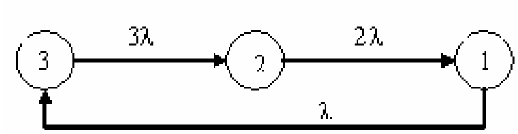

Figure 5: Simplified Markov Chain of Triple Redundant System

The state probability equations for this chain are

$$\frac{dp_1(t)}{dt} = -\lambda p_1(t) + 2\lambda p_2(t)$$

$$\frac{dp_2(t)}{dt} = -2\lambda p_2(t) + 3\lambda p_3(t)$$

Using the conservation equation $p_1 + p_2 + p_3 = 1.0$ we have $p_3 = 1 - p_1 - p_2$. The above is a set of two coupled ordinary differential equations. To solve it one may use many techniques such as the Laplace transform technique [5].

The result for the renewal rate is obtained then as

$$m(t) = \lambda * p_1(t) = \frac{6\lambda}{11}\left[1 - e^{-3\lambda t}(\cos\sqrt{2}\lambda t + \frac{3}{\sqrt{2}}\sin\sqrt{2}\lambda t)\right]$$

The average probability per flight is then

$$P_{avg} = \frac{t}{T}\int_0^T m(x)dx = \frac{6\lambda t}{11}\left[1 - \frac{6}{11\lambda T} + \frac{e^{-3\lambda T}}{11\sqrt{2}\lambda T}\left\{7\sin\left(\sqrt{2}\lambda T\right) + 6\sqrt{2}\cos\left(\sqrt{2}\lambda T\right)\right\}\right] \quad (6)$$

For large T the average probability expression is close to $6\lambda t/11$ which equals *t/MTBF* of the system.

λ	λ*T	Numerical Markov Solver Eq. (1)	Eq. (6)
1.00E-01	1000	5.30692E-02	5.45157E-02
1.00E-02	100	5.41012E-03	5.42479E-03
1.00E-03	10	5.15570E-04	5.15702E-04
1.00E-04	1	2.62312E-05	2.62314E-05
1.00E-05	0.1	8.61827E-08	8.61827E-08
1.00E-06	0.01	9.85124E-11	9.85124E-11

Table 3: Equation (6) vs. Eq. (1)

Inspection interval T is set at 10000 hours and flight length t at 1.0 hour. The comparison in Table 3 is very good and shows that removing the restriction of repair/replacement on the ground only as assumed in Eq. (6) does not affect the result much except for large component failure rates.

Based on the complexity of system redundancy and the component inspection scenario the approach shown in this section can be used to obtain the average probability. For some cases one can use approximation techniques described in [6]. For large systems a Fault Tree model can be constructed and the minimal cutsets of the fault tree model can be treated as a dual or triple redundant system with latent failure events and the equations can be applied to each type of minimal cutest.

Aging System/Component Analysis

The failure characteristics of an aging component or system can be described by a Weibull distribution. The component is inspected at intervals and is replaced with a new component if found to have failed. The inspection does not detect wear; hence the Weibull age of the component doesn't change unless the component is replaced because of failure. The average probability of failure of the component over the life of the airplane is to be calculated.

The average probability of failure for a component with Weibull failure distribution in the absence of repair is first derived. The derivation is extended to the case in which the component is repaired periodically. The derivation does not require repair intervals to be uniform.

Average Unreliability with No Repair

The average **reliability** of a Weibull distribution is given by the integral

$$R_{AV}(T) = \frac{1}{T}\int_0^T e^{-\left(\frac{t}{c}\right)^\beta} dt \quad (7)$$

Where β is the shape factor and c is the characteristic life.

Making the substitutions in (7)

$$x = \left(\frac{t}{c}\right)^\beta, \; t = cx^{\frac{1}{\beta}}, \; dt = \frac{c}{\beta}x^{\frac{1}{\beta}-1}$$

gives

$$R_{AV}(T) = \frac{c}{T\beta}\int_0^{T_0} x^{\frac{1}{\beta}-1} e^{-x}dx, \quad \text{where} \quad T_0 = \left(\frac{T}{c}\right)^{\beta} \tag{8}$$

The integral is the lower incomplete gamma function

$$R_{AV}(T) = \frac{c}{T\beta}\Gamma\left(\frac{1}{\beta}, T_0\right) = \frac{c}{T\beta}\Gamma\left(\frac{1}{\beta}, \left[\frac{T}{c}\right]^{\beta}\right) \tag{9}$$

The average **unreliability** $U_{AV}(T)$ is the complement of the average reliability:

$$U_{AV}(T) = 1 - R_{AV}(T) \tag{10}$$

Average Unreliability with Periodic Repair (Renewal Case)

A more accurate approximation takes renewal into account – that is, failed components are inspected periodically and repaired when the inspection uncovers failure. The repaired components then contribute to system reliability. Again we calculate average reliability, and obtain average unreliability from Eq (10).

