THERMAL DESIGN UNDER UNCERTAINTIES OF AIRCRAFT STRUCTURES

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Abstract

Thermal problems related with heat transfer mechanisms appearing in aeronautic engineering take place in an environment where many parameters required for the mathematical formulation of the problem have not precise numerical values and some degree of uncertainties or inaccuracies exists.

In that regard, deterministic analysis may be unable to provide the amount of information required by the designer and thus other approaches taking into account uncertainties will be necessary. That circumstance also requires for the designer to be aware about the variations of the performance in their prototypes, considering these random variables. And, usually, an aim of robustness or, in other words, the idea of choosing designs that reduce the uncertainty of the performance is preferred.

In this paper, a revision of some existing procedures, their capabilities and their applicability to aeronautical problems will be carried out along with some practical examples showing the performance of methods used in the study.

1 Introduction

Two methodologies of design under uncertainties applied to heat transfer problems were considered in this study: The uncertainty quantification approach [1] and the robust design method, by Taguchi [2] and [3].

The following sections describe the work carried out using both procedures, along with the practical examples, to demostrate the capabilities of the approach.

2 Uncertainty quantification

Uncertainty quantification involves the propagation of probabilistic information from the input parameters to the response functions, based on a set of simulations which take into account the random information of the uncertain inputs. The flowchart of this approach is shown in Fig. 1.

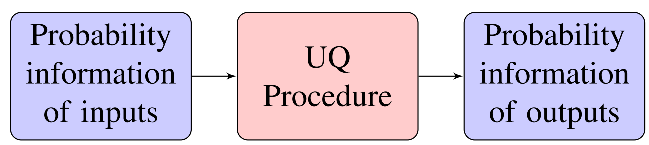

Fig. 1 Flowchart of uncertainty quantification

This method gives the designer some ideas about the significance of randomness in the response of the problem with regards to the input.

2.1 Description of the application example

A panel of 1 m^2 area and 6 mm thickness will be used to show the methodology of uncertainty quantification (Fig. 2). The following heat transfer phenomena are included in the analysis [4]:

- Solar flux.
- Conduction.
- Free convection to ambient.
- Radiation to ambient, radiation to ground and radiation to sky.

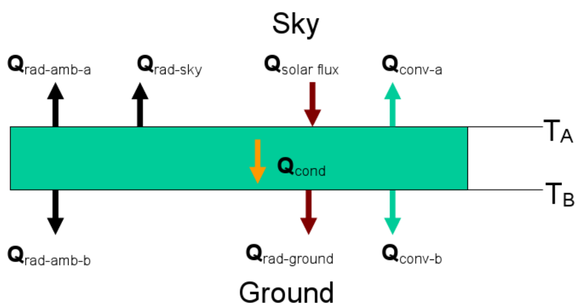

Fig. 2 Concept of panel and heat transfer mechanisms involved

2.2 Definition of random parameters

Up to fourteen random parameters were considered in this study. The list of those parameters grouped by concept is:

- Geometry: Panel thickness.
- Loads: Solar flux and its absorptivity.
- Conduction: Thermal conductivity.
- Free convection to ambient: Convection heat transfer coefficient.
- Radiation to ambient: View factor, surface emissivity and ambient temperature.
- Radiation to ground: View factor, surface emissivity and ground temperature.
- Radiation to sky: View factor, surface emissivity and sky temperature.

All of them were asumed to have normal distributions. The selected mean value μ was the most common value found in technical literature. For the standard deviation σ two different values were considered:

- $\sigma = 30$ % of the mean value for parameters having wide range of values.
- $\sigma = 5$ % of the mean value for parameters having narrow range of values.

The uncertainty is the coefficient of variation of the random parameters:

$$\delta = \frac{\sigma}{\mu} \tag{1}$$

The numerical values of mean and uncertainty of random input parameters are shown in Table 1. The probability density function (PDF) of one of them, the ambient temperature, is shown in Fig. 3. The output value of the UQ approach is the temperature value at the point located at the center of the panel.

2.3 Single random parameter uncertainty

Fourteen UQ studies considering separately each one of the random input parameters were carried out. In each of them, a hundred random processes were produced by using Monte Carlo simulations (MCS) [5].

Fig. 4 shows the temperature values obtained when uncertainty exists in the ambient temperature. A similar representation can be defined for the rest of simulations.

