PROBABILISTIC MODELING OF STRUCTURAL / AEROELASTIC LIFE CYCLE FOR RELIABILITY EVALUATION OF DAMAGE TOLERANT COMPOSITE STRUCTURES

Andrey V. Styuart, Kuen Y. Lin, and Eli Livne
Department of Aeronautics and Astronautics, Campus Box 352400
University of Washington, Seattle, WA, 98195-2400

Keywords: *Damage Tolerance, Probabilistic Analysis, Aeroelasticity*

Abstract

Due to the large uncertainties related to material properties and damage of composites, current design practices require extra safety factors to cover such uncertainties, resulting in conservative designs and service guidelines for composite structures. In addition, current fatigue crack-growth-based structural maintenance methodology does not apply to composite aircraft.

This paper presents a probabilistic model for estimating structural and aeroelastic reliability of damage-tolerant composite aircraft structures that enables damage tolerant design, damage tolerance and reliability substantiation, and optimal maintenance planning based on reliability. This development is motivated by the increasing use of composite materials in civil aircraft structures.

The method is based on probabilistic structural lifecycle simulation with the consideration of all related parameters, such as material properties with their dispersions, material properties versus environmental variables, damage statistics (type, location, and size of damage), residual structural properties versus damage type and size, possible damage growth at high stress levels, probability of damage detection (POD), quality of repair, inspection intervals, etc.

The new and significant aspects of the work is that the reliability of a damage-tolerant composite structure can be assessed on a quantitative basis, covering structural as well as aeroelastic failure modes, thus allowing aircraft manufacturers, operators and flight certification authorities to establish the maintenance service guidelines. Engineers will be able to use this methodology in the future as a guide, to establish design and inspection criteria while considering structural risk and maintenance cost at the same time.

1. Introduction

The structural efficiency of aircraft structures is believed to be improved through the use of polymer matrix composites (PMC) based on thermoset/thermoplastic matrices continuously reinforced with carbon, glass, or some other fibers. These materials possess high specific strength/stiffness and high resistance to crack initiation and propagation under cyclic loads, but are rather sensitive to impact damages. These present challenges to designers, production engineers, inspection and maintenance personnel and regulators, because some principles that have been used for design and maintenance of metal structures are increasingly recognized as being unsuited to composite structures.

The current quasi-deterministic approach to providing damage-tolerant structures is unable to quantitatively address the unique ways in which composites respond to damages. It is the purpose of this paper to present a probabilistic methodology for evaluation of damage tolerance based on structural reliability criteria.

1.1 Damage Tolerant Design

Fatigue cracks in metals are initiated in places with high stress and then grow relatively slowly. Even when the uncertainty of the cracks behavior is quite big, sooner or later they are detected in fatigue tests and inspectors will know when and where to inspect. They will find how soon the crack will grow up to become dangerous. With metals, we can think deterministically, establishing inspection intervals which are short enough to enable detection of the developing crack before it reaches a critical size. This general damage-related approach – the "damage-tolerant design" - is well-known and assumes that if we can reliably observe damage growth, this damage can be tolerated until it becomes dangerous for operations. The designer should provide for the slow growth of the damage while maintenance should provide for damage detection and repair.

In application to composites, regulatory requirements for airworthiness are inherited from the design practice for metal structures. Unfortunately, the situation with damages of composite structures is quite different from metals and an application of such principles may not result in an efficient structure. Damage in composites doesn't grow slowly from invisible to critical size. Its size, which may already be greater than critical, is established at the instant of impact, with no intervening time between the damage event and its ultimate effect on residual strength. Further, damage events in composites occur randomly, and we can't always predict when and where damage may occur. Damages accumulate due to accidental impacts by birds, hail, gravel from the runway, sand, ground vehicle collision, and also due to aging. And such small damages, each independently causing only a small reduction in strength or stiffness, can exist undetected for a long time. Their cumulative effect can therefore be greater than that for larger, detectable damages.

1.2 Risk Analysis and Safety Factors

In the presence of such uncertainties, there is no alternative to the application of reliability-based methods. The problem cannot be solved without acknowledging the stochastic character of the factors that affect damage-tolerance in composites: the uncertainty in their mechanical properties, applied loads (including impacts, thermal effects, etc.), repair and environmental conditions.

