TOWARDS AERODYNAMIC OPTIMISATION OF AN AEROELASTIC WING USING DOMAIN ELEMENT PARAMETERISATION

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Abstract

Generic 'wrap-around' aerodynamic shape optimisation technology is presented, and applied to a modern commercial aircraft wing in transonic cruise. The method uses a novel domain element parameterisation linked to efficient global interpolation functions to deform both the surface geometry and corresponding CFD volume mesh, in a high quality and robust fashion, and is totally independent of mesh type (structured or unstructured). The technique also provides a method that allows geometries to be parameterised at various levels, ranging from gross three-dimensional planform alterations to detailed local surface changes. Optimisation independence from the flow solver (inviscid, viscous, aeroelastic) is achieved by obtaining sensitivity information for an advanced gradient-based optimiser (FSQP) by finite-differences. Results have been presented recently for two-dimensional aerofoil cases, and shown very impressive results; drag reductions of up to 45% were demonstrated using only 22 active design parameters. Hence, this paper presents the extension of these methods to three dimensions. Results are presented for highly constrained optimisation of a modern aircraft wing in transonic cruise, using global and local parameters combined, to give 388 design variables for the wing. The optimisation produces a shock-free geometry with an 18% reduction in drag, with the added advantage of significantly reduced root moments.

A validated aeroelastic CFD solver has recently been developed and presented, and currently the generic 'wrap-around' optimisation methods are being applied to the aerodynamic optimisation of an aeroelastic wing.

1 Introduction

Computational fluid dynamics (CFD) methods are now commonplace in aerospace industries, and at the forefront of analysis capabilities, providing a fast and effective method of predicting a design's aerodynamic performance. However, with ever increasing complexity of designs, engineers can often struggle to interpret the intricacies of the CFD results sufficiently to be able to manually alter the geometry to improve performance. Hence, there has been an increase in demand for intelligent and automatic shape optimisation schemes. This requires combining geometry control methods with numerical optimisation algorithms, to provide a mechanism to mathematically seek improved and optimum designs, using CFD as the analysis tool.

Optimisation requires consideration of three issues, each of which have numerous solutions: shape parameterisation including CFD surface and volume mesh deformation, computation of design variable derivatives, and effective use of these derivatives to improve design. Geometry parameterisation is critical for effective shape optimisation. This is the method of representing the design surface, and defines the degrees of freedom in which the geometry can be altered and, ideally, this should be linked with an effective method of deforming the CFD surface and volume mesh in a corresponding fashion. Parameterising complex shapes is a problem that remains a serious obstacle to both manual and automatic CFD-based optimisation. A wide variety of shape control and morphing methods have been developed, but many do not allow sufficiently free-form design, can produce infeasible shapes and do not allow the possibility of manual manipulation of the geometry. Furthermore, most methods do not have a suitable method to deform the CFD mesh once the surface has been changed, and regeneration is often required. This may not be a problem for simple geometries and/or small meshes, but can in some cases make automation of the optimisation process impossible. Those methods that do incorporate CFD mesh deformation techniques are often of poor quality, hence restricting the size of the allowable deformation, and/or are computationally expensive and impractical for large CFD meshes.

It is essential that designs can be deformed into arbitrary shapes, allowing exploration of the entire design space, but having an excessive number of deformation degrees of freedom (design variables) can often make optimisation impractically expensive. Hence, an efficient domain element shape parameterisation method has been developed by the authors, along with a high quality and robust mesh deformation scheme, and presented recently for two-dimensional CFD-based shape optimisation [1, 2]. The parameterisation technique, surface mesh motion and volume mesh motion are all accomplished through combined global interpolations using radial basis functions, such that when the positions of the domain element are altered, both the design surface and its corresponding CFD volume mesh are deformed in a high-quality fashion, and this is aimed at automating the entire process. This interpolation has been developed such that the domain element parameterisation method has no computational memory overhead that would restrict the size of the CFD volume mesh that can be used. The domain element parameterisation technique also allows for geometry control at various fidelity levels, ranging from gross three-dimensional planform alterations to fine, detailed surface geometry changes. Furthermore, it is totally independent of the CFD mesh type, removing any grid generation or flow solver dependence.