Symbol	*Description*
$t_o, t_1, \ldots, t_n$	Component put into service at time t_o, and inspected at times $t_1, \ldots, t_n$
$A_{k,n}$	Average reliability of a unit installed new at t_k in the interval $[t_k, t_n]$, considering renewal
$U_{k,n}$	Average unreliability of a unit installed new at t_k in the interval $[t_k, t_n]$, considering renewal
$R_{k,n}$	Average reliability of a unit installed new at t_k in the interval $[t_k, t_n]$, when renewal is not considered, represented by equation 6 with T set to $t_n - t_k$,
$u_{k,j}$	Probability that first failure of a unit that was new at time t_k occurs in interval $[t_{k+j-1}, t_{k+j}]$.

It is convenient to start with the final interval and work backwards. A unit that is installed new at T_{n-1} has average reliability in the interval $[t_{n-1}, t_n]$ given by

$$A_{n-1,n} = R_{n-1,n}$$

(The component installed at the start of the last interval, so no renewal is possible). A part installed new at time t_{n-2} has the possibility of renewal at t_{n-1}:

$$A_{n-2,n} = R_{n-2,n} + u_{n-2,1} A_{n-1,n}$$

(Reliability of equipment installed new at time t_{n-2}, corrected for renewal at t_{n-1} after failure in the interval $[t_{n-2}, t_{n-1}]$). Similarly, equipment installed new at time t_{n-3}, can have first renewal at t_{n-2} or t_{n-1} :

$$A_{n-3,n} = R_{n-3,n} + u_{n-3,1}A_{n-2,n} + u_{n-3,2}A_{n-1,n}$$

In general,

$$A_{n-j,n} = R_{n-j,n} + \sum_{k=1}^{j-1} u_{n-k,n} A_{n-j+k,n}, \tag{11}$$

where

$$u_{k,j} = e^{-\left(\frac{t_{k,j-1}}{c}\right)^{\beta}} - e^{-\left(\frac{t_{k,j}}{c}\right)^{\beta}}, \text{ and}$$

$$t_{k,m} = t_{k+m} - t_k.$$

Continuing in this manner we calculate $A_{0,n}$, the average reliability of a component that was installed new at t_0 if repair is considered. The average unreliability of the component is $1 - A_{0,n}$.

This calculation has complexity $O(n^2)$ and requires n evaluations of the incomplete gamma function.

For example if a component has a Weibull distribution with characteristic life c=10000 hours, and β = 2.0 and periodic inspection is done at 10000 hours for failure with replacement upon failure over an airplane life of 80000 hours, then the average probability of failure of this component over the life of the airplane is 0.3503 using Eq. (9) and (11).

2.2 Numerical Methods for more Complex Systems

For systems with complex maintenance scenarios for redundancy management, a general approach to obtaining P_{avg} is by numerical solution of Markov models for each flight or mission of the airplane and then using Eq. (1).

We will assume that the system failure/repair in any mission is modeled by a time-homogeneous continuous time Markov chain (CTMC) [5]. CTMCs are fully specified by the instantaneous transition rate matrices and initial probability vectors. The evolution of state probabilities within a specific flight is governed by the Chapman-Kolmogorov equations for CTMCs:

$$\frac{d\mathbf{p}(x)}{dx} = \mathbf{Q}^T \mathbf{p}(x); 0 \le x \le t \qquad (12)$$

The relation between the state probabilities at the start and end of a mission can be written:

$$\mathbf{p}_i^e = \mathbf{P}\mathbf{p}_i^0 \qquad (13)$$

Repairs are performed between flights. We assume that such repairs can be modeled by a linear mapping between the state probability vector at the end of one flight and the initial probability vector for the subsequent flight

$$\mathbf{p}_{i+1}^0 = \mathbf{G}_{(i,i+1)} \mathbf{p}_i^e \qquad (14)$$

By combining Eqs. (12)-(14) we can express the state probabilities at the end of any flight in terms of the first flight's starting state probabilities $\mathbf{p}_0$ as follows:

$$\mathbf{p}_i(T) = \left[\prod_{k=1}^{i-1} \mathbf{P}\mathbf{G}_{(i-k,i-k+1)} \right] \mathbf{P}\mathbf{p}_0; 1 \le i \le M \qquad (15)$$

$$\mathbf{P} = e^{\mathbf{Q}^T T} \qquad (16)$$

Eqs.(15)-(16) show that the state probabilities at the of end of any mission can be computed by a series of matrix-vector multiplications involving the state-transition probability matrix **P**, the inter-mission mapping matrices **G** and the first mission's initial state probability vector $\mathbf{p}_0$.

<u>Example</u>

Assume there are three components in the system: Component A is an Active unit providing some functionality in the system, Component B is the Backup unit for Component 'A', and Component 'M' is the Monitor unit which is constantly monitoring the health of the Backup unit. System is assumed to be functional as long as both Active and Backup units do not fail.

Maintenance scenario in this system is as follows:

- System failure (Active unit and Backup unit failure) is repaired at the end of every flight to full up state. At this point any Monitor unit failures are also repaired.
- Active unit is assumed to be repaired at the end of a flight, if failed.
- Backup unit is checked and repaired every ten flights if the Monitor is working.
- If Monitor is not working, Backup unit is not checked until there is a system failure or until the monitor gets fixed.