Risk analysis methods for determining aircraft reliability have been under development for more than 30 years and, because many key engineering parameters are probabilistic in nature, research has focused on the development of probabilistic methods. Despite their advantages over classical deterministic safety factors in the design process, design organizations have been reluctant to adopt even the standard structural reliability methods or to include them as part of their risk analysis capability. Reasons cited include: the complexity of failure modes; lack of available statistical data; and safety issues.

1.3 Probabilistic Reliability Assessment

Much research has been devoted to the development of a probabilistic approach to damage tolerance of metal structures. The methods developed by Shiao et. al. of the FAA [1] and its corresponding software AFGROW [2], Chamis et. al. of NASA [3], and the NESSUS [4] and DARWIN [5] software packages developed by the Southwest Research Institute (SWRI) cover all the essential uncertainties and allow one to assess the probability of failure (POF) for damage-tolerant metal structures. These and other probabilistic methodologies have been proposed and adapted for composite structures, incorporating micromechanics, laminate theory, manufacturing defects, operating environment, as well as impact damage, but few have addressed the importance of inspection intervals, damage-detection capabilities, and repair quality.

While research results regarding the effects of system uncertainty on the uncertainty of aeroelastic response have been reported over the years, airframe aeroelasticity issues related to uncertainties of composite structures, damages, repairs have not been considered yet in the available literature in a comprehensive and thorough way. A few general attempts to treat flutter and aeroelastic limit-cycle-oscillations (LCO) on a reliability basis were reported more recently [Refs. 6-7].

What is needed is a probabilistic approach that enables accidental, random damage events to be assessed quantitatively.

Several Fast Probability Integration Methods are available in [2-5] for estimating the probability of failure in similar conditions. However, most of them are insufficient for solving the time-dependent problem with both continuous and discrete random variables due to the presence of discrete variables and response surface with discontinuities. In these conditions the Monte-Carlo (MC) method with variations like importance sampling appears to be the most appropriate if not the only possible approach. Schemes of probabilistic events of almost any complexity are possible with MC, though, even with parallel computing, the time required to run the simulations may be prohibitive depending on the accuracy required for high-reliability structures.

2. Methodology

In order to address these issues, the **RE**liability-based **L**ifecycle **A**nalysis of **C**omposite **S**tructures (RELACS) method and corresponding software were developed using the reliability analysis formulation [9]. The RELACS capability was written in a general format to accommodate the design of various structural components and damage threats.

The approach, presented in detail in [9], combines the "Level of Safety" method proposed by Lin, et. al. [10] and the ProDeCompos method proposed by Styuart, et. al. [11]

2.1 Method Development and Validation

The method development began with the identification of the issues critical to a reliability analysis formulation: threat/damage types; environmental factors; structural failure mechanisms; inspection methods; relevant data; assumptions; and reliability criteria. These considerations were integrated into a framework which formed the basis of our analysis. This reliability analysis formulation was then used to determine optimum inspection intervals for structures subjected to accidental damage [12]. Alternative formulations based on statistical models were also explored.

2.2 Problem Definition and Assumptions

The reliability problem to be studied is the probability that a structure reaches ultimate failure during its life under time-variant load. The structure may be instantly damaged and/or its load-bearing capability may deteriorate due to aging, corrosion, etc. There may be various failure modes, damage types, inspection methods and methods of repair. Structural strength and stiffness may vary with structural temperature. Thermal stresses can also be taken into account. In general, the following failure modes are considered:

- "Static" failure: random load exceeds the strength of undamaged/damaged/repaired structure.
- Excessive deformations.
- Flutter: airspeed exceeds the flutter speed of damaged structure.
- High amplitude limit cycle oscillations: the acceptable level of vibrations is exceeded.

The following terminology is mostly related to the "static" failure, but, in extending failure modes to cover aeroelastic phenomena, residual stiffness or the "residual" dynamic properties are considered in a similar way to address the effects of deterioration and damage on aeroelastic characteristics. So the term "load" used here is a generalization for the governing external loading condition. It may be a combination of load factors - pressure, bending moment, shear force, etc. - as well as airspeed or gust velocity.

The technique applied is the simulation of complex probabilistic phenomena describing the system of events leading to structural failure. The residual strength lives, in particular, are simulated. The simulation, in theory, should take into account as many events and random variables as one can describe by means of probabilistic theory. However, such an unlimited system will be impractical in practice, so our considerations are limited to the few random variables that seem most important and for which we have—or otherwise must develop—a probabilistic description.