Independence from the flow-solver is achieved by obtaining the sensitivities required for optimisation via finite-difference. This allows numerous options in terms of optimisation approaches, and an advanced FSQP [3, 4, 5] gradient-based optimiser has been integrated in the framework. Independence from both grid generation approach and flow-solver ensures a totally ‘wrap-around’ tool has been developed. This domain element parameterisation, global interpolation-based CFD mesh motion, and advanced optimisation approach has been proven in two dimensions, demonstrating drag reductions of up to 45% for highly constrained aerofoil cases [1, 2].

The research presented in this paper is the three-dimensional extension of the ‘wrap-around’ shape parameterisation and optimisation method. Optimisation is applied here to the MDO wing (a large modern transport aircraft wing, the result of a previous Brite-Euram project [6, 7]) in the economical transonic cruise condition. Detailed results of optimisation performed with the highest fidelity so far used are presented. This is a combination of global and local surface geometry changes, resulting in 388 design variables, and the objective is drag minimisation.

The aerodynamic loads on a transport aircraft wing in cruise flight are considerable. In a steady situation, these aerodynamic loads cause the wing structure to deform aeroelastically to a stable state, in turn modifying the nature of the flow itself. The results from CFD solutions of a wing when allowed to deform aeroelastically alter dramatically from those when the structure is assume rigid. With regard to optimisation of the aircraft geometry, it is therefore imperative to consider the affects that geometry alterations have to the performance of the wing under aeroelastic conditions. Gumbert[8] and Jameson [9] have recently presented aerodynamic optimisations of three-dimensional wings under aeroelastic conditions. However, it should be noted that if attempting a gradient-based optimisation that is not independent of the CFD solver, it is extremely difficult to obtain sensitivities are truly aeroelastic (the aeroelastic nature of the optimisation usually comes in the form of the evaluation of the objective function, but not it's derivatives). The optimisation technique presented here 'wraps-around' the CFD solver thus allowing the accurate calculation of aeroelastic sensitivities.

A parallel structured multiblock upwind Euler code has been developed [10] and this has recently been coupled to a modal structural analysis to provide an aeroelastic solver. The aeroelastic solver has recently been validated [11, 12]. The long-term aim of the research presented here is the direct comparison of wing optimisations from both structurally rigid and aeroelastic environments. The objective of the optimisation will be aerodynamic in both cases, however, with the aeroelastic solver accounting for static aeroelastic deflection will give the optimisation increased accuracy and real world relevance.

2 Domain Element Parameterisation

Finite-differences are used here to compute design variable sensitivities, since a 'wrap-around' framework has been developed. Hence, the choice of parameterisation method is absolutely critical in terms of the computational cost of any optimisation, and an efficient method, i.e. as few design variables as possible, is essential. However, the method must still allow sufficient free-form design such that any likely optimum design that may exist is achievable.

Numerous parameterisation methods have been presented for CFD shape optimisation, and these can be split into those that parameterise the design geometry from which a mesh is generated, or those that parameterise the aerodynamic mesh itself. Geometry parameterisation methods are inherently linked with the grid generation package, and optimisation requires automatic grid generation tools. Methods of this nature include partial differential equation methods (PDE)[13, 14], polynomial or spline[15], CAD and recently CST [16, 17] methods. Grid parameterisation methods are generally independent of the grid generation package. This requires a mesh deformation algorithm, but allows the use of previously generated grids for optimisation. Methods of this nature include discrete[18, 19, 20], analytical, basis vector[21], free form deformation (FFD)[22], and domain element methods[23]. The reader is referred to [24, 25] for comprehensive reviews of parameterisation methods.

The novel parameterisation method developed and applied in this research links all the aerodynamic mesh points to a domain element that controls the shape of the design surface. A multivariate interpolation has been developed using radial basis functions, and this provides a unique mapping between the domain element, the surface geometry and the locations of the volume mesh points. All points are treated as point clouds, so the parameterisation technique is totally independent from the grid type and generation package. The mapping is only required once for the initial design as the values of the parametric coordinates of the grid points with respect to the domain element remain constant throughout the optimisation. Updates to the geometry and the corresponding mesh are provided simultaneously by application of the multivariate interpolation; this is extremely fast and efficient and results in very high quality mesh deformation [11, 12].