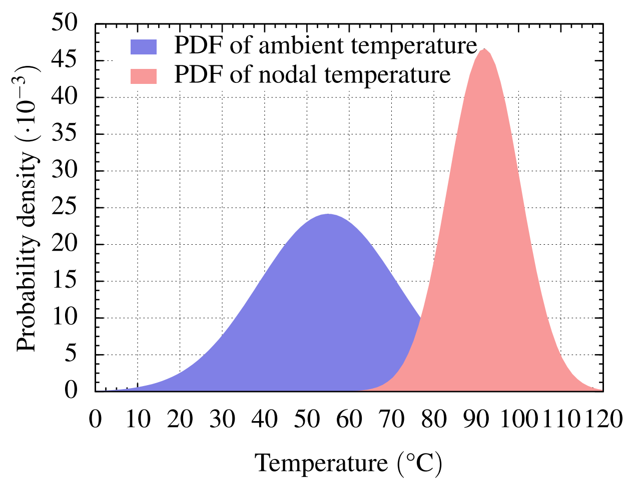

Fig. 3 PDF of ambient temperature (input) and nodal temperature (output)

The mean, standard deviation and the coefficient of variation, i.e., the uncertainty of the output, are calculated from those results. Fig. 3 show the PDF of the output with ambient temperature uncertainty and the Table 1 lists the values of δ for all the parameters sorted by their uncertainty level from higher to lower values.

It can be concluded that for all random parameters, the degree of randomness decreases from input to output. In other words, in this heat transfer problem, the level of uncertainty is at-

Parameter	Mean μ	Uncertainty δ (%) Input	Output
Solar flux $\left(\frac{W}{m^2}\right)$	1350	30	10.5
Ambient temperature (°C)	55	30	9.3
Ground temperature (°C)	83	30	6.3
Convection heat transfer coefficient $\left(\frac{W}{m^2 °C}\right)$	7	30	5.7
Solar flux absorptivity	0.8	5	1.8
Sky temperature (°C)	27	30	1.1
Thermal conductivity $\left(\frac{W}{m °C}\right)$	0.9	30	0.6
Radiation view factor in radiation to sky	0.8	5	0.6
Surface emissivity in radiation to sky	0.8	5	0.6
Panel thickness (m)	0.006	30	0.4
Surface emissivity in radiation to ambient	0.8	5	0.2
Radiation view factor in radiation to ambient	0.2	5	0.2
Radiation view factor in radiation to ground	0.8	5	0.1
Surface emissivity in radiation to ground	0.8	5	0.1

Table 1 Uncertainty values in input and output

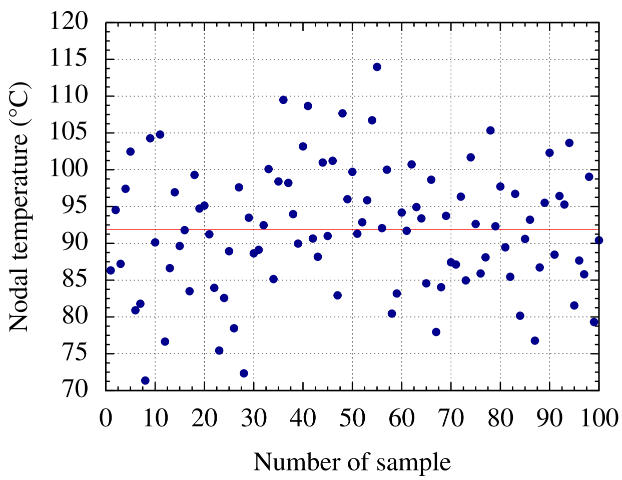

Fig. 4 MCS of temperature at panel center with ambient temperature uncertainty

tenuated due to the nature of the problem. It can be observed that solar flux is the parameter for which randomness is maintained to the largest degree and radiation to ground is the least significant random parameter.

2.4 Multiple random parameters uncertainty study

After the uncertainty studies using only a single random parameter, another study was carried out considering several random parameters aiming to find out how the agrupation of uncertainties affects the response. In order to make the process more consistent, the methodology used is characterized as follows:

1. The set of random parameters were distributed among five concepts: geometry, thermal load, conduction, convection and radiation. Statistic properties of each random parameter were the same as used in the single random parameter study.