2.3 The Reliability Model

Error! Reference source not found. illustrates the idea of a probabilistic model. Let us assume that every structural component in an infinite fleet of aircraft has residual strength history *R*. In the case of one damage occurrence, the residual strength life history consists of three intervals t_i, each of constant strength R_i. The initial strength is R_0, and at the instant t_0 a damage event occurs resulting in a damage of size *D*, so that *R* is reduced to the value $R_i=R_D$. At the time instant $t=t_0+t_D$ the damage is repaired and the strength is restored to some value $R_i=R_R$.. Failure can occur only where the random external load exceeds the structural strength.

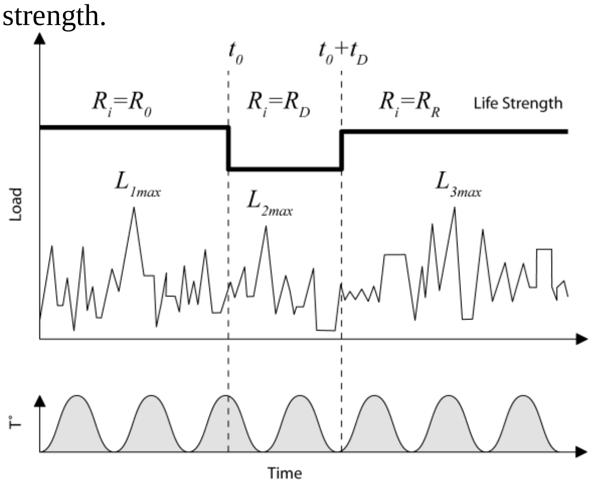

Fig. 1: Residual Strength History and Load

In general, the probability of failure can be expressed as an integral of a multivariate PDF of all random variables over the failure domain:

$$P_f = \int_\Omega f(N_j, \vec{D}, \vec{R}, t_i, t_D, \vec{L}, \vec{T}^\circ \,|\, TD, FM, T_1, T_2, T_3 ...)\, d\vec{v}$$

$$d\vec{v} = dN_j\, d\vec{D}\, d\vec{R}\, dt_i\, d(t_D)\, d\vec{L}\, dT^\circ, \qquad (1)$$

where Ω is a failure domain, TD is a type of damage, FM is a failure mode, T1, T2, T3… are inspection intervals. Vector notations are used here because of multiple failure modes and damage types. The most common numerical solution method for such integrals is Monte-Carlo simulation and was used consistently in the work reported here. MC integration makes it possible to check every step of the simulation and allows for the use of intermediate results, but its most important feature is its flexibility and its ability to accommodate future extensions to the model.

Given the load exceedance data or probability distribution function of maximum load per time unit and the residual strength history, expressed as a piecewise curve as shown on **Error! Reference source not found.**, one can obtain the probability of failure from the expression:

$$P_i^j = 1 - \{F_L[R_i^j(D_i^j) \,|\, \mu_L, \sigma_L]\}^{\frac{(t_{D_i}^j - t_i^j)}{Life}}, \qquad (2)$$

where μ_L, σ_L are the parameters of F_L and *Life* is a lifetime. Simulating *N* lives with random number of such intervals N_j, the average probability of failure P_f can be obtained from the relationships:

$$P_f = \frac{1}{N}\sum_{j=1}^{N} P_j; \quad P^j = 1 - \prod_{i=1}^{N_J}[1 - P_i^j(R_i^j, (td_i^j - t_i^j)];$$

$$N = f(\Delta);\ F_L = CPF\ of\ \max load\ per\ life; \qquad (3)$$

$$t_{D_i}^j = f[P_{Detect}(D_i^j), t_i^j]$$

where N depends on the required accuracy Δ.

2.4 Sources of Input Data

In-service damage data available in the public domain such as the FAA service difficulty report database was mined and sorted so that it could be readily used for the reliability analysis. The data included damage types, sizes, causes, and locations. The input probability density function of detected damage has been obtained from these data. Effects of environmental aging and chemical corrosion are taken into account and their effects have been incorporated into the software. Empirical data on residual strength degradation due to damages and environmental effects are used as an input.

The usual problem is that the empirical data are not sufficient to derive the statistical characterization of integrated structural parameters like static failure load of flutter speed. Actual tests of the additional structures just to measure the scatter are extremely expensive and therefore impractical. It is much easier and practical to use mathematical simulations – virtual testing – to study the effects of parameter variations on overall behavior of interest.