Figure 1 depicts the domain element parameterisation of the MDO wing considered in this paper. The method is an extension of that presented in Morris *et al* [2] for two-dimensional aerofoil, such that the three-dimensional element consists of an evenly distributed series of two-dimensional slices located according to local surface geometry. A hierarchy of intuitive shape deformation design variables have been developed, and three levels of design variables have so far been established. At the global level, design variables correspond to motions of all domain element nodes simultaneously, for example altering wing angle of attack, sweep, or dihedral.

At the intermediate level design variables control the twist, chord and thickness of each two-dimensional domain element slice separately, and at the local level very small groups or individual domain element nodes are altered to provide detailed and local shape changes.

An example deformation to a change in a design variable is also depicted in Figure1 (in this case, the eighth domain element slice is perturbed in the local twist design variable). Selected planes of the volume mesh for the MDO wing and the perturbation are also shown. The interpolation method is seen to provide a smooth surface change, and smooth and very high quality mesh deformation, with grid motion contained within the RBF support radius (see later).

2.1 Parameterisation Formulation

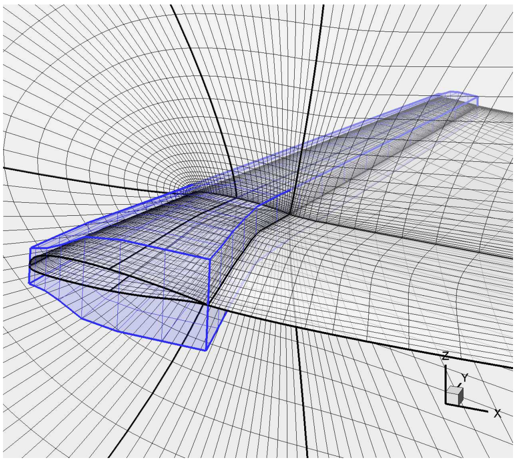
A global interpolation method using radial basis functions (RBFs) has been developed to provide a method of geometry parameterisation. The global dependence between the domain element nodes and the aerodynamic mesh points transfers a deformation of the element, due to a design variable change, to smoothly alter the aerodynamic shape and its corresponding CFD volume mesh. Using this method, only an initial mesh of the original design is required to allow optimisation. The interpolation method developed here requires no connectivity information, and can therefore be applied equally well to either structured and unstructured grid topologies. Domain element points and volume mesh points are simply treated as independent point clouds with the dependence matrix computed only once.

The general theory of RBFs is presented by Buhmann[26] and Wendland[27], and the method used here is comprehensively detailed in Allen and Rendall[11, 12]. It is sufficient here to state that the positions of the aerodynamic mesh points, given by the vectors $\mathbf{X}_a$, $\mathbf{Y}_a$ and $\mathbf{Z}_a$, can be directly computed by:

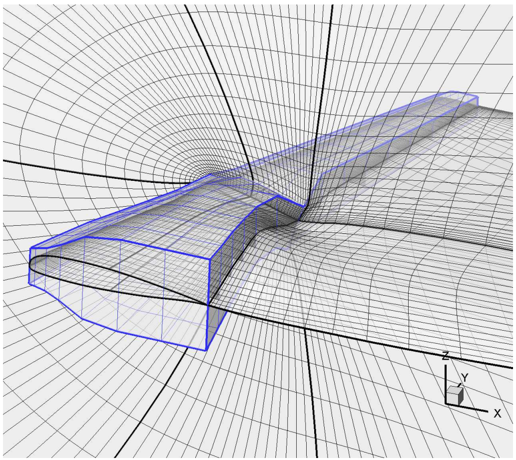
$$\mathbf{X}_a = \mathbf{A}\,\mathbf{a}_x = \mathbf{A}\,\mathbf{C}^{-1}\mathbf{X}_{DE} = \mathbf{H}\mathbf{X}_{DE} \tag{1}$$

