RESEARCH ON FAILURE CRITERION FOR COMPOSITE MATERIALS IN OPTIMIZATION

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Abstract

China Aviation Industry is face to seriously challenge and good opportunity in high performance computing fields, such as CFD computing, multi-objective optimization for composite structures under multiple constrains, etc. A remote multi-cluster grid optimization system (RMGOS) is developed in our previous research to solve the time-consuming aircraft design problems and proved to be efficient. On the other hand, the failure criterion for composite materials has its limitation in application for solve the strength problem in engineering design. In this paper, the four classic failure criterions have been discussed with respect to the enveloping range and an Equivalent Strain criterion is suggested as an alternative method. The Finite Element Analysis (FEA) model for the sandwich honeycomb composite structure has been established and optimized under various failure criterions through the grid optimization system. The optimization results have been verified with the experiment results. The results show that the RMGOS can solve the composite structure optimization problem efficiently and the TsaiWu criterion is the best in application.

1 Background

The application of grid technology in high performance computing fields as the tool to solve resource share and effectiveness problem, such as aeronautics, astronautics, meteorology, medicine, etc, has become more and more mature with the development of basic knowledge in grid technology[1,2]. Especially in aircraft structure design, the simulation platform for structure optimization has been developed by integrating Genetic Algorithm (GA)[3] with Grid Technology, which has become one of the new development directions in aircraft structure design. In previous research, the RMGOS has been developed to offer the way to solve composite structure optimization problem with constrained of various different conditions.
In this paper, the difference between various failure criterions for composite material has been analyzed and a new failure criterion based on Equivalent Strain has been suggested. Through the RMGOS, the FEA model for sandwich honeycomb composite structure has been established and optimized for the minimum weight under constrains of stiffness in each failure criterion. The optimization results also have been verified with experiments.

2 Failure Criterions for Composite Structure

2.1 Traditional Failure Criterions

The four traditional failure criterions for composite materials in plane stress state include[4,5]:

2.1.1TsaiHill Criterion

$$\frac{\sigma_1^2}{X^2} - \frac{\sigma_1\sigma_2}{X^2} + \frac{\sigma_2^2}{Y^2} + \frac{\tau_{12}^2}{S^2} = 1 \qquad (1)$$

2.1.2 Hoffman Criterion

$$\frac{\sigma_1^2}{X_t X_c} - \frac{\sigma_1\sigma_2}{X_t X_c} + \frac{\sigma_2^2}{Y_t Y_c} + \frac{X_c - X_t}{X_t X_c}\sigma_1 + \frac{Y_c - Y_t}{Y_t Y_c}\sigma_2 + \frac{\tau_{12}^2}{S^2} = 1 \qquad (2)$$

2.1.3 TsaiWu Criterion

$$F_{11}\sigma_1^2 + 2F_{12}\sigma_1\sigma_2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + F_1\sigma_1 + F_2\sigma_2 = 1 \qquad (3)$$

2.2 Application Range for Failure Criterions

The application range for four failure criterions is shown in Fig 1. The enveloping surface of TsaiHill criterion is fully in the coverage range of maximum stress. The enveloping surface of Hoffman criterion exceeds the coverage range of maximum stress in the third and fourth quadrants. The transverse compression stress in G position of the ellipse is great than F position, however the longitudinal compression stress has made an increase of 38%, which has exceeded the longitudinal compression strength value. And the value in D position is also greater than E position of 28%.

In the third quadrant, the coverage range of TsaiWu criterion is surpassing the other criterions. Especially in B position, the transverse compression stress is greater than C position of 30%, however the longitudinal compression stress is 1325MPa, which exceeds the longitudinal compression strength value. The same, the transverse compression stress in A position is 158MPa, however the longitudinal compression stress is greater than F position of 125%, which is abnormal. Hence, in the third quadrant, the Max-Stress criterion is suggested.