3. Software development

In order to obtain the statistical properties of integrated structural parameters, the Virtual Test Laboratory (UWVTL) tool has been developed at the University of Washington Aeronautics and Astronautics department. It allows virtual proof-testing of designs prior to or in parallel with their structure development. The major applications driving development of the UWVTT are advanced composite structures of civil aircraft.

As the mathematical model is usually of the Finite Element (FE) type, changing input parameters can be time consuming due to the complexity of the FE package and models used. Computational time per case, which adds up rapidly when models' complexity and detail rise, is another factor to consider. The UWVTL is designed to accelerate analysis and input data variation and to be truly interactive. The two main UWVTT parts are the module for virtual aeroelastic/structural tests (VASTM) and the RELACS module for time-dependent virtual tests.

3.1 Stochastic FE Simulation via Virtual Aeroelasticity/Strength Test Module (VASTM)

VASTM is a virtual aeroelasticity/strength test module designed primarily for characterization of some statistics-based variables required by RELACS input. The most demanded RELACS variables are the "static tests" failure load and flutter speed. These input data are not routinely obtained in current design and substantiation processes. Some data at coupon and sub-component level may be obtained through extensive and expensive testing programs. The accuracy of probabilistic analysis relies heavily on the accuracy of input. When experimental data is not available from full-scale physical testing, results obtained by Finite Element Analysis with random inputs may be used instead as virtual experiments.

The current VSTM simulation procedure consists of seven macro steps:

1. Firsts step is a common procedure of designing the FEA aeroelastic or stress analyses model. Those two models may be different for the same structure. This step is made outside of VASTM software and may involve such facilities as MSC PARTAN, FEMAP or any other FEA preprocessors. In a current study MSC/MD PARTAN has been used for structural model preparation and post-processing. NASTRAN input file for modal analysis is prepared based on nominal or average (preferable) thicknesses and material properties. In general, each finite element of the model should have its own property card and material card.
2. Having this structural model, the FEA model is constructed. The current study used the NASTRAN structural code as a solver.
3. In order to obtain the randomly selected test structure, the property and materials cards of that input file are changed by the appropriate VASTM-1 module.
4. The solver runs using randomized input file. The output data along with some input data are accumulated in a database. The steps 3 and 4 are repeated the many times to obtain the statistics of output variable.
5. After solver (NASTRAN) finishes its job, the appropriate output files are analyzed by VASTM-2 module which generates the probability density function of output variable.

It should be mentioned that the similar multi-run procedure is now routinely used by aircraft engineers e.g. to obtain the envelopes of static/dynamic loads or flutter speed for multiple configurations and flight parameter combinations. This differs from the VASTM procedure by the quasi-deterministic way of selection of configurations/parameters.

Apart from trivial independent sampling of random structural parameters, the UW VASTM incorporates the following features:

- Spatial correlation of properties. Within a subcomponent or panel, which is manufactured by a single process, the spatial variation of properties may not be independent. This may be due to the related tooling, manufacturing technique, raw material, etc. For example, thickness of metal panel may be correlated in a periodic fashion if it was formed by rollers; composite panels produced by automatic fiber placement may show consistent properties along the tape direction but display higher variability across different tapes; parts made by various resin transfer molding techniques may exhibit different thickness and resin volume variation according to the injection and overflow locations. VASTM is designed to take this into account and perform stochastic FEA with prescribed spatial correlations for selected properties.

- Correlation of different material properties. Traditionally, statistics on material elastic modulus, strength, fiber volume fraction, etc, are provided independent of each other. However, possible correlation between different properties are often not explored and even less often provided as data. Yet, these correlations can have significant impact on the performance of the final structure.
- Different panel-to-panel and point-to-point variations of properties. It is known that the variation of properties between composite panels is considerably greater than that for coupons cut from the same panel, which may have some impact on structural reliability.

3.2 RELACS: Time-Dependent Virtual Testing.

RELACS is created for virtual life testing. In this method, described in [9], life histories of the structural damage size (as well as material degradation and structural changes due to environmental effects) are simulated using randomized input parameters. These are converted into the histories of residual structural property subject to environmental exposures, repairs, and other factors. The residual property may be strength, stiffness, flutter speed, vibration level, etc. depending on the considered failure mode (see Figure 2). Random loading and flight conditions are also generated in the form of mechanical load / temperature / maneuver / airspeed histories and compared to the structural property statistics (stress, deformation, flutter speed, etc.) to find if failure can occur. That is, statistics of changes of structural behavior over time (with resultant change, in, say, flutter speed) are compared to statistics of loading and flight conditions (say, actual flight speeds and dynamic pressures) over time to evaluate the probability that failure (actual flight speed exceeds flutter speed) will occur. In another example, the statistics of residual strength are compared to actual loading histories to find the probability that actual loading will lead to stresses above buckling or yield limits). The number of failures per total number of simulated life histories yields the probability of failure (POF).