$$\mathbf{Y}_a = \mathbf{A}\,\mathbf{a}_y = \mathbf{A}\,\mathbf{C}^{-1}\mathbf{Y}_{DE} = \mathbf{H}\mathbf{Y}_{DE} \tag{2}$$

$$\mathbf{Z}_a = \mathbf{A}\,\mathbf{a}_z = \mathbf{A}\,\mathbf{C}^{-1}\mathbf{Z}_{DE} = \mathbf{H}\mathbf{Z}_{DE} \tag{3}$$



(a) MDO domain element and initial mesh planes



(b) Perturbed MDO domain element and mesh planes

Fig. 1 MDO parameterisation and RBF mesh deformation

where $\mathbf{X}_{DE}$, $\mathbf{Y}_{DE}$ and $\mathbf{Z}_{DE}$ are the vectors of domain element positions, $\mathbf{A}$ and $\mathbf{C}$ are matrices of basis functions, $\mathbf{a}_{x/y/z}$ are the vectors of coefficients, and $\mathbf{H}$ is the time-invariant dependence matrix.

Features of the RBF interpolation developed here worth noting are:

- The interpolation is independent of mesh type or structure
- The interpolation is unique.
- The interpolation in all three coordinate directions is independent.

- The interpolation is time-invariant, and so only needs to be computed once.
- The interpolation is perfectly parallel.

3 Optimisation Method

When considering practical optimisation of aerodynamic performance of a solid body, there are usually constraints which need to be imposed (minimum thickness, minimum volume, minimum lift, maximum moment etc), and constrained gradient-based optimisers are fast and efficient at providing solutions to local optimisation problems [28, 5]. Unconstrained optimisations can incorporate constraints by using a penalty function for designs that are near or beyond the constraint boundary, but these methods are now considered inefficient and have been replaced by methods that focus on the solution of the Kuhn-Tucker equations.

The solution of these equations forms the basis of the nonlinear programming algorithm. The constrained quasi-Newton method guarantees superlinear convergence by accumulating second-order information relating to the Kuhn-Tucker equations using a quasi-Newton updating procedure, i.e. at each major iteration, an approximation is made of the Hessian of the Lagrangian function. This is then used to generate a quadratic programming (QP) subproblem where the solution is used to form a search direction for a line search procedure. This forms the basis of the sequential quadratic programming (SQP) algorithm. Schittkowski [29] has implemented and tested an SQP algorithm that outperforms all other tested methods in terms of efficiency, accuracy, and percentage of successful solutions, over a large number of test problems. The feasible sequential programming (FSQP) algorithm used in the current research was originally developed by Zhou, Tits, Lawrence, and Panier [3, 4, 5]. The feasibility aspect of the optimiser relates to a generated design satisfying all constraints, i.e. if an initial design does not satisfy the specified constraints, the optimiser first achieves a satisfactory design, and then all subsequent iterates generated also satisfy all constraints simultaneously. This particular algorithm has been implemented across a wide range of optimisation problems, most relevant and notable is the work of Qin and Le Moigne [30, 31, 32, 33] where the algorithm is used for CFD constrained optimisation of a blended wing body using an inviscid adjoint solver to obtain the sensitivities.

The development of generic optimisation tools encompassing a wide range of applicability has been the principle aim of the current research. This has ultimately required the use of a finite-difference technique for evaluating gradients to enable independence from the flow-solver; the sensitivity for each design variable is easily obtained by the relative change in the value of steady-state objective function due to a geometric perturbation. To ensure no biasing towards one direction, and to increase accuracy, a second-order accurate finite-difference stencil is used.

4 3D Aerodynamic Optimisation

The MDO wing corresponds to a typical, traditional design of a large modern transport aircraft wing, with its primary design point being that of transonic cruise flight efficiency. At this design point the objective of optimisation is minimum drag, but this must be achieved without detriment to other aerodynamic, structural and geometric quantities. Hence, four constraints are imposed on each optimisation;

[1] Total lift $\geq$ Total lift of initial wing.