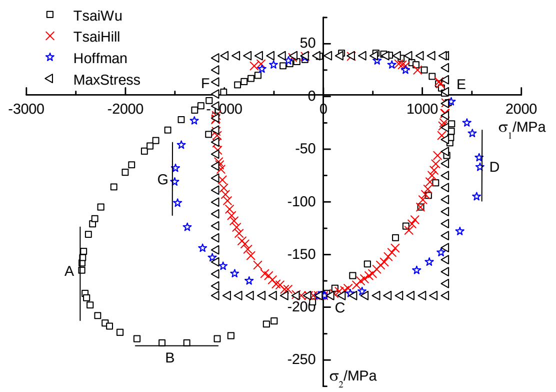

Fig 1 Comparison of enveloping surface for various failure criterions in the state of σ_1-σ_2

In this paper, the T700/ QY8911 has been chosen as the composite materials, and the above curves in Fig 1 are drawn according to the following properties shown in Table 1.

Table 1 The properties of T700/QY8911

Modulus	E_L	E_T	G_{LT}	v	ρ(g/cm³)
/GPa	125	7.2	4.1	0.3	1.6
Strength	X_t	X_c	Y_t	Y_c	S
/MPa	1239	38.7	1081	189	81

2.3 Equivalent Strain Failure Criterion

In engineering design, the failure criterion is usually based on strain, but less considering of the coupling between all the strain components. On the other hand, the above four classic failure criterions are based on the phenomenological theory, which usually deals with the stress instead of strain.

In this paper, a new failure strain criterion is suggested by using Equivalent Strain ε_{von} for engineering design, and in this criterion, the coupling effect of strain components has been considered. The Equivalent Strain criterion is described as follows:

$$\varepsilon_{von} / [\varepsilon_{von}] = 1 \qquad (4)$$

The working strain ε_{von} can be calculated by the flowing equation：

$$\varepsilon_{von} = \frac{2}{3}\sqrt{\frac{1}{2}\left[(\varepsilon_1 - \varepsilon_2)^2 + \varepsilon_1^2 + \varepsilon_2^2\right] + 3\gamma_{12}^2} \qquad (5)$$

Here, $[\varepsilon_{von}]$ is allowable strain calculated by the allowable strain components with the above equation.

3 Composite Structure Optimization based on Grid Platform

3.1 Remote Multi-cluster Grid Optimization System (RMGOS)[1]

In this paper, RMGOS is developed by integrating the grid technology and commercial FEA software to the GA. The commercial FEA software is utilized as the analysis tool to achieve the accuracy for the engineering requirement. The system is applied to solve multi-objective composite structure optimization problem. The flow chart of the optimization process is shown in Fig 1. In the flow, the neural-network is utilizing for predicting the search zone of next optimization loop to reduce the optimization time.

The algorithm of RMGOS includes (Fig 2): input the initial information including FE model, variables and constrains, etc; in GA module A, the objective function information is deal with by Pareto GA, and the population chromosomes are generated randomly or from the information of the previous generation; the datum files (feasible project set) for FEA are formed from the chromosome information; in the grid computation environment, the feasible project set are divided into some job units and dispatched to local/remote computation nodes through the grid Job Schedule Server; the numerical results are sent back to the Server after parallel computations; after having collected the results of all job units, the next generation will be generated by the GA operations in GA module B which performs the operations of selection, crossover and mutation and obtains the useful information, then the results of model B will return to GA module A. Therefore, the computation could be completed and the satisfactory solutions can be obtained after necessary or certain loops.

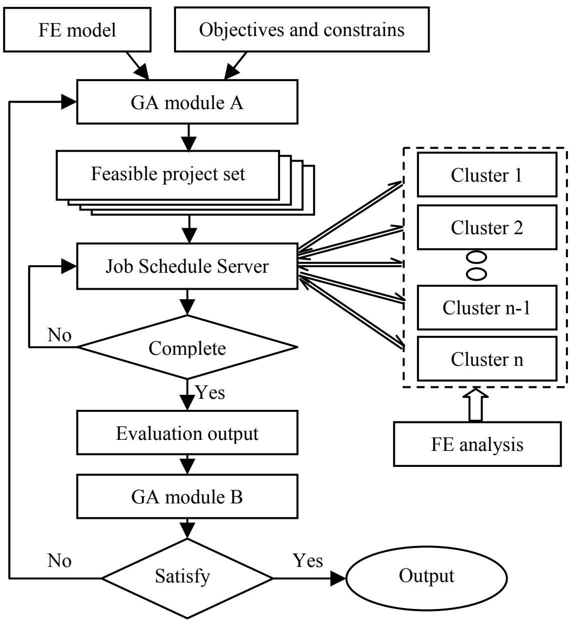

Fig 2 Flow chart of the optimization process

3.2 Optimization for Honeycomb Sandwich Composite Structures

To verify the RMGOS and Equivalent Strain Failure Criterion, a FEA model has been established to optimize the minimum weight under constrain of stiffness. The problem is described as follows:

$$
\begin{aligned}
&\text{Min} \quad W \\
&\text{s.t.} \quad \eta < 1 \\
&\qquad -90 \le \alpha \le 90 \qquad\qquad (6) \\
&\qquad x \in X \\
&\qquad t \ge t_{\min}
\end{aligned}
$$