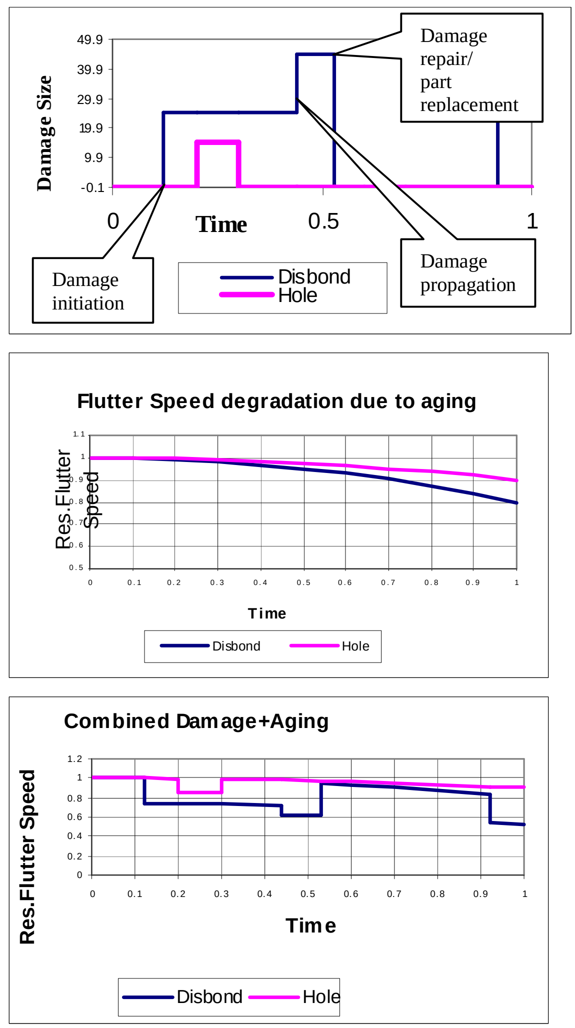

Fig. 2; Simulated Histories of residual flutter speed variation with time, as affected by damage, aging, and repair

The RELACS method and associated software can be used to determine any controllable parameters in the structural system required to achieve a prescribed level of reliability. Design parameters that affect structural reliability can be actual sizes (thicknesses) of structural elements, topology and load paths in the structure, material type, joint type and strength, etc. But important design parameters that affect the reliability of the system are also inspection/repair methods, their time and place.

4. Example Problems

So far several practical and important real-case example problems have been solved, in particular: Reliability-based damage-tolerance of a composite aileron of a commercial aircraft [9]; probabilistic disbond propagation for typical bonded skin-stringer panel of an aft fuselage of a commercial aircraft [15]; Probability of failure due to flutter for a realistic damage–tolerant composite flaperon on a supersonic fighter jet [13]; optimum maintenance planning for a composite aileron of commercial aircraft [12]. The following sections cover the broadest study conducted so far using UWVTL.

4.2 The Uncertain Aeroelastic Vertical Tail / Rudder System

A realistic NASTRAN model of a composite vertical tail / rudder system of a passenger airplane (but not representing any actual flying vehicle) is presented in Fig. 3. The nodes at the root end of spars were fixed.

The model random input is characterized by the data shown in Table 1. The model was modified with VASTM to allow every structural and mass element to have its own property and material card. The elements belonging to structural panels which are manufactured separately were united into groups to represent panel-to-panel variability.

Table 1

Number of grid points	1268
Number of CBAR elements	309
Number of CBUSH elements	45
Number of CONM2 elements	28
Number of CQUAD4 elements	1409
Number of CROD elements	1056
Number of CSHEAR elements	91
Number of CTRIA3 elements	187
Number of RBE2 elements	16
Number of RBE3 elements	28

Attention was paid to adequate simulation of the composite skin panels where impact damages were expected. Those structural panels were simulated using NASTRAN SHELL finite elements with randomized thickness and three random material properties: G11, G12, and G22. Since the model arrived from industry with lumped masses representing mass distribution for dynamics purposes, variations in material density were simulated as included in those lumped masses. The correlation between thickness and structural mass was not simulated due to the lack of appropriate information for this particular model. Average panel geometric and materials properties were simulated independently, while those of individual finite elements belonging to each panel were simulated using the Markov random field in the same way as for the simple delta wing model of reference [15].