2. Those concepts were grouped to form seven stochastic processes with increasing number of random parameters starting from two and finishing with fourteen. The concepts for each process are listed in Table 2

Up to a hundred MCS were carried out for each multiple random parameter process, and from those results, the statistic properties of nodal temperature at panel center were characterized.

Table 2 shows the number of parameters included in the random processes and their levels of uncertainty. It can be concluded that the level of uncertainty propagation from input data to output temperature increases as it does the number of random parameters. However, the degree of uncertainty in the panel center temperature is lower than the uncertainty of random parameters. This is the same kind of behaviour observed in the single random parameters study described previously.

3 Robust Design

Robustness is the aim of obtaining a design that is, as less as possible, insensitive to variation in the parameters. In robust analysis, given a target value for a design performance, the preferred candidate is not the one closest to the target value, but the one least sensitive to parameter variations.

Robust analysis improves the capacity of UQ technique because it incorporates a function representing robustness, which is optimized in the procedure. The flowchart of this methodology is shown in Fig. 5.

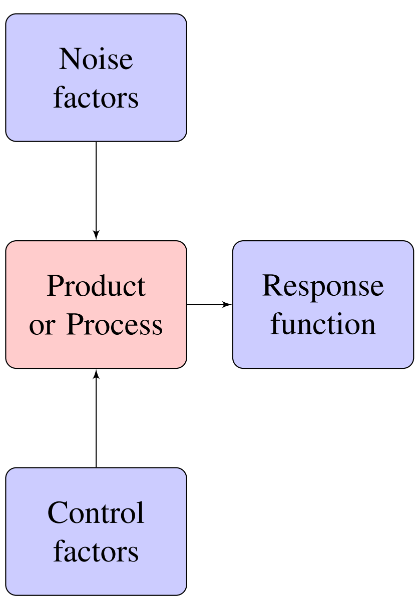

Fig. 5 Flowchart of robust design

Genichi Taguchi, the pioneer of robust design, said [6]: "... *robustness is the state where a technology, product or process performance is minimally sensitive to factors causing variability"*. Robustness has two goals: on-target behavior and low variation.

In Taguchi method, robustness of a design is determined by the value of a function defining the quality loss of the design. The best design is the one having the lowest quality loss. Taguchi method formulates the robust design problem in an uncertain environment as follows:

- A set of noise factors: Parameters related to the problem that have a range of values not controlled by the designer.
- A set of control factors: Parameters related to the problem that have a range of values that can be selected by the designer.
- A response of the problem, with a preferred value choosen by the designer, that is the target of the problem. Any difference between such value and the performance of a given design is considered to be decreasing in its adecuacy. In that regard, the so-called quality loss function is defined to measure such circumstances.

The strategy of Taguchi method for finding a robust design is to identify the proper values of the control factors that minimize the quality loss function, while taking into account the variability of the noise factors. In summary, the steps of Taguchi method are:

1. Define the vector of noise factors and their levels.
2. Define the vector of control factors and their levels.
3. Define the orthogonal arrays, which are related to the number of experiments to perform.
4. Carry out thermal analyses.
5. Define the quality loss function.

Concept	Random parameters	Uncertainty of output δ
Geometry, load, conduction, convection and radiation parameters	14	0.169
Geometry, load and radiation parameters	11	0.155
Conduction, convection and radiation parameters	11	0.130
Load, geometry, conduction and convection	5	0.124
Radiation parameters	9	0.111
Geometry and load	3	0.104
Conduction and convection	2	0.058

Table 2 Multiple random process results

6. Observe the effect of each control factor in the quality loss function by using analysis of mean method (ANOM).

7. Identify the best level of each control factor, and therefore the most robust design.

Three different classes of problems can be solved by Taguchi methods:

- Problems having a particular value for the response. They are called nominal the best type problems (NTB).

- Problems aiming for the lowest possible value. They are called smaller the better type problems (STB).

- Problems aiming for the greatest possible value. They are called larger the better type problems (LTB).

3.1 Application of Taguchi method to thermal problems

Thermal robust analysis can be carried out using Taguchi method formulation. The parameters included in the problem can be interpreted as noise factor or control factors. For instance:

- Set of possible noise factors: Solar flux, ambient temperature, sky temperature,...