[2] Internal volume $\geq$ Internal volume of initial MDO wing.

[3] Root bending moment $\leq$ root bending moment of the initial MDO wing.

[4] Root torsion moment $\leq$ root torsion moment of the initial MDO wing.

These constraints ensure that the results of any optimisation represent practical solutions, and that any improvements achieved can be attributed solely to improvements in geometric design.

The economical cruise flight Mach number for the MDO wing defined by Allwright [6, 7] is 0.85, with the wing trimmed to obtain a lift coefficient of 0.452. This design case is well suited to inviscid flow analysis by solution of the Euler equations, since induced and wave drag form a major part of the total drag. Furthermore, two-dimensional aerofoil optimisations have shown previously that the improvements achieved through inviscid optimisations in transonic Mach numbers are also realised in viscous analysis [1, 2]. The grid used in each optimisation is a 330,000 point structured multiblock mesh [34]. Flow solutions are provided by an inviscid, structured multiblock finite volume upwind solver [10, 35, 36, 37] using the flux vector splitting of van Leer[38, 39], and incorporating multigrid acceleration [42].

4.1 Results

The optimisation was run with the following design parameters:

1. **Global;** Only two issues are considered, anhedral/dihedral, and sweep. There are 15 domain element slices, and the root section is fixed in position and, hence, there are 14 sweep design variables, and 14 anhedral variables. 28 parameters.
2. **Intermediate;** Each domain element slice has two local design variables, the (x,z) location of the centre of rotation. 30 parameters.
3. **Detailed;** Each two-dimensional domain element slice also has the full set of 22 active design variables developed for free-form aerofoil design[1, 2]. 330 paramters.

The three levels of parameters above are combined to gives a total of 388 active design variables. It should be noted that if level 1 parameterisation was used alone, angle of attack and a twist parameter would need to be added, and level 2 parameterisation would also include angle of attack and chord and thickness for each domain element, but these are already included in the 22 parameters per section in level 3, so are not required when combined.

The results of this optimisation are given in Table 1. Fully free-form control of aerofoil profile geometries is combined with design variables that enable truly three-dimensional planform alterations, and this achieves a reduction in drag coefficient of over 18%. This is a significant reduction and may be the global minimum for such an optimisation problem (discussed in detail later).

Figure 2 shows the optimisation history of objective function and constraints, showing only 30 evolutions are required. Furthermore, this could be halted even after only a few evolutions and significant improvements would still have been obtained; even after only 15 evolutions, a 15% drag reduction would still have been achieved. Optimisation has had the beneficial effect of significantly reducing both root bending and root torsion moments.

	Initial	Optimised	%Diff
Cl	0.4523	0.4530	+0.14
$Cm_{bending}$	0.1340	0.1004	-25.03
$Cm_{torsion}$	-0.0547	-0.0471	-13.93
Volume	387.14	401.60	+3.73
Cd	0.02780	0.02287	**-18.29**

Table 1. Wing optimisation results

Initial and optimised domain element and wing geometries are depicted in Figure 3. The most notable change to the optimised wing is to the sweep distribution; not only has sweep been increased significantly, but the leading edge is no longer straight such that there is increased sweep angle towards the tip. This is surprising; root torsion moment is rigidly constrained and an increased sweep angle normally impacts negatively on this. Observation of Figure 6 demonstrates that loading has moved significantly inboard such that sweep angle can be increased in an effort to reduce drag with no penalty to root torsion moment. In fact both root torsion and

