AUTOMATIC FLIGHT CONTROL USING SAMPLED-DATA MIXED H_2 / H_∞ OPTIMIZATION

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Keywords: *automatic flight control, sampled-data, H_2 / H_∞ performance*

Abstract

The purpose of the present paper is to provide a design methodology for digital automatic flight controllers. The main idea of the proposed method is based on the mixed H_2 / H_∞ optimization which allows the simultaneous attenuation of both deterministic bounded power and stochastic bounded spectrum exogenous inputs. The paper mainly brings two contributions. The first is that the considered H_2 / H_∞ optimization problem is formulated in the sampled-data framework. The motivation for this statement comes from the fact that automatic control laws are extensively implemented using digital equipment. In order to model the hybrid dynamics obtained by coupling the sampled-data control system to the continuous-time dynamics of the aircraft, a representation based on systems with finite jumps has been used. The second contribution consists in introducing a multiplicative noise component in order to improve the stability robustness performances with respect with random parametric variations. The theoretical results are illustrated by a case study for the design of an automatic landing system.

1 Introduction

Although the aircraft flight control design has been dominated for a long period of time by classical control methods, over the last decades a major attention was devoted to "unconventional" design techniques able to provide effective solutions for control problems with high level of complexity. This interest has been motivated by the aim to enhance the functionality and the safety of the aircraft over an increased range of operating conditions characterized by severe external disturbances and parameters variations. Optimal control techniques are among the most widely studied modern design methodologies due to their large area of applications in aerospace engineering. The norm minimization of the mapping from certain exogenous inputs to appropriate regulated outputs played a crucial role in many applications including tracking problems, robust control, filtering, identification, fault detection. A powerful approach combining the advantages of the well-known linear quadratic optimization with the ones of the H_∞-norm minimization is the mixed H_2 / H_∞ optimization method. The solution of this problem provides an effective procedure to attenuate simultaneously the effects of stochastic disturbances and of deterministic input variables. Such requirements often arise in aviation applications (see for instance [5], [11], [16], where examples like automatic landing, control under aeroelastic effects and robust fault detection can be found).

Another important issue for the performance of the automatic control system is the implementation of the control law. Over the last decades the discrete-time control algorithms became dominant due to the intensive use of the digital devices. There are two common ways to design discrete-time control laws. The first one is to design a continuous-time controller and then to discretize it for a small enough sampling period. A second usual method is to determine a discretized model of the dynamics aircraft and then, using a discrete-time synthesis approach, to obtain the discrete-time control system. There is also a third alternative methodology which takes into account the hybrid character of the closed loop configuration obtained by coupling the discrete-time control system with the continuous-time dynamics of the airplane. In this framework in which the discretization approximations are eliminated, one expects to improve the stability and the disturbance attenuation performances and to point out a more direct relationship between the sampling period and these performances. For such hybrid dynamical systems, well-known in the literature as *sampled-data systems*, many useful results for optimal control are actually available. Some of these results can be found in [1], [2], [8], [9], [10], [17], [18].

An important aspect taken into account in the control system design is the robustness performance with respect with modeling uncertainties of the airplane dynamics. Besides the well known representation of the modeling uncertainty (see *e.g.* [12]), many other applications in aviation deal with models in which some of the parameters have nonuniformly distributed random variations. Such behavior naturally leads to stochastic models with multiplicative noise which have been intensively studied both in the continuous-time case (see *e.g.* [15] and the applications therein) and in the discrete-time framework (*e.g.* [7]).

In the present paper a mixed H_2/H_∞ type control problem for sampled-data systems corrupted with multiplicative white noise is considered. Its formulation given in the next section takes into account the specific of some usual applications in aviation. Section 3 is devoted to the computation of the performance index associated with the considered H_2/H_∞ control problem. Using the result derived in this section, the state-feedback H_2/H_∞ control problem for stochastic systems with jumps corrupted with multiplicative white noise is solved in Section 4. The results proved in Sections 3 and 4 are expressed in terms of the solutions of some specific systems of coupled continuous-time and discrete-time Riccati equations. In the final part of Section 4 an iterative procedure to compute the stabilizing solution of such systems is presented. An illustrative example is given in Section 5 for the design of an automatic landing system of an airplane. The paper ends with some concluding remarks.