Monitor unit itself is checked every hundred flights. So, at the end of 100th flight, all the components are effectively checked and any failures are repaired. This implies that system reliability/unreliability profile repeats itself every hundred flights and the Average system unreliability per flight can be computed by averaging over any cycle of hundred flights.

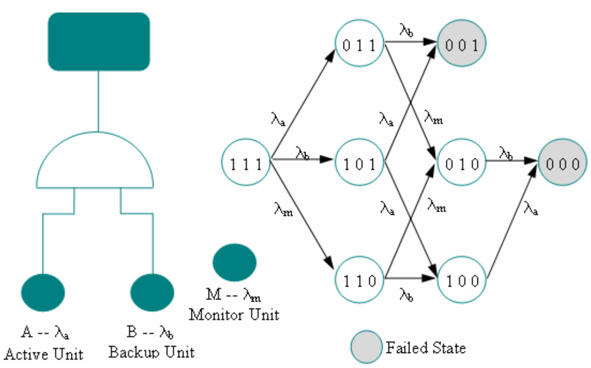

Figure 6. System Model for an Active and a Hot Spare Backup Unit

Figure 6 shows the Markov chain model of the system. Suppose the state tuples are numbered as shown in Table 4. Then the flight to flight mapping matrices $\mathbf{G}_{(i-1,i)}$ for this system that model the repair and replacements between flights are as shown below:

$$\mathbf{G}_{(i,i+1)} = \begin{bmatrix} \mathbf{e}_1^T + \mathbf{e}_4^T + \mathbf{e}_7^T + \mathbf{e}_8^T \\ \mathbf{e}_2^T + \mathbf{e}_6^T \\ \mathbf{e}_3^T \\ \mathbf{0}^T \\ \mathbf{e}_5^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \end{bmatrix}; i \geq 1; i \neq 10k \forall k \geq 1 \qquad (17)$$

$$\mathbf{G}_{(i,i+1)} = \begin{bmatrix} \mathbf{e}_1^T + \mathbf{e}_3^T + \mathbf{e}_4^T + \mathbf{e}_7^T + \mathbf{e}_8^T \\ \mathbf{e}_2^T + \mathbf{e}_6^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{e}_5^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \end{bmatrix}; \qquad (18)$$

$$i = 10j;\ j \geq 1; i \neq 100k \forall k \geq 1$$

$$\mathbf{G}_{(i,i+1)} = \begin{bmatrix} \mathbf{e}_1^T + \mathbf{e}_2^T + \mathbf{e}_3^T + \mathbf{e}_4^T + \mathbf{e}_5^T + \mathbf{e}_6^T + \mathbf{e}_7^T + \mathbf{e}_8^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \\ \mathbf{0}^T \end{bmatrix};$$

$$i = 100j;\ j \geq 1; \qquad (19)$$

In Eqs (17), (18) and (19), the matrices $\mathbf{G}_{(i-1,i)}$ are (8x8) in size, $\mathbf{e}_i^T$; i = 1,...,8 are (1x8) row vectors where the i th entry equals 1.0 and all other entries in the row are 0.0, and $\mathbf{0}^T$ is a (1x8) row vector all of whose entries equal 0.0. The flight to flight mapping matrices are very sparse and they also have the property that their columns sum to 1.0, i.e. $\mathbf{e}^T\ \mathbf{G}_{(i-1,i)} = \mathbf{e}^T$ where $\mathbf{e}^T$ = (1, 1, 1, ..., 1) to conserve probability between flights. The tool HIMAP [1] starts with the initial probability vector and marches across flights recursively applying Eqs. (13) and (14).

When the system is analyzed for one hundred missions in the tool HIMAP [1] with the component characteristics as shown in Table 5, we obtain the system unreliability time plot shown in Figure 7.

State Number	State Tuple (state of A, state of B, state of M)
1	(1,1,1)
2	(1,1,0)
3	(1,0,1)
4	(0,1,1)
5	(1,0,0)
6	(0,1,0)
7	(0,0,1)
8	(0,0,0)

Table 4: State Tuples and State Numbering

Component	**Failure Rate**	**Inspection Interval (Flights/Missions)**
Active Unit (A)	1.2e-4	1
Backup Unit (B)	1.2e-4	10 – If Monitor is working
Monitor Unit (M)	1.2e-6	100

Table 5: Component Failure Rates

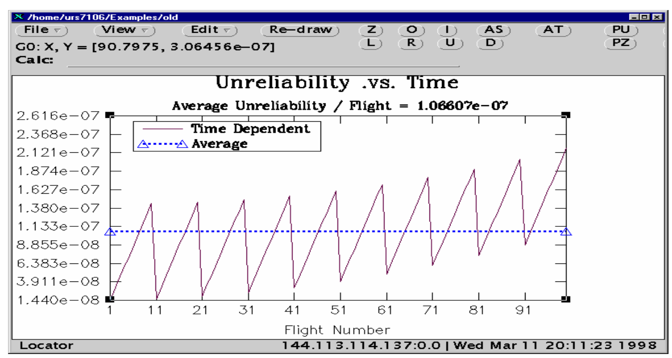

Figure 7. Probability of Failure and Average

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