Where W represents the weight for composite structure, a is adding angle, x is adding layer position, t and $t_{\min}$ are the skin thickness, the minimum skin thickness respectively.

In the flow process, the optimization starts from the initial panel skin with a minimum thickness, such as 1mm. The optimization is step by step to develop itself. In each step, 1% composite weight is dispersed and added in the optimum position with best lay-up angle. The optimization process keep running until reach the convergence, including reach preassigned step or judging if the failure efficiency η is less than zero. In the optimization, four failure criterions including Tsaihill, Hoffman, Tsaiwu, Equivalent Strain, are chosen as determinant conditions

3.3 The Numerical Results

After the optimizations, four honeycomb sandwich composite structures have been designed related to the four failure criterions. Each structure has met the design requirement, but the skin layouts are different. The failure indexes and design variables in various criterions are shown in Fig 3. In the initial state,

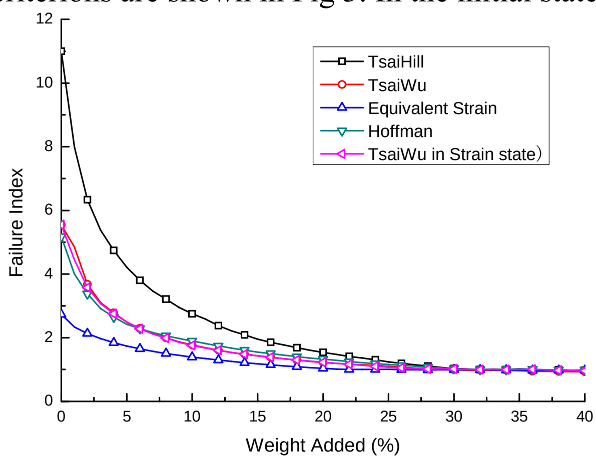

Fig 3 Optimization process in four failure criterions

the difference between failure indexes in all failure criterions is great, however, with the optimization process, when adding 26-33% weight, the maximum failure index in model are all less than 1.0, which meet the design requirement. In this circumstance, the structure is safe.

The failure indexes of the four sets of optimization results under four different failure criterions are shown in Fig 4. In the initial state, the failure index for TsaiHill is maximizing, and the one for Equivalent Strain is minimize. The other two is similar. When adding some certain percent weight, failure index in each criterion reach 1.0. Especially using TsaiWu criterion, the consistency is best and the results optimized by Equivalent Strain criterion has some error.