- Set of possible control factors: Thickness of structure, material conductivity,...

The type of response appropriate for each kind of problem is:

- Nominal the best type: Temperature value at some selected points.

- Smaller the better type: Lowest temperature value at a point in a domain.

- Larger the better type: Maximum temperature change between two domains.

3.2 Description of the application example

A box having two cavities subjected to several heat transfer phenomena will be used as an application example. Figs. 6 and 7 describe the geometry of the box and the heat transfer phenomena taken into account in the problem. In this example, nodal temperature at central points of each cavity were chosen to define the design quality level (Fig. 7).

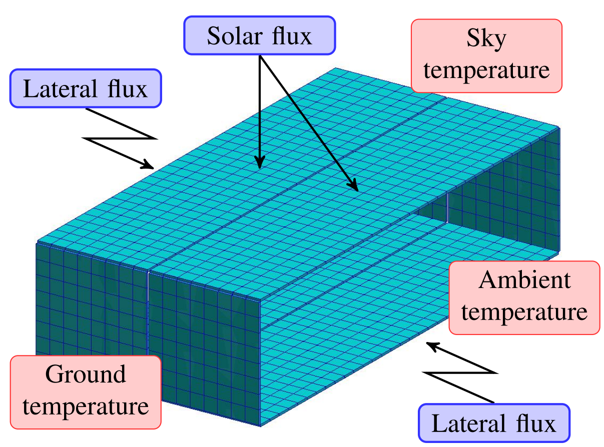

Fig. 6 Box with two cavities subjected to solar flux, radiation, convection and conduction.

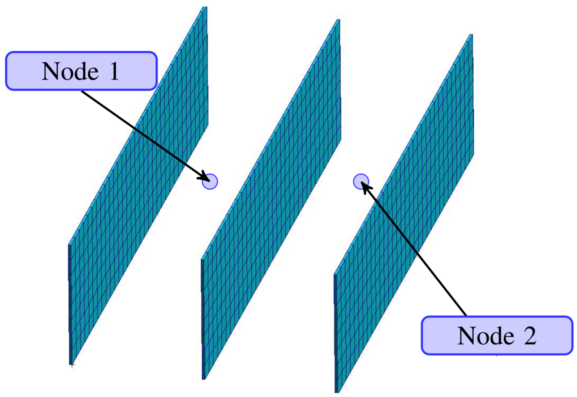

Fig. 7 Nodal points selected for temperature output

Seven noise factors were included in the study, having a mean value μ and percentage of variation ν. From these two statistical parameters, the value of standard deviation σ can be evaluated as

$$\sigma = \frac{\mu\nu}{3} \quad (2)$$

For each noise factor, two different values, namely $\mu - \sigma$ and $\mu + \sigma$, were considered in the study. According to the terminology used in Taguchi method, these two values will be referred to as level 1 and level 2. The complete information for the definition of noise factors appears in Table 3.

In the case of control factors, up to four different characteristics of the thermal problem were chosen. In each of them, up to three different levels were selected, which are shown in Table 4.

After the noise and control factors were defined, the next step is to define the set of orthogonal arrays. The idea is to predict a set of analysis unbiasing the significance of any noise factors and, therefore, to assign the same number of events to all of them. Table 5 shows a matrix containing the levels selected for each noise factors. Each one of the eight combinations is called experiment in Taguchi's terminology (EXP). In the same way, an orthogonal array needs to be defined for control factors. Table 6 describes the levels selected for the nine experiments.

We should keep in mind that, for each experiment included in the array of control factors, the eight cases considered in the orthogonal arrays

EXP	NF1	NF2	NF3	NF4	NF5	NF6	NF7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Table 5 Orthogonal array of noise factors

EXP	CF1	CF2	CF3	CF4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

Table 6 Orthogonal array of control factors

of noise factors need to be carried out, which results in up to seventy two solutions. Each of them corresponds to different cases of the thermal problem, therefore up to seventy two thermal analyses with the numerical values of the levels of noise factors and control factors used in the study were performed. For each of them, the nodal temperature fields at each point were obtained and amongst them, temperature values of central points of both cavities were used in the study. Figs. 8 and 9 show an example of numerical results obtained for each case.

