RESEARCH ON CONTROL ALLOCATION FOR AIRCRAFTS WITH MULTIPLE CONTROL EFFECTORS

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Abstract

This paper discusses the control allocation principle and solution for aircrafts with multiple control effectors from the angle of flight performance. ADMIRE developed by FOI is taken as example, and the control selector for ADMIRE is redesigned based on the control allocation principle and solution developed in this paper. The trimmed lift to drag ratio (L/D) and Combat Cycle Time (CCT) of ADMIRE with its original control selector and with the selector redesigned in this paper are compared, the results indicate that the principle and solution built in this paper are reasonable.

1. Introduction

The aerodynamic configuration of aircraft depends on the performance requirements. Many aircraft were being given new control surfaces that were intended to provide additional capabilities. [1] To obtain high AOA maneuverability, canards are introduced. To obtain post stall maneuverability and controllability, thrust vectoring is introduced. To improve performances during take off and landing, high-lift devices are introduced. To improve the stealthy, tails are modified or canceled. To improve the controllability of tailless aircrafts, innovative control effectors such as all moving tips [2] are introduced. The aerodynamic surfaces can be grouped together to form a set of aerodynamic effectors, and the components of the propulsion system can be grouped together to form a set of propulsion effectors. [3]

For traditional aircrafts, there are equal number of effectors and desired control parameters, and one unique solution exists. But for aircrafts with multiple control effectors, the number of effectors is larger than the number of control parameters, and there are an infinite number of solutions. The control effectors must be allocated properly to get an optimal solution.

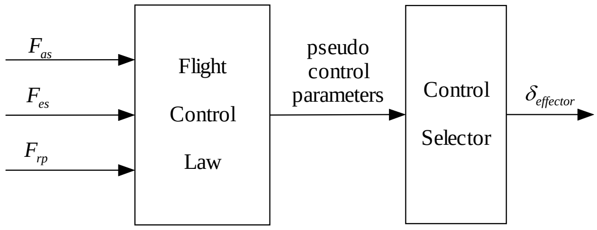

Fig.1 FCS for Aircrafts with Multiple Control Effectors

As shown in Fig.1, Flight Control System (FCS) of aircrafts with multiple control effectors consists of two parts: Flight Control Law (FCL) which transfer pilot inceptor into pseudo control parameters, and the Control Selector which transfer pseudo control parameter into deflections of control effectors. For same aircraft with same FCL, the flight and control performances depend on the Control Selector.

Recent researches on control allocation are mostly concentrated on optimizing control performances, influences of control allocation on aerodynamic performances are not fully considered. Hence, the performance potentials of almost all the aircrafts with multiple control effectors were not fully utilized. [3]

2. Control Allocation

2.1 The Control Allocation Problem

Control allocation problem is to determine how the control effectors should be positioned to produce desired effect. [1] The input of control allocation is the desired control effect $v(t) \in R^k$ to be produced. The output is the real control input $u(t) \in R^m$, where $m > k$.

For linear system:

$$Bu(t) = v(t) \tag{1}$$

Here, B is the control effectiveness matrix.

The actuator position constraints:

$$u_{\min} < u(t) < u_{\max} \tag{2}$$

The actuator rate constraints:

$$\rho_{\min} < \dot{u}(t) < \rho_{\max} \tag{3}$$

Since the control selector is part of a digital control system, it is reasonable to approximate the time derivative as

$$\dot{u}(t) \approx \frac{u(t) - u(t-T)}{T} \tag{4}$$

T is the sampling time. Combining Equation (2)-(4) yields

$$\underline{u}(t) \le u(t) \le \overline{u}(t) \tag{5}$$

where

$$\begin{aligned} \underline{u}(t) &= \max\{u_{\min}, u(t-T) + T\rho_{\min}\} \\ \overline{u}(t) &= \min\{u_{\max}, u(t-T) + T\rho_{\max}\} \end{aligned} \tag{6}$$