2 Preliminary Results and Problem Formulation

The sampled-data mixed H_2/H_∞ optimization problem analyzed in this paper has the following simplified configuration:

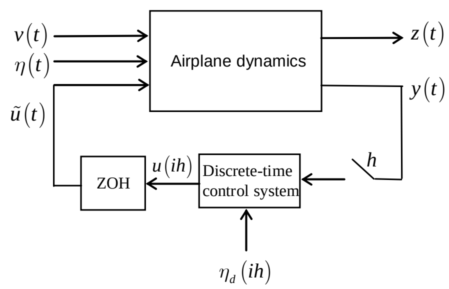

Fig. 1. Sampled-data closed loop configuration

where $v(t)$ is a continuous-time bounded in power input, $\eta(t)$ is a white noise process with unitary intensity, $z(t)$ denotes the continuous-time controlled output and $y(t)$ is the continuous-time measured output which is sampled with the constant period $h>0$ in order to be processed by the digital controller. η_d denotes a discrete-time noise approximated by a sequence o independent random vectors. The constant piecewise control $\tilde{u}(t)$ is obtained as the output of the zero-order hold (ZOH) element, that is $\tilde{u}(t)=u(ih)$, $ih<t\le(i+1)h$.

An effective method to represent the hybrid configuration in Figure 1 which includes the continuous-time dynamics of the airplane and the discrete-time controller together with its corresponding ZOH is based on the representation using the *systems with finite jumps*. Thus, following the idea used in [9], the constant piecewise control $\tilde{u}(t)$ can be represented as the solution $\bar{x}(t)$ of the system with jumps:

$$\begin{aligned} \frac{d\bar{x}(t)}{dt} &= 0, \text{ for } ih < t \le (i+1)h \\ \bar{x}(ih^+) &= u(ih), \text{ at } i = 0,1,\ldots \end{aligned} \quad (1)$$

where the following notation has been used $f(\alpha^+) := \lim_{\varepsilon \searrow 0} f(\alpha + \varepsilon)$. The above equations suggest to consider the following structure of a system with jumps corrupted with multiplicative white noise:

$$\begin{aligned} dx(t) &= (A_0 x(t) + Hv(t))dt + A_1 x(t) d\xi(t) \\ &\quad + G\eta(t),\ t \ne ih \\ x(ih^+) &= A_d x(ih) + H_d v_d(i) \\ &\quad + G_d \eta_d(i),\ i = 0,1,\ldots \\ z(t) &= Cx(t) \\ z_d(i) &= C_d x(ih) \end{aligned} \quad (2)$$

where $x \in \mathbb{R}^n$ denotes the state vector, $h > 0$ is the sampling period, v and v_d are continuous-time and discrete-time bounded in power inputs, respectively. The random variables $\xi(t) \in \mathbb{R}, t \ge 0$ and $\eta(t) \in \mathbb{R}^r, t \ge 0$ are such that the pair $(\xi(t), \eta(t))$ is an $r+1$-dimensional standard Wiener process and $\eta_d(i) \in \mathbb{R}^{r_d}$, $i = 0,1,\ldots$ is a sequence of independent random vectors on a probability space $(\Omega, \mathcal{P}, \mathfrak{F})$. It is assumed that $\xi(t), \eta(t), t \ge 0$ and $\eta_d(i), i = 0,1,\ldots$ are independent stochastic processes with zero mean and unitary second moments. The outputs z and z_d are the continuous-time and the discrete-time controlled outputs, respectively. By virtue of standard results from theory of stochastic differential equations (see *e.g.* [6]), the system (2) has a unique $\mathfrak{F}_t$-adapted solution for any initial condition $x(0)$, $\mathfrak{F}_t$ denoting the σ-algebra generated by the random vectors $\xi(s), \eta(s)$ and $\eta_d(i)$, $0 \le s \le t, 0 \le ih \le t$. This solution is almost surely left continuous.

Definition 1. *The system* (2) *with jumps and with multiplicative white noise is exponentially stable in mean square (ESMS) if for* $v(t) = 0, \eta(t) = 0, t \ge 0$ *and* $v_d(i) = 0$, $\eta_d(i) = 0, i = 0,1,\ldots$ *there exist* $\alpha > 0$ *and* $\beta > 0$ *such that* $E\left[|x(t)|^2\right] \le \beta e^{-\alpha t} |x(0)|$ *for any initial condition* $x(0) \in \mathbb{R}^n$ *for all* $t \ge 0$, *where* $E[\cdot]$ *denotes the mean of the random variable* $[\cdot]$ *and* $|q|^2 := q^T q$.

