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A SYSTEMATIC PROCEDURE FOR DETERMINING SEPARATION MINIMA

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Abstract

This paper presents a build-up approach to determining appropriate separation minima. In this approach, the influences of relevant factors such as aircraft wake vortices, surveillance errors, and flight technical errors on separation requirements are analyzed and their combined effects are derived. Semi-analytical expressions are obtained for the rapid and convenient analysis of separation requirements. Applications of the proposed procedure are illustrated. The proposed procedure presents a systematic and coherent methodology for the appropriate definitions and analyses of safe reductions of separation minima.

1. Introduction

Aircraft must maintain adequate separations in order to ensure safety. On the other hand, excessive separation requirements can overly constrain the traffic flow and cause unnecessary delays in the air traffic system. The Next Generation Air Transportation System (NextGen) may accommodate up to three times the current traffic demand by the year 2025 [1]. The safe reduction of separation standards should therefore be carefully examined [2].

The present separation standards were established several decades ago based on much older technologies. Worse yet, the basis of these separation standards was not well documented. It appears to have been based upon “radar accuracy, display target size, and controller and pilot confidence” [3]. Except for oceanic separation standards, separation standards for parallel runway operations, and reduced vertical separations [4], domestic separation standards have by and large been unchanged.

Ever since the 1960’s, researchers have sought to establish theoretical foundations for separation minima [5,6]. In 1977, Holt & Marner discussed elements that might influence separation requirements [7]. More recently in 2000, Reynolds & Hansman identified factors involved in defining aircraft separation standards and discussed the importance of accurate state information for controllers in maintaining separation standards [8]. In Ennis & Zhao [9], an integrated procedure was proposed for a formal analysis of separation requirements. Also related are collision risk models that calculate probabilities of aircraft collisions per unit of a certain activity under specified conditions [10], though these models may not directly determine separation minima.

Wake vortex is an essential ingredient to the required separation between two aircraft. Efforts to investigate the characteristics of wake vortices are exemplified by works in [11-13]. Also, works in [14-16] explore effective methods to reduce separation requirements caused by the presence of wake vortices.

For a given aircraft, the effective danger region of its wake vortices also depends on the performance capability of a trailing or crossing aircraft. Works in [17-21] present methods for assessing the risks in flying through a wake vortex and providing estimates of the effective vortex region behind an aircraft that should be avoided by other aircraft.

In addition, considerable research has been conducted on conflict detection and resolution strategies with specified separation standards

[22]. In particular, practical algorithms have been developed for the detection and resolution of aircraft conflicts in the Center/TRACON Automation System (CTAS) [23-25], and for the detection and avoidance of close encounter collisions in the on-board Traffic Alert and Collision Avoidance System (TCAS) [26-27].

Despite these advances, a systematic and transparent process for the determination of appropriate separation minima is still lacking.

In this paper, a protected zone is defined as a region around a given aircraft that, if violated, the safety of either the given aircraft, or an intruding aircraft, or both, could be compromised. The protected zone defines minimum separation standards among aircraft.

For example, in the current air traffic control (ATC) system, the protected zone is a cylindrical shaped region centered on the aircraft that varies in size in different flight phases and/or altitudes. For en route flight, the cylinder has a radius of 2.5 nm and a height of 1,000 ft or 2,000 ft depending on the altitude.

2. Problem Statement

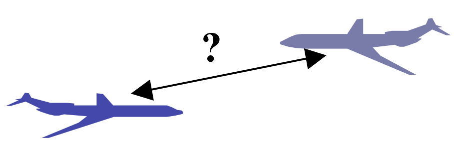

Fig. 1 Aircraft Separation Requirement

The fundamental question in this paper is what is the minimum separation between two aircraft that can prevent them from inadvertently collide with one another under normal operations?

This paper seeks to establish a systematic and transparent methodology for determining the minimum safe separation requirement between a pair of aircraft under specified conditions. In doing so, it employs a step-by-step build-up approach that properly integrates the effects of key relevant factors in a conflict detection and avoidance process. It extends the results of Ennis & Zhao [9] by refining the concepts and the integration of these factors. In addition, this paper presents some semi-analytical expressions for rapid solutions.