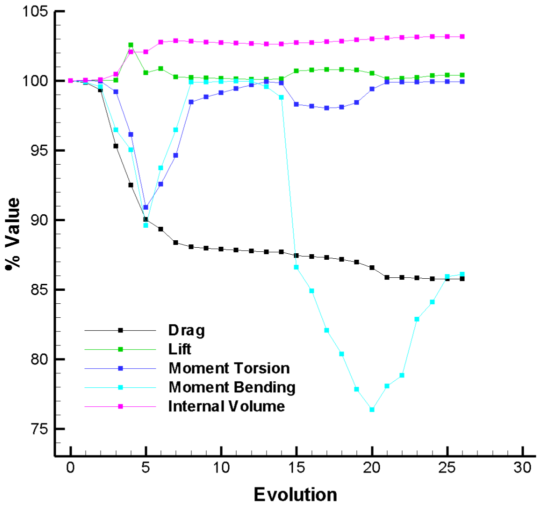


Fig. 2 Optimisation history

root bending moments have been reduced significantly when compared to the initial MDO wing. Although cruise flight is not usually the determining case for structural wing box design, reduced root aerodynamic moments could provide possible weight savings. Figure 6 also shows that drag has increased slightly inboard due to the increased loading there, but at all outboard locations the drag is significantly reduced.

Figure 4 depicts views of CFD surface and volume meshes corresponding to initial and optimised wing geometries. Even though the deformation is dramatic, the parameterisation method maintains a high quality of CFD mesh.

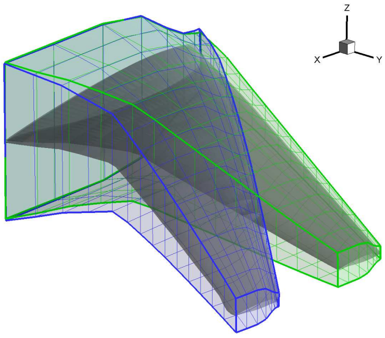


Fig. 3 Domain element and wing geometries (initial MDO-green, optimised-blue)

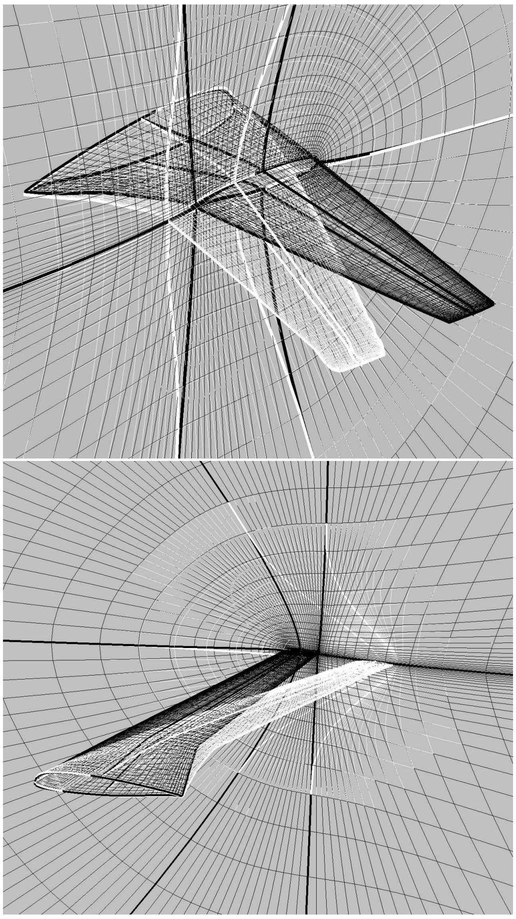
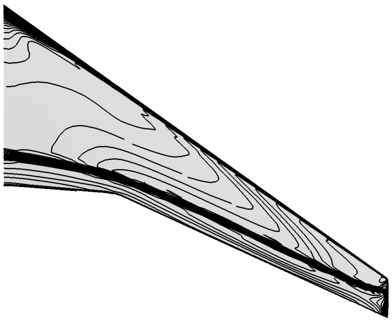


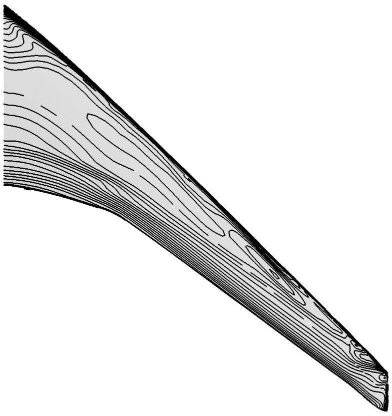
Fig. 4 CFD volume mesh deformation. Initial MDO mesh-black, optimised mesh-white

Figures 5 depict contours of coefficient of pressure (Cp) for the initial MDO and optimised wing geometries, showing the large change in flow structure. The MDO wing in its economical cruise flight exhibits a strong shock along the entire length of the span. The free-form design control allowable by the parameterisation developed here achieves a reduction in drag of over 18% and results in a completely shock-free wing. This is a considerable result considering the constraint on a high value of lift and at a high transonic Mach number. Given the inviscid nature of the optimisation this may be a truly optimum geometric design given the rigid constraints.