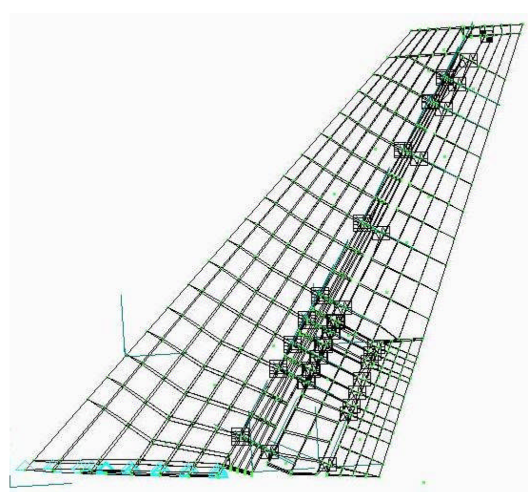

Fig. 3: Representative Vertical Tail FEA Model

Unsteady aerodynamic modeling based on the Doublet Lattice Method of NASTRAN SOL 145 was used. Because of the focus here on structural uncertainty, random variations of aerodynamic parameters were not considered.

The average flutter frequency was about 13 Hz. Actual natural modes contributing to the flutter mechanism can vary, depending on variations in the structure and possible switching of flutter mechanisms, depending on the magnitude and combination of structural perturbations.

The following results were obtained using the data on variances and correlations for elements with PSHELL properties given in Table 2.

Table 2

Property	Panel-to-panel C.O.V.	Element-to-element C.O.V.	Radius of Correlation, mm
Thickness *t*	**0.03**	**0.01**	**25**
G11	**0.05**	**0.02**	**250**
G22	**0.05**	**0.02**	**250**
G12	**0.05**	**0.02**	**250**
σ11	**0.07**	**0.03**	**250**
σ22	**0.07**	**0.03**	**250**
σ12	**0.07**	**0.03**	**250**

The typical coefficients of variation (C.O.V). values of Table 2 parameters were taken from MIL-HDBK-17. Theoretically the radius of correlation could be also evaluated from the data of the MIL-HDBK-17, if the appropriate supporting information like panel size, coupon size were present there. The C.O.V. of thickness and Young's modulus for PBAR and PROD properties were assumed equal to 0.02. The same value has been used for CONM2 mass elements.

4.2.1 Virtual Static Test Results

Virtual static tests were conducted for the Tuned Gust Load case obtained using NASTRAN SOL 146. Simplified material failure criteria have been used. Fig. 4 shows the cumulative distribution function of the tail failure load. The results are shown here for demonstration of software capability.

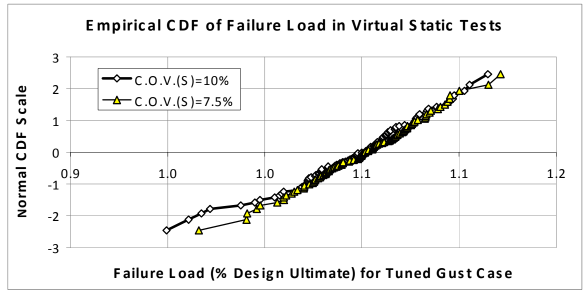

Fig. 4: Virtual Static Test results

4.2.2 Virtual Flutter Test Results

The most interesting results of the virtual tests are shown in Fig. 5 and Fig. 6. Fig. 5 shows the empirical CDF of flutter velocity. It is obvious that the corresponding PDF is bimodal.