3. Basic Protected Zone

Fig. 2 Basic Protected Zone: a 3D View

If the position of an aircraft can be precisely known from now to some future time, a basic protected zone can be defined that would consist of just two regions (Fig. 2): a safety buffer and the trailing vortex. The safety buffer is needed around the aircraft body that no other aircraft should ever penetrate. In comparison, the trailing vortices originating from the wing tips of the aircraft create dangerous rolling moments, which could potentially overpower the roll control of a following aircraft, or cause vertical turbulence powerful enough to risk the safety of passengers or even cause damage to the following or a cross trail aircraft.

In an emergency situation such as a close encounter when maneuver options are limited, the basic protected zone may also be used as a separation requirement, such as for a collision avoidance system like TCAS.

Strictly speaking, the violation of the wake vortex region does not pose any danger to the generating aircraft. It should be avoided by a trailing or crossing aircraft. As a result in Fig. 2, the safety buffer is encircled with solid lines, whereas the wake vortex is represented by a shaded area.

Furthermore, the risk level of a given wake vortex not only depends on the flight speed, weight, and attitude of the generating aircraft; it also depends on the performance capabilities of the trailing/crossing aircraft, as well as the encounter geometry and ambient conditions. As a result, the effective shape and size of the vortex range depends on all of these factors. Determining the shapes and sizes of a generic trailing vortex region requires the proper assessment of risks, as discussed in [17-21]. Particularly, a simplified hazard area prediction (SHAPE) model is proposed [18] in which rectangular cross sections are used to model the wake vortex region.

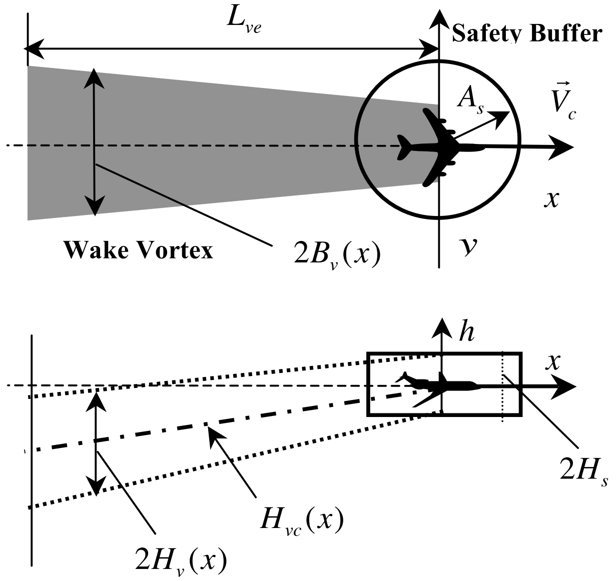

Fig. 3 Basic Protected Zone: Top and Side View

In this paper, a structure of the basic protected zone is defined as follows. At some initial time, a target aircraft is located at $[x_c, y_c, h_c]$and is flying with the velocity of $\vec{V}_c$. The safety buffer region is represented by a cylinder. The vortex region is assumed to have rectangular cross-sections as proposed in the SHAPE model.

While the simplified structure of the basic protected zone is assumed for the convenience of discussions, the build-up approach in this paper would be applicable to general shapes.

4. Protected Zone in Normal Operation

During normal operations, the definition of a protected zone must to take into consideration uncertainties in both current and future aircraft positions. Particularly, errors of surveillance, intent estimation, and onboard flight systems must all be considered.

4.1 Nominal Predicted Trajectory

At some time t_0, the surveillance system produces an estimate of the most likely current position and velocity of a target aircraft.

$$x_{n0}, y_{n0}, h_{n0}, V_{n0}, \Psi_{n0}, \dot{h}_{n0} \tag{1}$$

Based on filed flight plans or other sources of information, an estimate of the flight intent can also be made in terms of velocity components.

$$\dot{V}_n(t), \dot{\Psi}_n(t), \ddot{h}_n(t) \tag{2}$$

Finally, measurements of the wind field are obtained as

$$W_{x,n}(t), W_{y,n}(t), W_{h,n}(t) \tag{3}$$

If any piece of the above information is missing, some nominal value, such as zero, may be used.