Equation (1) constrained by (5) constitute the standard formulation of the linear control allocation problem. [4]

$$\begin{aligned} &Bu = v \\ &\underline{u} \le u \le \overline{u} \end{aligned} \tag{7}$$

2.2 Principle of Control Allocation

The characteristics of the aircraft and the control effectors must be fully considered first in control allocation. Each untraditional control effectors introduced to aircrafts with multiple control effectors has its original purpose, not always for control, although the control effectors may have the ability to control. Take the close-coupled canard as example, the introduction of it is to increase the lift at high AOA by the interference and their breakdown between canard and wing vortices. [5] Since the canard moment arm is short, and the interference between canard and wing vortices will be affected by the deflection of canard, the close-coupled canard isn't fit for control. Hence, new control effectors can take part in control allocation only after their original purpose of new control effectors being ensured. This ensures the original purpose of the control effectors, and simplifies the control allocation problem.

For different aircrafts, because of the differences between their aerodynamic configurations and missions, the control allocation principles are different. The control allocation principle should be selected on the basis of fully analysis of the aerodynamic characteristics.

For different flight phases of the same aircraft, the performance requirements are different; the control principle should be different too. The control allocation principle of cruise and air-combat will be presented below.

2.2.1 Cruise

The optimal object of cruise is maximum cruise range or longest cruise time. To get the maximum cruise range, we should keep the cruise parameter maximum, while to get the longest cruise time, keep L/D maximum. No matter what the optimal object is, the L/D should be as large as possible.

Since the lift equals to weight during cruising, the object of control allocation can be transferred into minimum drag.

At trim condition, the lift and pitch moment coefficient are as follows: [6]

$$\begin{cases} C_m = C_{m0} + C_{m_\alpha}\alpha + C_{m_\delta}\delta = 0 \\ C_L = C_{L0} + C_{L_\alpha}\alpha + C_{L_\delta}\delta \end{cases} \tag{8}$$

The trim AOA α_{trim} and control effectors deflection δ_{trim} are:

$$\alpha_{trim} = \frac{C_{m0}C_{L_\delta} + C_{m_\delta}C_{Ltrim}}{\Delta} \tag{9}$$

$$\delta_{trim} = -\frac{C_{m0}C_{L_\alpha} + C_{m_\alpha}C_{Ltrim}}{\Delta} \quad (10)$$

Where

$$\Delta = C_{L_\alpha}C_{m_\delta} - C_{L_\delta}C_{m_\alpha} \quad (11)$$

From equation (9, 10, 11) we get the trimmed lift curve:

$$C_{Ltrim} = -\frac{C_{m0}C_{L_\delta}}{C_{m_\delta}} + \frac{\Delta}{C_{m_\delta}}\alpha_{trim} \quad (12)$$

And the slope is given by

$$\left(\frac{dC_L}{d\alpha}\right)_{trim} = C_{L_\alpha} - \frac{C_{L_\delta}}{C_{m_\delta}}C_{m_\alpha} \quad (13)$$

The induced drag of modern fighters increases very fast as AOA increases. In order to get large L/D, the trim AOA should be small, and the trimmed lift-curve slope should be as large as possible. The selection of trim control effectors should be based on the longitudinal static stability of the aircraft and the control efficiency of each control effectors. If the aircraft is stable, control effectors ahead of the control fixed neutral point, such as canard, should be chosen. Otherwise, control effectors behind the control fixed neutral point, such as elevon, should be chosen.

Since the deflection angles and rates of control effectors are small, the limits of control effectors can be ignored.

2.2.2 Air Combat

The superiority of modern air-combat depends on the maneuverability of the aircrafts. Several performance metrics are presented here.