The next result is well-known in the literature as the Itô's formula and it plays a key role in the proof of the results presented in the following sections.

Proposition 1 ([6]). *Let* $v(t,x)$ *be a continuous function in* $(t,x) \in [0,T] \times \mathbb{R}^n$. *If* $x(t)$ *is a solution of the stochastic differential equation* $dx(t) = a(t)dt + \sigma(t)dw(t)$, *then*

$$\begin{aligned} dv(t,x(t)) &= \left[\frac{\partial v}{\partial t}(t,x(t)) + \left(\frac{\partial v}{\partial x}(t,x(t))\right)^T a(t)\right. \\ &\quad \left. + \frac{1}{2} Tr\, \sigma^T(t) \frac{\partial^2 v}{\partial x \partial x}(t,x(t)) \sigma(t)\right] dt \\ &\quad + \left(\frac{\partial v}{\partial x}(t,x(t))\right)^T \sigma(t) dw(t) \end{aligned}$$

where Tr *denotes the trace of the matrix.* □

Necessary and sufficient conditions under which the system (2) is ESMS are derived in [4] using some appropriate Lyapunov operators.

Throughout the paper $L^2[0,\infty)$ denotes the space of measurable functions $f(t), t \ge 0$ with

$$\| f \|_{L^2}^2 := E\left[\int_0^\infty |f(t)|^2 dt\right] < \infty$$

and ℓ^2 denotes the space of the random vectors $g(i), i = 0,1,\ldots$ with the property

$$\|g\|_{\ell^2}^2 := \sum_{i=0}^\infty E\left[|g(i)|^2\right] < \infty .$$

If the additive white noise components in (2) are missing, namely if $G = 0$ and $G_d = 0$, one obtains a system which will be denoted by T with the inputs v and v_d. If the system (2) is

ESMS then for all $v \in L^2[0,\infty)$ and $v_d \in \ell^2$ its solution $x(t), t \geq 0$ has the property that $x(t) \in L^2[0,\infty)$ (see *e.g.* [13]), $z(t) \in L^2[0,\infty)$ and $z_d(i) \in \ell^2$. Then one can consider the input-output operator $\mathfrak{T}$ associated with the system T defined as $\mathfrak{T} : L^2[0,\infty) \times \ell^2 \to L^2[0,\infty) \times \ell^2$, $(v, v_d) \mapsto (z, z_d)$. The norm $\|\mathfrak{T}\|$ of this input-output operator will be analyzed in the next section.

3 The H_2/H_∞ Performance for ESMS Systems with Jumps and Multiplicative White Noise

As it is already known from the deterministic framework, the computation of the mixed H_2 and H_∞ performance ([1], [2]) implies to solve the following optimization problem:

$$J_0 = \sup_{(v,v_d)} \left(\|z\|_{L^2}^2 + \|z_d\|_{\ell^2}^2 - \gamma^2 \left(\|v\|_{L^2}^2 + \|v_d\|_{\ell^2}^2 \right) \right) \tag{3}$$

where $(v, v_d) \in L^2[0,\infty) \times \ell^2$ and $\gamma > \|\mathfrak{T}\|$, $\mathfrak{T}$ denoting the input-output operator defined above. Note that if v and v_d are ignored, J_0 corresponds to the norm induced from (η, η_d) to (z, z_d) which represents the H_2-type norm of the system (2). Similarly, if η and η_d are ignored then J_0 corresponds to the H_∞ performance of (2).

The main result of this section is given by the following theorem:

Theorem 1. *The optimum in* (3) *is given by:*

$$J_0 = Tr\left(G_d^T X(ih^-) G_d\right) + \frac{1}{h}\int_0^h Tr\left(G^T X(t) G\right) dt \tag{4}$$

where $X(t)$ *denotes the stabilizing solution of the system of coupled Riccati-type equations:*

$$\begin{aligned}
-\dot{X}(t) &= A_0^T X(t) + X(t) A_0 + A_1^T X(t) A_1 \\
&\quad + \gamma^{-2} X(t) HH^T X(t) + C^T C, \quad t \neq ih \\
X(ih^-) &= A_d^T X(ih) A_d + A_d^T X(ih) H_d \\
&\quad \times \left(\gamma^2 I - H_d^T X(ih) H_d\right)^{-1} H_d^T X(ih) A_d \\
&\quad + C_d^T C_d, \quad i = 0,1,\ldots
\end{aligned} \tag{5}$$