Based on the above information, a likely future trajectory of the target aircraft over some time interval $[t_0, t_0 + T_n]$, called a nominal predicted trajectory, can be constructed. Symbolically, this nominal predicted trajectory is expressed as,

$$x_n(t), y_n(t), h_n(t), V_n(t), \Psi_n(t), \dot{h}_n(t) \tag{4}$$

In the current air traffic control system, controllers build this trajectory mentally. Decision Support Tools (DSTs) employ trajectory predictors to generate this trajectory.

For example, assuming constant velocity, constant wind, and small flight path angles, a nominal predicted trajectory is given by

$$x_n(t) \approx x_{n0} + (V_{n0} \sin \Psi_{n0} + W_{x,n0})(t - t_0) \tag{5a}$$
$$y_n(t) \approx y_{n0} + (V_{n0} \cos \Psi_{n0} + W_{y,n0})(t - t_0) \tag{5b}$$
$$h_n(t) = h_{n0} + \dot{h}_{n0}(t - t_0) \tag{5c}$$

$$V_n(t) = V_{n0}, \Psi_n(t) = \Psi_{n0} \quad (5d)$$

$$\gamma_n(t) = \frac{\dot{h}_{n0} - W_{h,n0}}{V_{n0}} \quad (5e)$$

4.2 Commanded Trajectory

In actual flights, the aircraft is tracking a certain commanded trajectory that implements some velocity at the current time and some acceleration vector in the near future.

$$V_{c0}, \Psi_{c0}, \dot{h}_{c0}; \dot{V}_c(t), \dot{\Psi}_c(t), \ddot{h}_c(t) \quad (6)$$

If the **actual** current position of the aircraft is

$$x_{c0}, y_{c0}, h_{c0} \quad (7)$$

and the onboard knowledge of the wind field is

$$W_{x,c}(t), W_{y,c}(t), W_{h,c}(t) \quad (8)$$

the commanded trajectory over $[t_0, t_0 + T_n]$ can also be constructed. Symbolically,

$$x_c(t), y_c(t), h_c(t), V_c(t), \Psi_c(t), \dot{h}_c(t) \quad (9)$$

Again for constant commanded velocity, constant wind components, and small flight path angles, the commanded trajectory is given by

$$x_c(t) \approx x_{c0} + (V_{c0} \sin\Psi_{c0} + W_{x,c0})(t - t_0) \quad (10a)$$
$$y_c(t) \approx y_{c0} + (V_{c0} \cos\Psi_{c0} + W_{y,c0})(t - t_0) \quad (10b)$$
$$h_c(t) = h_{c0} + \dot{h}_{c0}(t - t_0) \quad (10c)$$
$$V_c(t) = V_{c0}, \Psi_c(t) = \Psi_{c0} \quad (10d)$$
$$\gamma_c(t) = \frac{\dot{h}_{c0} - W_{h,c0}}{V_{c0}} \quad (10e)$$

4.3 Surveillance and Flight Technical Errors

Differences between actual current positions and estimated likely current positions reflect surveillance errors

$$x_{c0} = x_{n0} + \Delta x_{S0} \quad (11a)$$
$$y_{c0} = y_{n0} + \Delta y_{S0} \quad (11b)$$
$$h_{c0} = h_{n0} + \Delta h_{S0} \quad (11c)$$

Differences between currently commanded velocity and the estimated current velocity reflect the combination of both surveillance errors and flight technical errors.

$$V_{c0} = V_{n0} + \Delta V_{S0} + \Delta V_{FTE,0} \quad (12a)$$
$$\Psi_{c0} = \Psi_{n0} + \Delta\Psi_{S0} + \Delta\Psi_{FTE,0} \quad (12b)$$
$$\dot{h}_{c0} = \dot{h}_{n0} + \Delta\dot{h}_{S0} + \Delta\dot{h}_{FTE,0} \quad (12c)$$

4.4 Intent Errors

Finally, differences between estimated intents and actual intents reflect intent errors.

$$\dot{V}_c(t) = \dot{V}_n(t) + \Delta\dot{V}_I(t) \quad (13a)$$
$$\dot{\Psi}_c(t) = \dot{\Psi}_n(t) + \Delta\dot{\Psi}_I(t) \quad (13b)$$
$$\ddot{h}_c(t) = \ddot{h}_n(t) + \Delta\ddot{h}_I(t) \quad (13c)$$

In general, flight intents may be expressed in different forms. For short-term trajectory predictions, acceleration intents are convenient.