(1) Specific Excess Power (P_s)

Specific excess power represents an aircraft's ability to change its specific mechanical energy either by changing altitude or airspeed. PS can be calculated as [7]

$$P_S = \frac{[T\cos(\alpha - \phi_T) - D]V}{W} \quad (14)$$

Here, *V*, *T*, *D* and *W* are the aircraft's velocity, thrust, drag and weight respectively. AOA is α and ϕ_T is the angle between thrust line and body X axis.

For same aircraft at same velocity, P_S depends on the drag. In order to get larger P_S, the drag should be as small as possible.

(2) Ability to Change Flight Direction

Maximum Instantaneous Turn Rate

$$|\dot{\psi}| = \left|\frac{d\psi}{dt}\right| = \frac{g}{V}\sqrt{n_{z\max}^2 - 1} \quad (15)$$

Maximum Vertical Flight Path Angle Rate

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{g}{V}(n_{z\max} - \cos\gamma) \quad (16)$$

Where the yaw angle is ψ, γ is the flight path angle, g is the acceleration due to gravity, $n_{z\max}$ is the maximum normal load factor.

Generally speaking, larger lift and smaller drag produces better maneuverability. But it is usually unlikely to get both of them.

During high AOA maneuver, the drag increases dramatically, and the velocity falls very fast, which is adverse to air-combat. Hence, drag is as important as lift to maneuverability in air-combat.

2.3 Control Allocation Solution

After over ten year's efforts, many kinds of control solutions have been presented. These solutions can be classified into four categories: Generalized Inverse, Daisy Chaining, Direct Allocation, and Mathematical Programming. [8]

All these solutions have their advantages and disadvantages.

This paper will take the combination of linear programming (one kind of mathematical programming) and daisy chaining as the solution of control allocation: while AOA is far below stall, only aerodynamic surfaces take part in control allocation by linear programming; while AOA approaches stall, both aerodynamic surfaces and thrust vectoring take part in control allocation still by linear programming; after stall, the only control effectors is thrust vectoring.

Conventional aerodynamic control surfaces will be fully utilized by this combination. Because thrust vectoring will take part in control allocation before the failure of aerodynamic control surfaces, saturation is not probably happening.

A standard linear programming problem consists in finding a vector *x* such that [9]

$$J = f^T x \tag{17}$$

is minimized, subject to

$$x_{\min} \le x \le x_{\max}\,,\ Ax = b \tag{18}$$

3. Results and Discussion

This paper will take the ADMIRE (Aero-Data Model in Research Environment) developed by FOI, as example. ADMIRE, as shown in Fig.2, is a generic model of a small fighter with delta-canard configuration. [10]

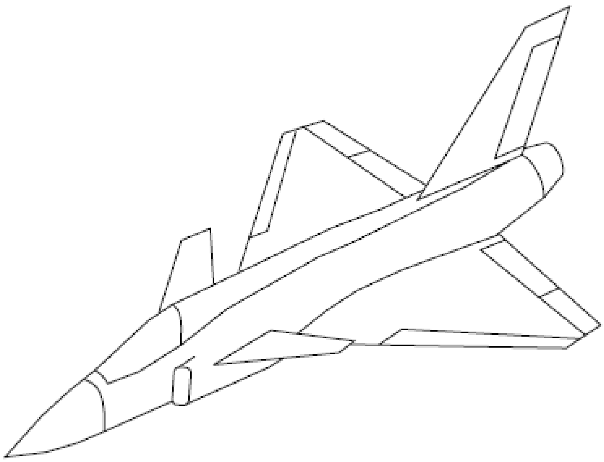

Fig.2 configuration of ADMIRE

The control surfaces of ADMIRE include:

- left canard (δ_{lc})
- right canard (δ_{rc})
- left inner elevon (δ_{lie})
- right inner elevon (δ_{rie})
- left outer elevon (δ_{loe})
- right outer elevon (δ_{roe})
- leading edge flap (δ_{le})
- rudder (δ_r)
- horizontal thrust vectoring (δ_{th})
- vertical thrust vectoring (δ_{tv})