3.3 Problem formulation: Nominal the best

In this variant of Taguchi method, temperature values T_1 and T_2 at nodes 1 and 2 located in the center of the cavities will be chosen as targets of the design. Problem will be formulated aiming to minimize variations of T_1 or T_2 due to noise factors. As the variation is an indication of quality

Noise factors	Mean μ	Variation ν (%)	σ	Level 1	Level 2
Ambient Temperature (°C)	48	10	1.6	46.4	49.6
Ground Temperature (°C)	75	10	2.5	72.5	77.5
Sky Temperature (°C)	24	10	0.8	23.2	24.8
Heat transfer coefficient $\left(\frac{W}{m^2 °C}\right)$	12	10	0.4	11.6	12.4
Thermal conductivity $\left(\frac{W}{m °C}\right)$	150	20	10	140	160
Solar flux $\left(\frac{W}{m^2}\right)$	1350	5	22.5	1327.5	1372.5
Lateral flux $\left(\frac{W}{m^2}\right)$	500	15	25	475	525

Table 3 Noise factors formulation

Control factors	Level 1	Level 2	Level 3
Solar absorptivity of top panels	0.2	0.5	0.8
Emissivity of left box in radiation to ambient	0.1	0.5	0.9
Emissivity of right box in radiation to ambient	0.1	0.5	0.9
Emissivity of two boxes in radiation in cavities	0.1	0.5	0.9

Table 4 Control factors formulation

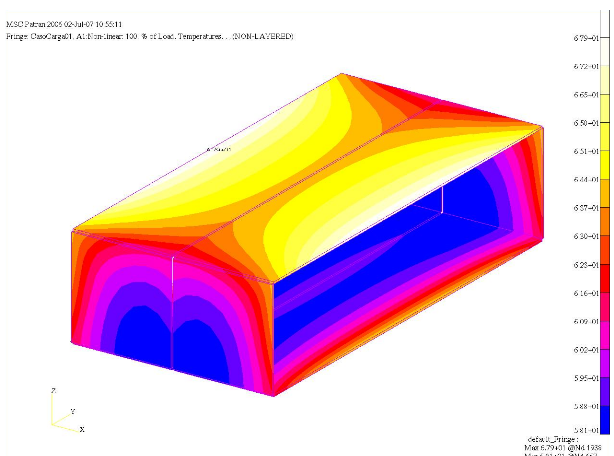

Fig. 8 Temperature field in external walls

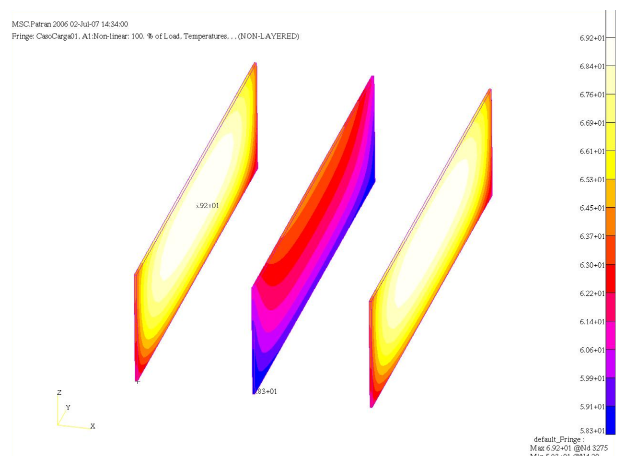

Fig. 9 Temperature field in internal walls

loss, the problem represents the lower the difference, the better the behaviour. Therefore, this formulation corresponds to the definition of nominal the best type. According to this approach, signal to noise (S/N) function is defined as follows:

$$\eta = 10 \log_{10}\left(\frac{\mu_t^2}{\sigma_t^2}\right) \quad (3)$$

In the expression above, μ_t is the mean value and σ_t is the standard deviation of temperature at any node in the thermal experiments associated to orthogonal array of control factors. Obviously, in this example, only temperature at nodes 1 and 2, or in other words T_1, and T_2 will be of interest. It can be observed that function η includes the standard deviation in the denominator, which means that small values of σ_t are equivalent to similar values in the temperature field. The lower the value of σ_t, implies the greater the value of η, which means, the quality of the design. Therefore, minimization of quality losses can be interpreted as maximization of such function η.