Remark 1. (i) *By definition, a solution* $X(t), t \geq 0$ *of* (5) *is called stabilizing if it is symmetric and it satisfies the following conditions:*

(a) $\gamma^2 I - H_d^T X(ih) H_d > 0, \quad i = 0,1,\ldots$

(b) *The linear system with jumps and with multiplicative white noise:*

$$\begin{aligned}
dx(t) &= \mathcal{A}_0 x(t) dt + A_1 x(t) d\xi(t), \ t \neq ih \\
x(ih^+) &= \mathcal{A}_d x(ih), \ i = 0,1,\ldots
\end{aligned} \tag{6}$$

is ESMS, where

$$\mathcal{A}_0 := A_0 + \gamma^{-2} HH^T X(t)$$

$$\mathcal{A}_d := A_d + H_d \left(\gamma^2 I - H_d^T X(ih) H_d\right)^{-1} \times H_d^T X(ih) A_d .$$

(ii) *The stabilizing solution of* (5) *is* h*-periodic and right continuous.*

Theorem 1 was proved using the Itô's formula applied for the function $v(x(t)) = x^T(t) X(t) x(t)$ where $x(t)$ is the solution of (2) and $X(t)$ denotes the stabilizing solution of the system of Riccati-type equations (5).

4 Mixed H_2/H_∞ Optimal State-Feedback Control for Stochastic Systems with Jumps and Multiplicative White Noise

Based on the result presented in the previous section one can state the following state-feedback H_2/H_∞ optimization problem: given the system with jumps and with multiplicative white noise

$$dx(t) = (A_0x(t) + Hv(t))dt + A_1x(t)d\xi(t) + G\eta(t),\ t \neq ih$$

$$x(ih^+) = A_dx(ih) + B_du(i) + G_d\eta_d(i),\ i = 0,1,... \quad (7)$$

$$z(t) = Cx(t)$$

$$z_d(i) = u(i),$$

determine the discrete-time control $u(i) = Fx(ih),\quad i = 0,1,...$ such that the closed loop system

$$dx(t) = (A_0x(t) + Hv(t))dt + A_1x(t)d\xi(t) + G\eta(t),\ t \neq ih$$

$$x(ih^+) = (A_d + B_dF)x(ih) + G_d\eta_d(i),\ i = 0,1,... \quad (8)$$

$$z(t) = Cx(t)$$

$$z_d(i) = Fx(ih)$$

is ESMS and the following conditions are fulfilled:

(i) The input-output operator obtained for $\eta(t) \equiv 0$ and $\eta_d(i) \equiv 0$ has the norm less than a given $\gamma > 0$;

(ii) The performance index:

$$J_F = \sup_{v \in L^2[0,\infty)} \left(\|z\|_{L^2}^2 + \|z_d\|_{\ell^2}^2 - \gamma^2 \|v\|_{L^2}^2 \right) \quad (9)$$

is minimized.

Remark 2. *In the first equation of the system* (7) *the continuous-time control* $u(t)$ *is missing. This particular structure of the system* (7) *corresponds to the one obtained when representing a sampled-data system as a system with jumps. Such representation can be obtained for the continuous-time stochastic system corrupted with multiplicative white noise:*

$$dx(t) = (A_0x(t) + Bu(t))dt + A_1x(t)d\xi(t)$$

with the piecewise-constant control

$$u(t) = Fx(ih), t \in (ih, (i+1)h],\ i = 0,1,...$$

by introducing the additional state $\bar{x}$ defined in (1) with $u(i) = Fx(ih),\ i = 0,1,..$ *Thus, using the continuity of* $x(t)$ *at* $ih,\ i = 0,1,...$ *one gets the extended system with jumps and multiplicative white noise:*

$$dx_e(t) = A_{0e}x_e(t)dt + A_{1e}x_e(t)d\xi(t),\ t \neq ih$$
$$x_e(ih^+) = A_{de}x_e(ih) + B_{de}u(i),\ i = 0,1,... \quad (10)$$

with

$$x_e := \begin{bmatrix} x \\ \bar{x} \end{bmatrix},\ A_{0e} := \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, A_{1e} := \begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{de} := \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, B_{de} = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

in which the continuous-time control variable is missing.

The solution of the state-feedback to the mixed H_2/H_∞ optimisation problem formulated above is given by the following result.