The aerodynamic surfaces can be grouped together to form 7 aerodynamic control effectors:

$$\begin{cases} \delta_{ei} = (\delta_{lie} + \delta_{rie})/2 \\ \delta_{ai} = (\delta_{lie} - \delta_{rie})/2 \\ \delta_{ey} = (\delta_{loe} + \delta_{roe})/2 \\ \delta_{ey} = (\delta_{loe} + \delta_{roe})/2 \\ \delta_{ne} = (\delta_{lc} + \delta_{rc})/2 \\ \delta_{na} = (\delta_{lc} - \delta_{rc})/2 \\ \delta_r = \delta_r \end{cases} \tag{19}$$

Since post-stall maneuverability doesn't belong to the research area of this paper, thrust vectoring will not be discussed.

The lateral control surfaces include elevon and canard. Since the lateral control efficiency of canard is much lower than elevon before stall, and its deflection will influence the flow of wing, canard doesn't fit for lateral controlling. Hence, elevon is the lateral control surface of ADMIRE, outer and inner elevon will take part in control with the same deflection.

3.1 Cruise

Besides thrust vectoring, the longitudinal control surfaces of ADMIRE include: canard, leading-edge flap and elevon. The baseline ADMIRE is statically unstable, from equation (13) we get: in order to get larger L/D, control surfaces behind the control fixed neutral point should be chosen. Since the lift, drag and pitch control efficiency of inner and outer elevon are not equal, they must be treated as two different control effectors.

The control allocation of cruise phase may be determined by solving the following linear programming problem:

$$\left.\begin{aligned} \min f^T u \\ Bu = v \end{aligned}\right\} \tag{20}$$

Where, $u = [\delta_{ei} \quad \delta_{ey}]^T$

Matrix $B = [C_{m_{\delta ei}} \quad C_{m_{\delta ei}}]$ is the control effectiveness matrix.

Vector $v = -C_{m0} - C_{m_\alpha} \cdot \alpha$ is the moment coefficient of control effectors at trim condition.

Vector f is used to weight the control variables. Here,

$$f = \begin{bmatrix} a \cdot \Delta C_{T_{\delta ei}} & -\Delta C_{N_{\delta ei}} \\ a \cdot \Delta C_{T_{\delta ey}} & -\Delta C_{N_{\delta ey}} \end{bmatrix}$$

$$a = \begin{cases} K, \text{ largest cruise range} \\ MK, \text{longest cruise time} \end{cases}$$

Since the deflection angles and rates are small, the constraints can be ignored.

No matter the optimal object is largest cruise range or longest cruise time, the L/D should be as large as possible. The trimmed L/D of ADMIRE is shown in Figure 3. The black solid line indicates ADMIRE using control allocation results solved in this paper; and the red dash line indicates ADMIRE using its original control allocation results. The trimmed L/D of the former is larger than the latter, which indicates the control allocation principle and solution of cruise phase are reasonable.

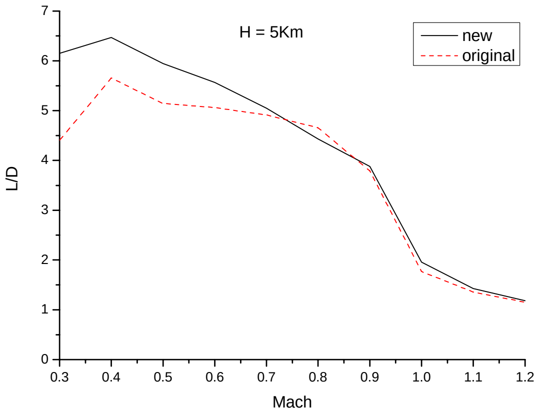

Fig.3 Trimmed Lift to Drag Ratio at H=5Km

3.2 Air Combat

Since lift and drag are both very important to maneuverability in combat, control allocation in combat may be transformed into the following linear programming problem:

$$\left.\begin{aligned} &\min f^T u \\ &Bu = v \\ &\underline{u} \le u \le \overline{u} \end{aligned}\right\} \qquad (21)$$

Where, $u = \begin{bmatrix} \delta_n & \delta_{ei} & \delta_{ey} \end{bmatrix}^T$

Matrix $B = \begin{bmatrix} C_{m_{\delta n}} & C_{m_{\delta ei}} & C_{m_{\delta ey}} \end{bmatrix}$ is the control effectiveness matrix.