Correct determinations of aircraft's flight intents are crucial to accurately predicting its future flight trajectories. In the current ATC system, intents are typically known through pre-filed flight plans and the mandatory compliance with controller commands. Sometimes, a situation could arise where the intents are unclear.

There can be two approaches to handling intent errors in defining protected zones: incorporating intent errors directly into protected zones or considering intent errors through conflict resolution strategies.

Incorporating intent errors into the construction of a protected zone can drastically increase its dimensions, since the lack of knowledge of intents greatly magnifies the state-uncertainty region defined below. For the efficiency of airspace operations, protected zones in normal operation conditions should be defined under most likely conditions. In other words, the acceleration intent errors should be assumed reasonably small in defining protected zones.

4.5 Actual Trajectory

Actual trajectories result when pilots or autopilots seek to follow commanded trajectories in the presence of modeling uncertainties and disturbances. Actual trajectories can be represented by

$$x(t), y(t), h(t), V(t), \Psi(t), \dot{h}(t) \tag{14}$$

Differences between actual and commanded trajectories constitute flight technical errors (FTE). In general, FTEs depend on

- Navigation errors
- Characteristics of pilot feedback control
- Aircraft performance
- Command dynamics
- Flight environment

FTEs may be estimated via Monte Carlo simulations [9].

4.6 State Uncertainties

In order for air traffic controllers and/or other pilots to maintain sufficient inter-aircraft separations, they need to identify a region in which a target aircraft is most likely to be, say with a specified probability, over a certain time interval $[t_0, t_0 + T_n]$. This region is called a ***state-uncertainty region*** for the given aircraft.

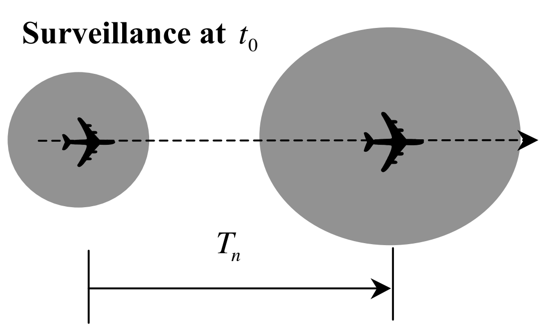

Fig. 4 State Uncertainty Region

The state uncertainty region is caused by the differences between actual and nominal predicted trajectories. Fig. 4 illustrates the concept.

Mathematically, the state uncertainty region can be bounded in three dimensions by

$$a = \max_{[t_0, t_0+T_n]} |x(t) - x_n(t)| \tag{15a}$$
$$b = \max_{[t_0, t_0+T_n]} |y(t) - y_n(t)| \tag{15b}$$
$$c = \max_{[t_0, t_0+T_n]} |h(t) - h_n(t)| \tag{15c}$$

which may be estimated from

$$|x - x_n| \le |x - x_c| + |x_c - x_n| \tag{16a}$$
$$|y - y_n| \le |y - y_c| + |y_c - y_n| \tag{16b}$$
$$|h - h_n| \le |h - h_c| + |h_c - h_n| \tag{16c}$$

4.7 Length of the Nominal Interval T_n

Whenever a new surveillance measurement becomes available, a different nominal predicted trajectory could be defined. On the other hand, the protected zone is fundamentally defined to prevent aircraft from colliding with each other. The response time of pilots in avoiding each other must be considered.

When another aircraft is trying to avoid a target aircraft, it may not be able to change its maneuver plans easily once it initiates the maneuver. Therefore, it would need a stable estimate of the protected zone for the target aircraft during the course of an avoidance maneuver.