After carrying out each set of eight thermal analyses for each control factors experiment, the values of η are shown in Table 7.

EXP	Control factor				Nominal the best Loss of quality	
	1	2	3	4	Node 1	Node 2
1	1	1	1	1	30.99	30.99
2	1	2	2	2	31.71	31.71
3	1	3	3	3	32.04	32.04
4	2	1	2	3	32.00	32.23
5	2	2	3	1	32.63	32.74
6	2	3	1	2	32.56	32.21
7	3	1	3	2	32.71	33.15
8	3	2	1	3	32.75	32.46
9	3	3	2	1	33.38	33.24
			Mean		32.31	32.31

Table 7 Values of η at nodes 1 and 2

Then, the analysis of mean step is carried out. In such step, the mean value of η for the three experiments corresponding to the same level of each control factor is evaluated. For instance, control factor 2 has level 3 in experiments 3, 6 and 9 and then:

$$\eta_{23} = \frac{32.04 + 32.56 + 33.38}{3} = 32.66 \qquad (4)$$

In this formula, sub-index 2 and 3 represents the control factor and the control factor level, respectively. Table 8 shows the complete set of values for nodes 1 and 2.

Numerical results of ANOM can be shown graphically. For example, those values corresponding to node 1 appears in Fig. 10.

It is already known that maximizing η, is equivalent to decreasing quality losses. Therefore Fig. 10 indicates clearly which level of each control factor produces the best values of η and thus the set of selected levels can be easily chosen. The only indeterminacy occurs in control factor 4, when levels 1 and 2 produce the same η result of 32.33. In that circumstance, any of them can be selected.

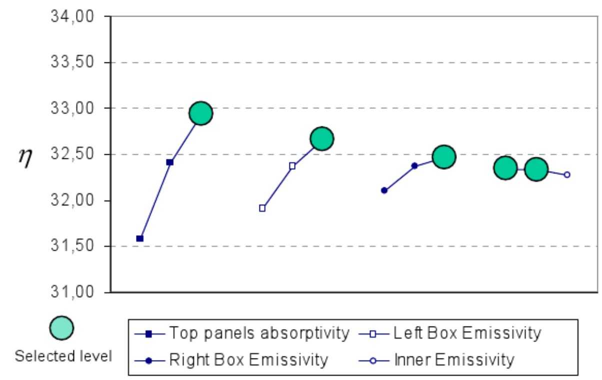

Fig. 10 Results of ANOM in node 1 and selected control factor levels

It can be observed that the set of control factor levels corresponding to robust design does not exist in the orthogonal array of Table 7. This circumstance is one of the powerful characteristics of Taguchi method, as it can identify combinations of control factors with better performance than those included in the set of experiments carried out.

The last step in robust design is to confirm that the set of control factor levels selected behaves better than the set of experiments carried out. This phase is called usually experiment verification. This is done by comparing the mean value and the standard deviation of the seventy two experiments defined by the orthogonal arrays of control and noise factors, named starting condition, and the mean value and the standard deviation corresponding to the eight experiments carried out considering the selected set of control factors at robust design and the orthogonal array of noise factors. Table 9 shows the numerical values obtained at node 1. It can be concluded that not only function η has a larger value, meaning lower quality losses, but also the variance decreases showing an improvement of 26.15 %.

3.4 Problem formulation: Smaller the better

Another suitable formulation of the thermal robust analysis is trying to have the lowest possible temperature value at nodes 1 and 2 and, of course, having the smallest variation as possible. This corresponds to the smaller the better type of

Nominal the best node 1				Nominal the best node 2			
Factor	Level			Factor	Level		
	1	2	3		1	2	3
1	31.58	32.40	32.94	1	31.58	32.40	32.95
2	31.90	32.36	32.66	2	32.12	32.30	32.49
3	32.10	32.36	32.46	3	31.88	32.39	32.64
4	32.33	32.33	32.26	4	32.32	32.36	32.24

Table 8 Results of ANOM

	Starting Condition	Robust Design	Improvement (%)
η	32.31	33.46	
Variance	2.75	2.18	26.15

Table 9 Nominal the best. Numerical values of experiment verification

formulation of robust design. In this formulation, the signal to noise (S/N) function η describing quality losses is to be maximized and defined as:

$$\eta = -10\log_{10}\left(\frac{1}{n}\sum_{i=1}^{n} t_i^2\right) \quad (5)$$

Where t_i is the temperature value at any point and the parameter n represents the number of experiments in the orthogonal array of noise factors. In our example, n = 8. As η is a negative number, maximizing this expression can only be done by decreasing temperature values at the nodes. The set of control and noise factors in this problem are the same used in the previous nominal the best type problems. Therefore orthogonal arrays in this problem formulation are those appeared in Tables 5 and 6. After calculation of function η for each experiment, the numerical values obtained are shown in Table 10. Carrying out the analysis of mean technique, the results obtained are shown in Table 11.