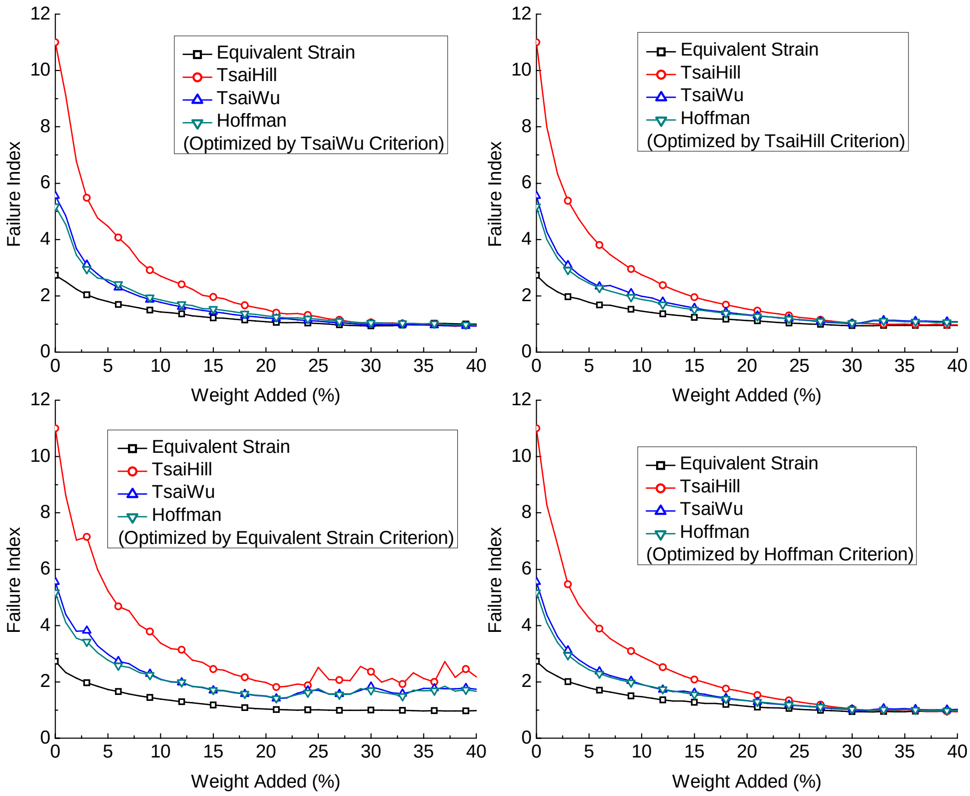

Fig 4 Flow chart of the optimization process

3.4 Equivalent of Raw Optimization Results

In the raw optimization results obtained after the RMGOS calculation, each lay-up has different thickness and different angles, which needs to be transferred to standard lay-up for engineering application by using the equivalent equation of three directions stiffness.

3.4.1 The Equivalent of Three Direction Stiffness

The mechanical derive can proof that, no matter how choose the lay-up angle, the total thickness remain the same, when the stiffness in the longitudinal, transverse and shear directions has been determined. Hence, in the initial design for laminate, the three direction stiffness can be calculated according to the structure design, then the lay-up angle and thickness can be calculated consequently.

The longitudinal, transverse and shear stiffness is the basic characteristic for laminate. When designing one symmetric laminate, the lay-up number in every set of angle can be calculated, if the two axial rigidity $E\delta_1$, $E\delta_2$ and shear rigidity $G\delta$ has been given. The following is the equation:

$$E\delta_1 = A_{11} - \frac{A_{12}^2}{A_{22}}; E\delta_2 = A_{22} - \frac{A_{21}^2}{A_{11}}; G\delta = A_{66} \qquad (7)$$

Assume that the three lay-up angle α, β, θ are 0°, 45°, 90° respectively. The corresponding lay-up number is N_1, N_2, N_3. When the three direction stiffness is given, the lay-up number can be calculated out by the following equation:

$$
\begin{aligned}
E\delta_1 &= (Q_{11}^{\alpha}N_1 + Q_{11}^{\beta}N_2 + Q_{11}^{\theta}N_3)TB - \frac{(Q_{12}^{\alpha}N_1 + Q_{12}^{\beta}N_2 + Q_{12}^{\theta}N_3)^2 TB}{Q_{22}^{\alpha}N_1 + Q_{22}^{\beta}N_2 + Q_{22}^{\theta}N_3} \\
E\delta_2 &= (Q_{22}^{\alpha}N_1 + Q_{22}^{\beta}N_2 + Q_{22}^{\theta}N_3)TB - \frac{(Q_{12}^{\alpha}N_1 + Q_{12}^{\beta}N_2 + Q_{12}^{\theta}N_3)^2 TB}{Q_{11}^{\alpha}N_1 + Q_{11}^{\beta}N_2 + Q_{11}^{\theta}N_3} \qquad (8) \\
G\delta &= (Q_{66}^{\alpha}N_1 + Q_{66}^{\beta}N_2 + Q_{66}^{\theta}N_3)TB
\end{aligned}
$$

Here, *TB* represents the thickness in each lamina.

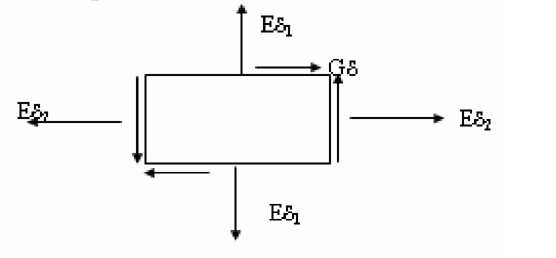

Fig 5 3D stiffness state in composite panel

3.4.2 The Equivalent of Three Direction Stiffness

By using the tool Equivalent Equation of Three Directions Stiffness, the raw optimized results has been transferred to the standard lay-up laminate met the manufactory requirement, as shown in Fig 6. The different color area represents different lay-up thickness and angles.