Sectional slices through the transformed wing are shown in Figures 7 and compared to the initial MDO wing geometry. Significant aerofoil section changes (and twist distribution) are clearly

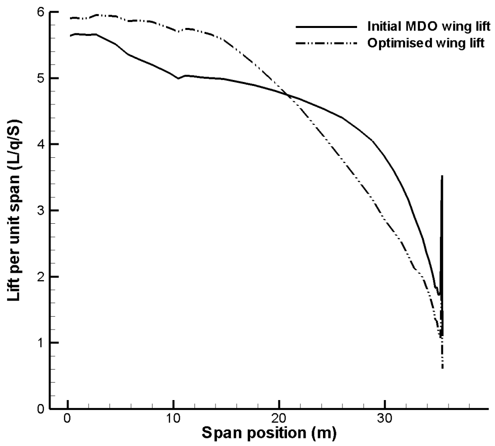


(a) Initial MDO

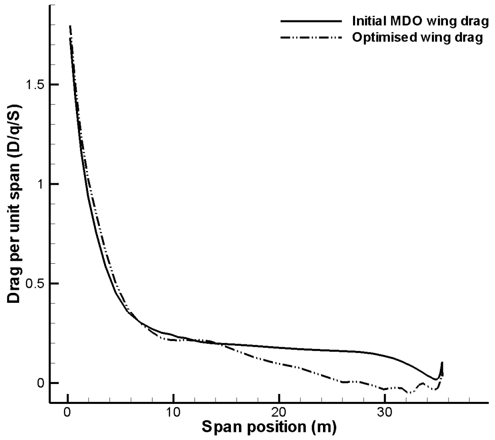


(b) Optimisied

Fig. 5 Cp distributions



(a) Lift



(b) Drag

Fig. 6 Spanwise distributions

seen. Root incidence is increased, but with a larger wash-out, highlighting that inboard sections are more highly loaded with relief towards the tip.

5 3D Aeroelastic Optimisation

The CFD solver is a structured multiblock finite volume upwind unsteady code[37, 42] with the flux vector splitting of Van Leer[38, 39] and the implicit pseudo-time stepping scheme of Jameson[41, 40]. The flow solver has been parallelised and shown to have very good scaling properties[37]. A modal structural solution is used, and weak coupling adopted between fluid and structure each time step since exact synchronisation of fluid and structural models is unimportant for a steady result. Use of an approximately critical amount of damping ensures that the wing converges relatively quickly to its deformed equilibrium position.

The structural grid consists of only the upper and lower portions of the wing box, so there is a significant gap between the fluid and structural grids, necessitating a robust and accurate interpolation between the two. This interpolation is dealt with using radial basis functions and is formulated to conserve total force, total moment and energy between the two grids[11].

Typical deformed shapes of the MDO wing are shown in fig8 for Brite-Euram case III. These are compared to previously published results to confirm the accuracy of the solver. Details of the solver can be found in Rendall [11].