Therefore, an appropriate choice of the nominal interval length should be the larger one of the surveillance interval T_S and a typical maneuver time T_M needed for conflict avoidance.

$$T_n = \max\{T_S, T_M\} \tag{17}$$

According to the FAA [28], the pilot response time in case of an emergency avoidance is on the order of $T_R = 12.5$ sec. This is supported by a human operator model by Hess [29]. A typical maneuver time can therefore be estimated as

$T_M = (2 \sim 3)\, T_R \approx 40$ sec. This time frame is consistent with the alert time used in TCAS design [26,27].

Currently, en-route radar system updates every 12 sec so $T_n = T_M$ should be used. For oceanic flights, on the other hand, $T_S > T_M$ and the surveillance interval T_S should be used for estimating the state-uncertainty region.

4.8 Estimate of Protected Zone

Because every point inside the state-uncertainty region represents a likely aircraft position in actual flight, the basic protected zone in Fig. 2 must be placed at each point of the state-uncertainty region; resulting in a protected zone in normal operations, as shown in Fig. 5.

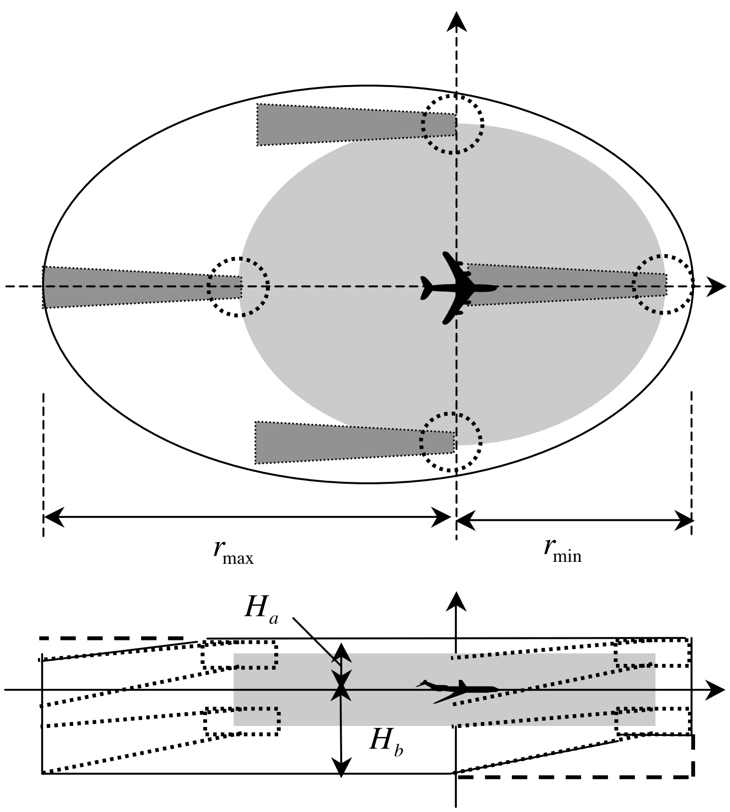

Fig. 5 Protected Zone: Top and Side View

4.9 A Special Case Solution

Assuming constant flight velocity, constant wind components, and negligible acceleration intent errors, appropriate bounds on the along-track, cross-track, and vertical dimensions of the state uncertainty region can be obtained as

$$a = \left(\Delta\xi_{FTE} + \Delta\xi_{S0} + [\Delta V_{S0} + \Delta V_I + \Delta W_t]\tau\right)_{\max}$$
$$b = \left(\Delta\eta_{FTE} + \Delta\eta_{S0} - [V_{n0}(\Delta\Psi_{S0} + \Delta\Psi_I) - \Delta W_c]\tau\right)_{\max}$$
$$c = \left(\Delta h_{FTE} + \Delta h_{S0} + [\Delta\dot{h}_{S0} + \Delta\dot{h}_I]\tau\right)_{\max} \quad (18a,b,c)$$

for $\tau \in [0, T_n]$. Details of the solution process are omitted.