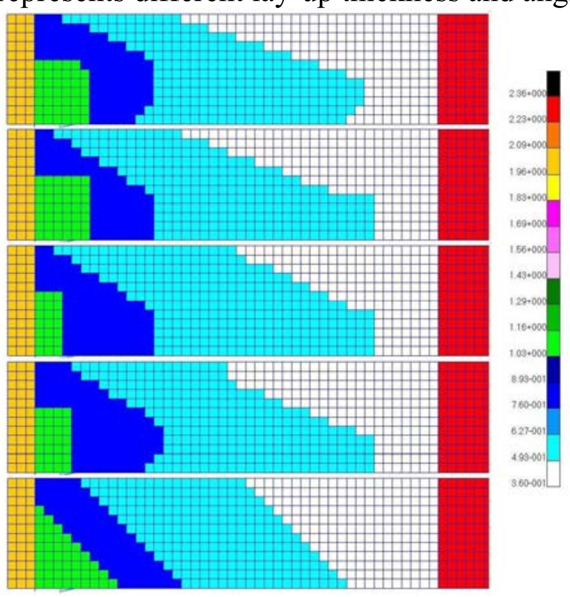

Fig 6 Skin lay-up after optimization for four criterions

4. The Experiment for Optimization Results

4.1 The Experiment Design

The experiment samples are a group of symmetric imbalance honeycomb sandwich composite structure, as shown in Fig 7. The samples skin is composite material with optimized lay-up for four criterions. The core is paper honeycomb. There are 5 groups of samples with two same samples in each group.

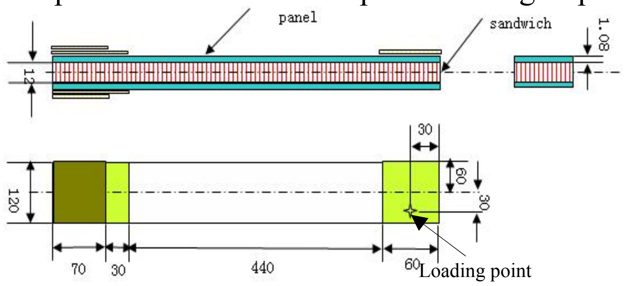

Fig 7 Experiment design

4.2 The Experiment Results

4.2.1 The Failure Load

The experiment data of the samples is shown in Table 2 and Fig 8, including the loading and failure process for each sample. When the sample 1-1 reaches failure condition, it is far away from the design strength value without consideration of the release force between the aluminum reinforcement in root and panel. Hence 6 self-plugging rivets are used to strength the structure in each sample in order to avoiding the failure result from the release force. After reach the maximum loading, the curve drop sharply.

Table 2 The failure load of samples

Value/N	TsaiHill	TsaiWu	EquStrain	Hoffman	Strandard
Design	1000	1000	1000	1000	1000
Exp Samp1	699.5	1322.8	1280.72	1286.9	1204.5
Exp Samp2	1129.7	1122.6	996.9	1321.9	1392.7

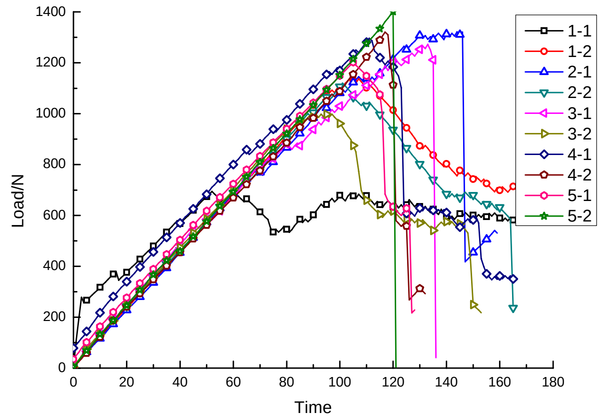

Fig 8 Flow chart of the optimization process

4.2.2 The Failure Mode

There are three kinds of failure mode, including: The sample 1-1 failed because of the release strength between the aluminum and the composite panel (top fig in Fig 9). In this mode, the insufficient of adhere release strength result in the failure when adding 70% design load.