Graphical results of ANOM for node 1 appears in Fig. 11 and also the selected values of control factors that maximize function η and, therefore, minimize quality losses.

Finally, the numerical values obtained for node 1 in the experiment verification are shown in Table 12. It can be observed that the robust design produces diminution of temperature by about 16 %, while at the same time function η is increased. Such behaviour is equivalent to decreasing quality losses, which is the aim of robust design.

EXP	Control factor				Smaller the better Loss of quality	
	1	2	3	4	Node 1	Node 2
1	1	1	1	1	-36.21	-36.21
2	1	2	2	2	-35.73	-35.73
3	1	3	3	3	-35.42	-35.42
4	2	1	2	3	-37.08	-36.84
5	2	2	3	1	-36.48	-36.29
6	2	3	1	2	-36.47	-36.91
7	3	1	3	2	-37.78	-37.23
8	3	2	1	3	-37.69	-38.00
9	3	3	2	1	-37.00	-37.23
				Mean	-36.65	-36.65

Table 10 Values of η at nodes 1 and 2

	Starting Condition	Robust Design	Improvement (%)
η	-36.65	-35.41	
Variance	68.26	58.93	15.83

Table 12 Smaller the better. Numerical values of experiment verification

Smaller the better node 1				Smaller the better node 2			
Factor	Level			Factor	Level		
	1	2	3		1	2	3
1	-35.79	-36.68	-37.49	1	-35.79	-36.68	-37.49
2	-37.02	-36.63	-36.30	2	-36.76	-36.67	-36.52
3	-36.79	-36.60	-36.56	3	-37.04	-36.60	-36.31
4	-36.56	-36.66	-36.73	4	-36.58	-36.62	-36.75

Table 11 Results of ANOM

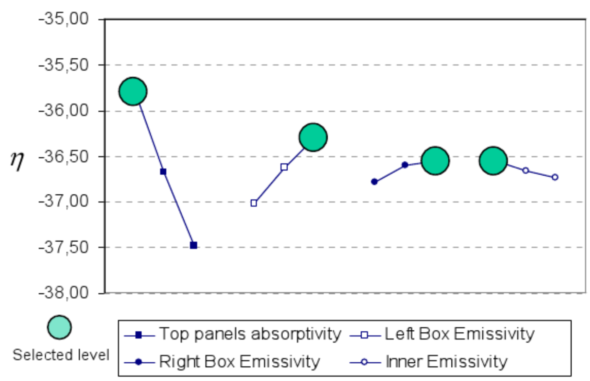

Fig. 11 Results of ANOM in node 1 and selected control factors levels

4 Conclusions

1. Fourteen stochastic processes with single random variables have been carried out. In all of them, the level of uncertainty decreases for the whole set of paramenters selected.

2. It must be remembered that the amount of standard deviation chosen could not be realistic and more appropiate values should be preferred. Nevertheless, the methodology carried out is completely general.

3. The set of random variables have been distributed among five concepts: geometry, thermal load, conduction, convection and radiation.

4. Seven stochastic processes with increasing number of random variables starting form two random variables and finishing with fourteen random variables have been carried out.

5. The level of uncertainty propagation from input data to output temperature values in the thermal problem increases as it does the number of random variables. Some tables are enclosed to show the relative importance of each stochastic process.

6. Taguchi method is suitable to obtain robust designs in thermal problems.

7. The approach proceeds by carrying out a discrete search in the range of variation of noise factors and control factors.

8. Several type of problems can be defined to be solved: nominal the best, smaller the better and larger the better.

9. Two examples have been solved to demonstrate the methodology using a selected set of noise and control factors. Other diferent sets can be defined as the method is completely general.

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