5. Approximate Shapes

For the convenience of practical applications, the above estimated protected zone may be approximated with regular shapes. In the horizontal plane, it may be approximated by an elliptical shape with the aircraft in the focal point where,

$$r_{\min} = a + A_s \quad (19a)$$
$$r_{\max} = a + L_{ve} \quad (19b)$$

The semi-major, the semi-minor axis, and the eccentricity of the horizontal ellipse are respectively,

$$a_e = a + \frac{L_{ve} + A_s}{2} \quad (20a)$$
$$b_e = \sqrt{(a + L_{ve})(a + A_s)} \quad (20b)$$
$$e = \frac{L_{ve} - A_s}{2a + L_{ve} + A_s} \quad (20c)$$

In the vertical plane, it may be approximated by a rectangular shape with

$$H_a = c + H_s \quad (21a)$$
$$H_b = c + H_s + H_v(L_e) + H_{vc}(L_e) \quad (21b)$$

6. Examples

Four numerical examples are now provided to illustrate the impacts of various factors in shaping the protected zones. In these examples, it is assumed $A_s = 600\ ft$, $\Delta\xi_{FTE} = 1{,}000\ ft$,

$\Delta V_I = 0, \Delta W_t = 10\ kts, T_n = 40\ s$, and $\Delta V_{S0} = 15\ kts$.

Case 1: $L_{ve} = 2\,nm, \Delta\xi_{S0} = 1\,nm$

$$r_{\min} = 1.541\,nm,\ r_{\max} = 3.442\ nm \tag{22}$$

Case 2: $L_{ve} = 2\,nm, \Delta\xi_{S0} = 0.3\,nm$

$$r_{\min} = 0.841\,nm,\ r_{\max} = 2.742\ nm \tag{23}$$

Case 3: $L_{ve} = 2\,nm, \Delta\xi_{S0} = 10\,m$

$$r_{\min} = 0.546\,nm,\ r_{\max} = 2.448\ nm \tag{24}$$

Case 4: $L_{ve} = 1\,nm, \Delta\xi_{S0} = 1\,nm$

$$r_{\min} = 1.541\,nm,\ r_{\max} = 2.442\ nm \tag{25}$$

Case 1 imitates the current en-route situation with radar surveillance. The maximum separation along the major axis is consistent with the current day standards. When the surveillance accuracy is improved in Case 2, which imitates the terminal area, the required separation decreases. When the surveillance accuracy drastically improves in Case 3, resembling the use of GPS, the required separation requirement decreases further but not drastically, because the wake vortex region still dominates. In comparison, when the wake vortex region is reduced as in Case 4 without improving the surveillance accuracy, the required separation decreases too but not too drastically either.

Numbers in these examples are not meant to accurately represent capabilities of the surveillance systems or dimensions of the wake vortex region. Nonetheless, these examples demonstrate that either a significant improvement in the surveillance accuracy or a significant reduction in the wake vortex region alone can only have a limited impact. A synergetic approach that improves all key aspects is more effective in significantly reducing the separation requirement.

Similar studies can be conducted to examine the influences of other factors, such as flight technical errors.

7. Another Separation Concept

Because of constraints on aircraft dynamics and delays of human operator responses, either one or both aircraft must start proper avoidance maneuvers before the minimum separation standards are violated. A required action range represents the smallest required separation in practical flights by which proper corrective actions must be taken in order to maintain the minimum separation. The required action range effectively defines the minimum separations between two aircraft that must be maintained in practical flights [30,31]. Fig. 6 illustrates the two concepts of separation in a potential head-on conflict.