Vector $v = \dfrac{\dot{q}_{des} I_y}{0.5 \rho V^2 S c} - C_{m0} - C_{m_\alpha} \cdot \alpha$,

$\dot{q}_{des}$ is the desired pitch acceleration, c is the mean aerodynamic chord.

Vector f is used to weight the control variables. Here,

$$f = \begin{bmatrix} a \cdot \Delta C_{T_{\delta n}} & -\Delta C_{N_{\delta n}} \\ a \cdot \Delta C_{T_{\delta ei}} & -\Delta C_{N_{\delta ei}} \\ a \cdot \Delta C_{T_{\delta ey}} & -\Delta C_{N_{\delta ey}} \end{bmatrix}$$

In this paper, $a = 1$

The CCT plot of the original ADMIRE and ADMIRE with the control selector designed in this paper is shown in Fig.4.

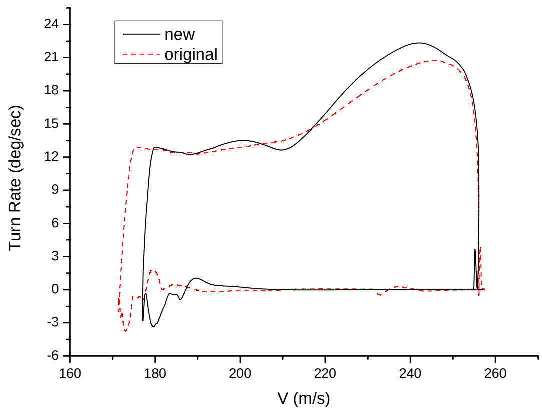

Fig.4 Combat Cycle Time Plot

The black solid line indicates ADMIRE using the new control selector designed in this paper; and the red dash line indicates ADMIRE using its original control selector. Both the turn rate and minimum velocity of ADMIRE with the new control selector are larger than the original aircraft. Because of the differences between the two control selectors, the lift coefficient of the aircraft with new control selector at high AOA is a little larger than the original aircraft, and the drag coefficient is much smaller than the original aircraft.

The CCT metrics is defined as the sum of $t_1 + t_2 + t_3 + t_4$ [11]

Where

t1 = time to pitch from one g to the limit normal load factor

t2 = time to turn to a specified new heading angle at maximum normal load factor

t3 = time to unload to a normal load factor of either one or zero g

t4 = time to accelerate to the original energy level

Table 1 Combat Cycle Time

Control Selector	$t_1+t_2+t_3$(s)	t_4(s)	CCT(s)
original	15.57	14.56	30.13
new	15.2	12.61	27.81

The CCT of ADMIRE with the new control selector is shorter than with the original one, which indicates the principle and solution built in this paper are reasonable.

4. Conclusions

Each untraditional control effectors of aircrafts with multiple control effectors has its original purpose, which should be ensured in control allocation.

This paper proposed the control allocation principle from the angle of flight performance, and took the combination of daisy chaining and linear programming as the control allocation solution.

By Comparing the trimmed L/D and CCT of the original ADMIRE and ADMIRE with the new control selector designed in this paper, we can get that the principle and solution built in this paper are reasonable from the angle of flight performance.

But, this paper only discussed the primary research. There are still many detailed problems, such as, the influence of weights of the object function on flight performance, thrust vectoring, etc. These detail problems will be the central work of next step.

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