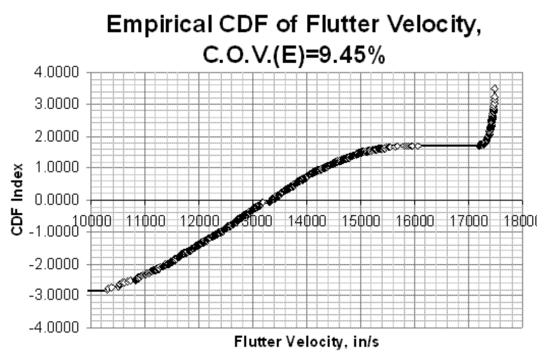

Fig. 5: CDF for Vertical Tail Flutter Velocity

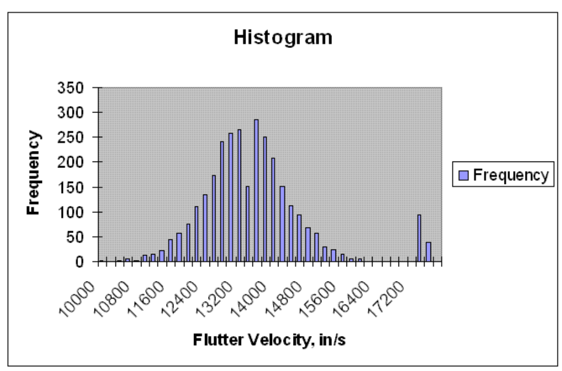

Fig. 6: Flutter Velocity Histogram

This fact is reflected in Fig. 6, where the corresponding histogram is shown. Practically this means that some aircraft in a fleet simulated using the assumptions listed above, may have flutter frequency and flutter mechanisms that are rather different from those of the main population. It is also evident that the variance of the second flutter mechanism is much smaller than the first one. This may be an evidence of the possibility of different uncertainty propagation for different failure modes mentioned earlier. In this particular case, the second mode of PDF is on the right tail of distribution which does not affect much the structural safety. But there may be situations when this mode may appear on the left tail. It is also obvious that the response function in this case has a discontinuity and that some popular fast reliability methods like SORM, FORM [3-4] may generally not be applicable to the probabilistic study of flutter and similar aeroelastic phenomena.

Another observation is that the variance of V_F is noticeably greater than variances for the input parameters shown in Table 2.

4.2.4 Damaged Structure Tests

Fig. 7 shows the empirical CDF of V_F obtained with VASTM under the condition that a randomly selected element belonging to the tail torsion box skin has big damage of the size of about 75 mm.

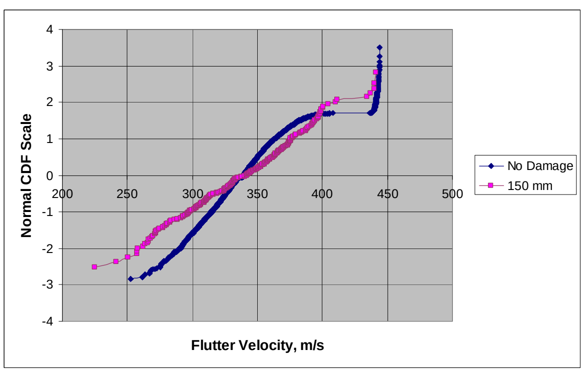

Fig. 7: Empirical CDF of VF for damaged and undamaged structure

Stiffness reduction due to damage was considered and estimated (by using analysis of the element itself) as the difference of average relative displacements of opposite nodes per given loading depending on damage size. The damage was assumed at a center of the element. The locations of damaged elements have been chosen randomly with uniform distribution over the tail box skin area.

The cross-section width of an element has been defined for each randomly selected element in the direction coinciding with an aircraft longitudinal axis at the position of element centroid.

The V_F CDF for the undamaged structure is shown in Fig. 7 for comparison. It is interesting to notice that the average values are almost the same, but the C.O.V of the damaged structure is much greater. This behavior will inevitably lead to lower reliability.

4.2.3 Time-dependent study with RELACS

The input data for RELACS were taken from Ref. 6. Panel weight change due to repair was not considered due to the lumped mass nature of both structural and nonstructural mass in the model provided for this work by industry. The V_F CDF for undamaged structure and damaged structure were taken by polynomial approximation of curves shown in Figure 7 and other curves obtained with VSATM for different damage sizes.

The following input data was used:

- Number of Design Cases = 1; Subsonic flight
- Number of Damage Types = 2; Hole and Delamination
- Number of Inspection Types =2; Visual and Instrumental
- The CDF of maximum airspeed per life is expressed by equation (3) of Ref. [15]
- The probability of damage detection model described in [9] was used.

The exceedance data of damage occurrence is taken from [11] and recalculated for 60000 flight hours and torsion box skin area. To introduce even more conservatism, the damage sizes in the calculations were twice larger than those in [11]. This might include the damages inflicted by uncontained turbine blades and similar cases.

Fig. 8 shows the Probability of Failure in flutter accounting for damage depending on the safety margins used for design. The POF without damages as a function of the safety margin used for design is also shown for comparison.