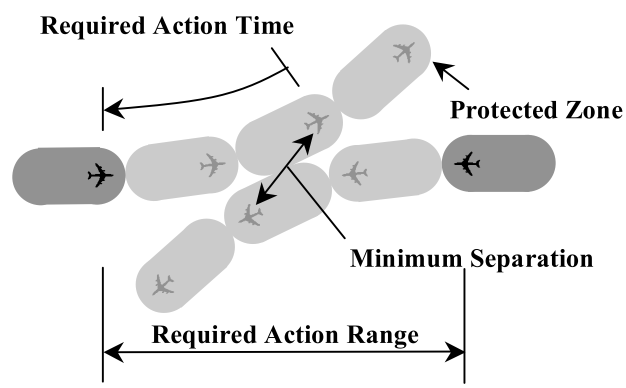

Fig. 6 Minimum and Required Separation

The concepts of protected zone and required action range are not independent. The division is nonetheless convenient. For example, the size and shape of the protected zone does not depend on the relative motion geometry of aircraft involved in a potential conflict, although the influence of the wake vortex does depend on the type of aircraft that follows and the approach geometry. In comparison, the required action range intimately depends on the geometry of relative aircraft motions, methods of conflict resolutions, as well as pilot and/or controller reaction times in normal flights.

Further discussions of the required action range are omitted because of the limitation of space. Some results can be found in [30,31].

8. Discussions

Fig. 5 yields a protected zone that is different from the current cylindrical disk shaped zone. In this newly developed zone, the target aircraft does not lie in the center. Instead, it lies at the focal point.

This is in fact more consistent with actual flights, because of the asymmetry of the protected zone introduced by the wake vortex region. Furthermore, because of the roughly elliptical shape of the protected zone, the required separation in the horizontal plan depends on encounter geometry. When the velocities of involved aircraft are aligned along the same track, a larger separation distance is required. This points to the possibilities of avoidance maneuvers that take advantage of encounter geometries.

Results of this paper further indicate that the protected zone for each aircraft pair can be unique. For example, the effective dimensions of the vortex region depend upon the relative characteristics of the aircraft behind a given aircraft. As a result, a heavy aircraft can follow a light aircraft at a closer distance than a light aircraft can follow a heavy aircraft. Therefore, further reductions of minimum separation requirements may be achieved through individually designed and/or dynamically varying protected zones.

In the current paper, dimensions of the protected zone are expressed deterministically. These dimensions can be interpreted as the worst-case values. The concept may also be studied through a probabilistic approach, where the dimension parameters are stated with certain probabilities.

Much additional work is needed in verifying and validating the procedure presented in this paper. The proper use of the elliptically shaped protected zone in conflict detection and resolution also awaits further investigation.

9. Conclusions

This paper presents a systematic, integrated procedure for determining appropriate separation requirements under specified conditions. It combines the effects of a wide range of uncertainties and error sources in a step-by-step build-up approach. Furthermore, semi-analytical expressions are provided for the rapid and convenient applications of this procedure.

The protected zone is defined as a region around a given aircraft that, it violated, would cause danger to either the own aircraft, or the intruder aircraft, or both. It is related to but different from the required action range, which is the least relative separation between two aircraft at which correct avoidance maneuvers must be initiated to prevent the violation of the protected zone.

A basic protected zone is first defined that consists of the vortex region and a safety buffer region. The basic protected zone represents the minimum separation requirement when the current and future positions of a target aircraft can be precisely known, or in an emergency situation such as a close encounter.

In normal operations, both current and future positions of an aircraft cannot be precisely known due to surveillance, intent, and flight technical errors. There is a state uncertainty region in which every point is a likely aircraft position over a specified time interval. Accordingly, the basic protected zone must be considered at all locations of the state-uncertainty region; resulting in an enlarged protected zone.

A unique feature of the derived protected zone is its approximately elliptical horizontal shape. It indicates that minimum separation requirements depend on the encounter geometry of aircraft. Expressions derived in this paper for the protected zone clearly indicate the effects of various error sources on the minimum separation requirements, and points to the fact that an integrated approach is desirable for significant reductions of separation minima.

NOMENCLATURE

A_s	radius of safety buffer region
B_v	half cross-track width of vortex
h	altitude
H_s	half height of safety buffer
H_v	half height of vortex region
H_{vc}	vertical central line of vortex
L_{ve}	effective length of vortex
V	true airspeed
T_n	nominal prediction time
(W_t, W_c)	(tail, cross) wind
(W_x, W_y, W_h)	wind in (East, North, Up)
(x, y)	aircraft positions in (East, North)
Δ	error term
η	cross-track trajectory deviation
γ	air-relative flight path angle
Ψ	heading clockwise from North
ξ	along-track trajectory deviation

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