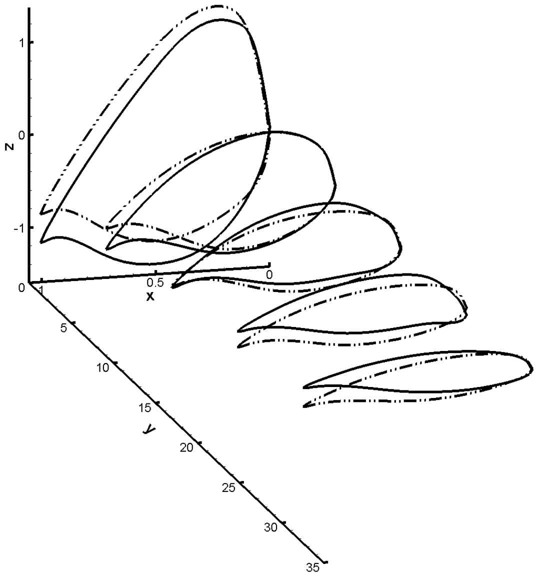
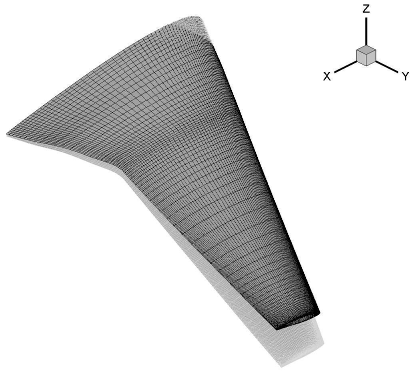


Fig. 7 Aerofoil profiles. Initial (dashed) and Optimised (solid).

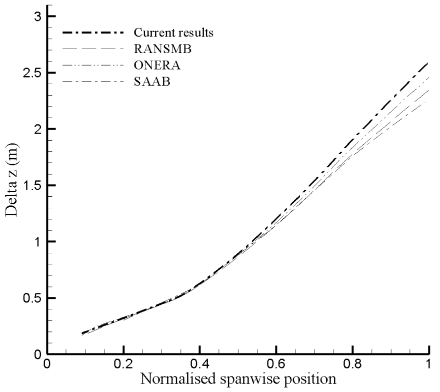
The aeroelastic optimisation of the MDO wing, again in transonic cruise flight, using the same parameterisation and active design variables as for the purely aerodynamic optimisation of the previous section is currently underway. The optimisation procedure is simply ’wrapping-around’ the aeroelastic solver instead of the rigid Euler solver. In this way both the optimiser has knowledge of both the objective function under aeroelastic conditions, and improves the geometry based on accurate knowledge of the aeroelastic sensitivities.

6 Conclusions

Generic ’wrap-around’ aerodynamic optimisation tools have been developed and applied here to three-dimensional wing optimisation. This comprises a new geometric parameterisation technique for application to CFD-based aerodynamic optimisation. The parameterisation uses radial basis functions to interpolate positions of the domain element and the grid coordinates to provide simultaneous deformation of the design surface and its corresponding aerodynamic mesh.



(a) Comparison between MDO jig-shape and aeroelastic flight-shape



(b) Trailing edge deflection comparison

Fig. 8 Aeroelastic Solver

The interpolation and updates to the geometry and its associated CFD mesh are of extreme high quality, robust, fast and efficient. This domain element technique is mesh topology and mesh generation package independent, requiring only an initial mesh.

The parameterisation technique allows the combination of variables of different scales and types with only a few parameterisation nodes, and this leads to a significantly reduced number of design variables for three-dimensional applications when compared to many other types of shape parameterisation method. Derivatives of these design parameters are computed via second-order finite-differences and fed into a feasible sequential quadratic programming (FSQP) gradient-based optimiser.

The optimisation tools have been applied to aerodynamic shape optimisation of the MDO wing in transonic cruise, with the objective function being drag, subject to strict constraints. 388 combined global and local parameters were used, and this results in a totally shock-free wing with over 18% reduction in inviscid drag, combined with significantly reduced root aerodynamic moments.

An aeroelastic flow solver has recently been developed, and the aeroelastic optimisation of the same MDO wing in cruise flight condition is currently underway. The objective of the optimisation is again aerodynamic, however, with the aeroelastic solver accounting for static aeroelastic deflection will give the optimisation increased accuracy and real world relevance.

It should be noted that although ’in-house’ CFD grid generation and flow solver codes are used here, the method is completely generic and can be wrapped around any appropriate tools. Furthermore, although an external aerodynamic design problem is presented, this is not a restriction, and the methods can be applied to any steady-state fluid dynamic design problem.

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