Fig. 8: Flutter POF vs. Safety Margin (Factor)

It should be mentioned that the representative vertical tail / rudder system here has about 57% safety margin above V_D by design and it is highly safe.

The probabilistic analysis shows that in order to ensure the same POF as in no-damage case, the safety margin with nominal stiffness should be at least 5% greater than that without damage considerations. This conclusion, it should be emphasized, is not general and is case dependent. Situations may occur, for some airframe designs, where damage or combination of damages that lead to partial local loss of stiffness or increased mass may lead to flutter failures. The simulation capabilities developed for this work and described above can be used to identify such cases and to study them and the consequences for design and maintenance.

5. Acknowledgments

This work was sponsored by the Federal Aviation Administration and The Boeing Commercial Airplane Group through the University of Washington's Center of Excellence, AMTAS. The FAA technical monitors for this work were Peter Shyprykevich, Curtis Davies, and Dr. Larry Ilcewicz. Their support and contribution are greatly appreciated. Thanks are also due to Boeing Commercial for their strong support, especially the provision of the vertical tail/rudder NASTRAN model used in here.

3.3 References

[1] Shiao M., Boyd K, and Fawaz S, A risk assessment methodology and tool for probabilistic damage tolerance-based maintenance planning", *8th Joint NASA/FAA/DoD Conference on Aging Aircraft*, 2005

[2] Harter J. AFGROW Users Guide and Technical Manual, AFGROW for Windows 2K/XP, Version 4.0009.12, June 2004.

[3] Chamis, C. Damage tolerance and reliability of turbine engine components, *NASA TP-1999-209878*, 1999.

[4] Riha D, Thacker B, Millwater H, Wu Y-T, and Enright M. Probabilistic engineering analysis using the NESSUS software, *41st Structural Dynamics and Materials Conference,* Paper 2000-1512, Atlanta, Georgia, 2000.

[5] Wu Y-T, Enright M, and Millwater H. Probabilistic methods for design assessment of reliability with inspection. *AIAA Journal*, Vol. 40, No. 5, pp. 37-946, 2002.

[6] Mahadevan S, and Dey A, Adaptive Monte Carlo simulation for time variant reliability analysis of brittle structures", *AIAA Journal*, Vol. 35, No, 2, pp.321-326, 1997.

[7] Pettit C. Uncertainty quantification in aeroelasticity: recent results and research challenges. *Journal of Aircraft*, Vol. 41, No.5, pp 1217-1229, 2004.

[8] Poirion, F. On some stochastic methods applied to aeroservoelasticity," *Aerospace Science and Technology*, Vol. 4, No. 3, pp. 201-214, 2000.

[9] Lin K and Styuart A, Probabilistic approach to damage tolerance design of aircraft composite structures. *Journal of Aircraft,* Vol. 44 No. 4, pp. 1309-1317, 2007.

[10] Lin K, Du J and Rusk D. Structural design methodology based on concepts of uncertainty. *NASA Rept*. CR-2000-209847, Langley, VA, 2000.

[11] Ushakov A, Stewart A, Mishulin I, and Pankov A. Probabilistic design of damage tolerant composite aircraft structures. *FAA report DOT/FAA/AR-01/55*, Washington DC, 2002.

[12] Styuart A and Lin K. Maintenance planning for aircraft damage tolerant composite structures based on risk assessment. *AIAA-2007-1979, 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Honolulu, Hawaii, 2007

[13] Styuart A, Mor M, Livne E, and Lin K. Aeroelastic failure risk assessment in damage tolerant composite airframe structures. *AIAA-2007-1981 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*, Honolulu, Hawaii, 2007

[14] Cheung C, Styuart A, and Lin K. Reliability of composite structures with damage growth consideration," *ASC 22nd Annual Technical Conference*, Seattle, WA, 2007.

[15] Styuart A, Demasi L, Livne E and Lin K. Probabilistic Modeling of the Aeroelastic Life Cycle for Risk Evaluation of Composite Structures. *AIAA-2008-2300 49th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference*. Schaumburg, IL, 2008.

Copyright Statement

The authors confirm that they, and/or their company or institution, hold copyright on all of the original material included in their paper. They also confirm they have obtained permission, from the copyright holder of any third party material included in their paper, to publish it as part of their paper. The authors grant full permission for the publication and distribution of their paper as part of the ICAS2008 proceedings or as individual off-prints from the proceedings.