Theorem 2. *Assume that the following system of coupled Riccati-type equations*

$$-\dot{X}(t) = A_0^TX(t) + X(t)A_0 + A_1^TX(t)A_1 + \gamma^{-2}X(t)HH^TX(t) + C^TC,\quad t \neq ih$$

$$X(ih^-) = A_d^TX(ih)A_d - A_d^TX(ih)B_d \times (\gamma^2I + B_d^TX(ih)B_d)^{-1} B_d^TX(ih)A_d,\quad i = 0,1,... \quad (11)$$

has a stabilizing solution $X(t)$. *Then the optimal solution of the state-feedback mixed* H_2/H_∞ *problem for the system* (7) *is given by* $u(i) = Fx(ih),\quad i = 0,1,...$ *with*

$$F = -(\gamma^2I + B_d^TX(ih)B_d)^{-1} B_d^TX(ih)A_d \quad .(12)$$

Proof. Note first that F given by (12) is constant since the stabilizing solution $X(t)$ is h-periodic. Further, one can prove that the stabilizing solution has the property

$$X(t) \leq X_{\tilde{F}}(t),\ t \geq 0 \quad (13)$$

where $X_{\tilde{F}}(t)$ denotes the stabilizing solution of the Riccati-type system of form (5) associated with the closed loop system (8) obtained for an arbitrary stabilizing state-feedback gain $\tilde{F}$, namely:

$$
\begin{aligned}
-\dot{X}_{\tilde{F}}(t) &= A_0^T X_{\tilde{F}}(t) + X_{\tilde{F}}(t) A_0 + A_1^T X_{\tilde{F}}(t) A_1 \\
&\quad + \gamma^{-2} X_{\tilde{F}}(t) HH^T X_{\tilde{F}}(t) + C^T C, \quad t \neq ih \\
X_{\tilde{F}}(ih^-) &= \left(A_d + B_d \tilde{F}\right)^T X_{\tilde{F}}(ih) \left(A_d + B_d \tilde{F}\right) \\
&\quad + \tilde{F}^T \tilde{F}, \quad i = 0,1,\ldots
\end{aligned} \tag{14}
$$

The proof of (13) is mainly based on the same arguments used in [2] to prove the similar result for systems without multiplicative white noise, and therefore it is omitted. On the other hand, by direct algebraic computations one can show that the Riccati-type system (11) coincides with (14) for $\tilde{F} = F$ with F given by (12). Then taking into account the expression (4) of the mixed H_2 / H_∞ performance one concludes that F minimizes (9). The fact that the input-output operator obtained for $\eta(t) \equiv 0$ and $\eta_d(i) \equiv 0$ has the norm less than γ is a direct consequence of the Bounded Real Lemma applied for (14) with $\tilde{F} = F$. □

A natural problem arising when determining the optimal state-feedback gain (12) is the computation of the stabilizing solution of (11). The next result provides an iterative procedure allowing to compute this stabilizing solution.

Proposition 1. *Assume that the system of coupled Riccati-type equations* (11) *has a stabilizing solution. Then it can be determined by the following iterative procedure:*

$$
\begin{aligned}
X_{k+1}(ih) &= e^{A_0^T h} X_{k+1}(ih^-) e^{A_0 h} \\
&\quad + \int_0^h e^{A_0^T s} \left(C^T C + \gamma^{-2} X_k(s) HH^T X_k(s)\right) e^{A_0 s} ds \\
&\quad + \int_0^h e^{A_0^T s} A_1^T X_k(s) A_1 e^{A_0 s} ds \\
X_{k+1}(ih^-) &= \left(A_d + B_d F_k\right)^T X_{k+1}(ih) \left(A_d + B_d F_k\right)
\end{aligned} \tag{15}
$$

with

$F_k = -\left(I + B_d^T X_k(ih) B_d\right)^{-1} B_d^T X_k(ih) A_d$,

$k = 0,1,\ldots$ *where* $X_0(t) = 0,\ t \in (0,h)$, F_0 *is a stabilizing gain for the system* (7). □

Remark 3. *In order to apply the iterative procedure given in the statement of Proposition 1 one first substitutes* $X_{k+1}(ih)$ *given by the first equation* (15) *into the second one which is solved with respect with* $X_{k+1}(ih^-)$. *Then* $X_{k+1}(t),\ t \in [0,h)$ *is obtained as the solution of the differential equation in* (11) *with the final condition* $X_{k+1}(ih^-)$.

5 A Digital Automatic Landing System Design

In this section one illustrates the previous theoretical developments by a case study in which the design problem of a digital automatic landing system is considered. It is a known fact that landing is the most demanding among all aircraft flight phases. The difficulties are determined by the fact that during landing the aircraft flies at a low speed and low altitude at which accidents are more likely to occur. In the same time important uncertain factors as atmospheric turbulence can severely influence the motion of the aircraft. The airplane landing can be accomplished by coupling an automatic landing system able to track the reference signals corresponding to the glide path. The aim of this case study is to design using the results derived in the previous sections, a digital automatic landing system providing together a zero order hold element a piecewise-constant control such that the aircraft tracks the desired trajectory. A linear continuous-time model of a modern aircraft with independent control surfaces has been used [14]. The state vector includes ten states and the control vector has five components. The state variables are assumed to be obtained either by direct or by indirect measurement. The aircraft dynamics has been augmented with integral and derivative components of the tracking errors e_β, e_h and e_{lat} representing the sideslip angle errors, altitude error and lateral error, respectively (for details, see [14]). One thus obtains an augmented system with the state vector $x_a \in \mathbb{R}^{15}$ and with six exogenous inputs (three for the reference signals and three for the atmospheric turbulence). A digital state-feedback controller has been then determined for this augmented system. Using the idea presented in Remark 2, one firstly determined a system with jumps for which the optimal solution to the mixed H_2/H_∞ problem was obtained by Theorem 2 and Proposition 1. Some of the simulation results under vertical wind shear and lateral turbulence obtained for the sampling period $h = 0.05\,\text{sec}$ are presented in Fig. 2-5.

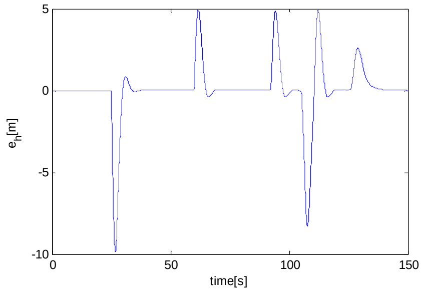

Fig. 2. Time response of altitude tracking error

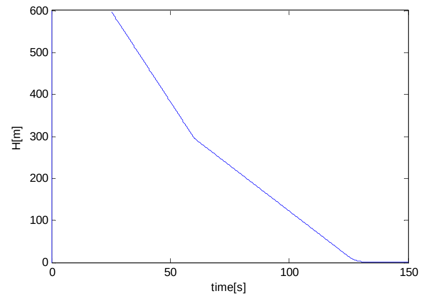

Fig. 3. Time response of altitude

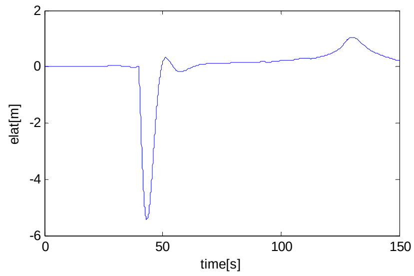

Fig. 4. Time response of lateral tracking error

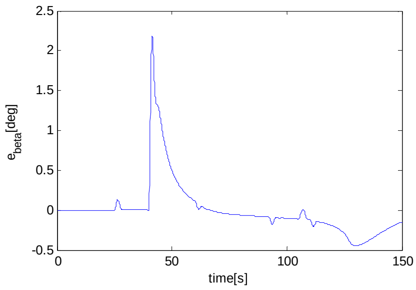

Fig. 5. Time response of the sideslip angle error

The above responses indicate a very good behavior of the aircraft on the glide path under strong vertical and lateral winds.

Then the influence of the sampling period h over the stability of the closed loop system has been analyzed. In Table 1 some comparative results are presented. They have been obtained with the state-feedback gains F computed for different sampling periods h using the results described in Section 4 and with the state-feedback gain F_c determined in [14] by a continuous-time design method.

Table1: stability of the closed loop system with respect to the sampling period

h[sec]	0.05	0.01	0.3	0.5	1
F_c	stable	stable	stable	unstable	unstable
F	stable	stable	stable	stable	stable

One can see that even for large values of the sampling period, the design procedure based on the representation of sampled-data systems by systems with finite jumps provides stabilizing solutions.

6 Conclusions

A mixed H_2/H_∞ state-feedback control problem for a class of systems with jumps and with multiplicative white noise has been analyzed. It was shown that the optimal solution of this problem depends on the stabilizing solution of a specific system of coupled Riccati-type equations that can be solved using an iterative procedure. To demonstrate the proposed approach a state-feedback automatic landing controller was designed and analyzed with respect to the stability and tracking performances.

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