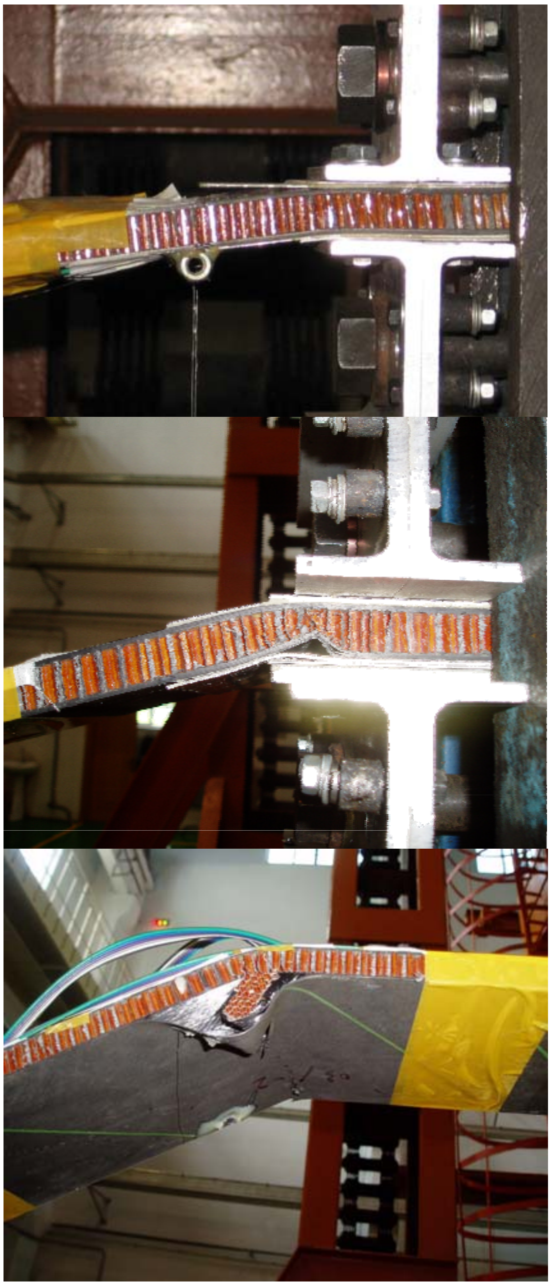

Fig 9 The failure modes of experiment samples

The sample 3-1 failed result from the collapse of honeycomb under the buckling loading (middle fig in Fig 9). In this mode, the reinforcement from the self-plugging rivet improved the bearing capacity of the composite panel, so the failure load is greater than the design load. On the other hand, the design strength of honeycomb is very lower, so the failure always focused on the honeycomb structure.

The sample 5-1 failed because of the crack in composite panel. In this mode, the standard without optimization failed in the middle part result from the insufficient strength in the relevant position.

In all samples, 1-1 is the 1st failure mode (lower fig in Fig 9). The sample 5-1 and 5-2 are the 3rd failure mode. The other samples are also the 2nd failure mode.

4.2.3 The Failure Strain

The comparison between experiment and computation value in test point is shown in Fig 10 and Fig 11, here, the five figures represent lay-up optimized with the standard, Hoffman, Equivalent Strain, TsaiHill, TsaiWu criterion respectively. The curve is made with the test number as abscissa and strain value as vertical coordinate. The "theory" curve describes the computation strain in all test position under design loading. The "test1" and "test2" curve show the actual experiment strain. The "loadcase1" and "loadcase2" curve represent the strain recalculated by adding the actual failure load in the FEA model.

- The experiment strain is less than computation strain in each criterion. The reason is that the reinforcement in root position is insufficient to offer enough support to the whole structure. Hence, the honeycomb structure has collapsed when the composite panel is far away from the failure.
- There exist some wave in each criterion; it means the design hasn't realized the equal strain state. The difficult to close agreement on the design with the manufacture process is the main reason. Another reason is that the stepped shape change of the lay-up results in the discontinuous strain distribution.
- The test position in standard lay-up wave more sharply than the other lay-up. It means

that it's difficult to obtain good lay-up by arbitrary design without optimization.

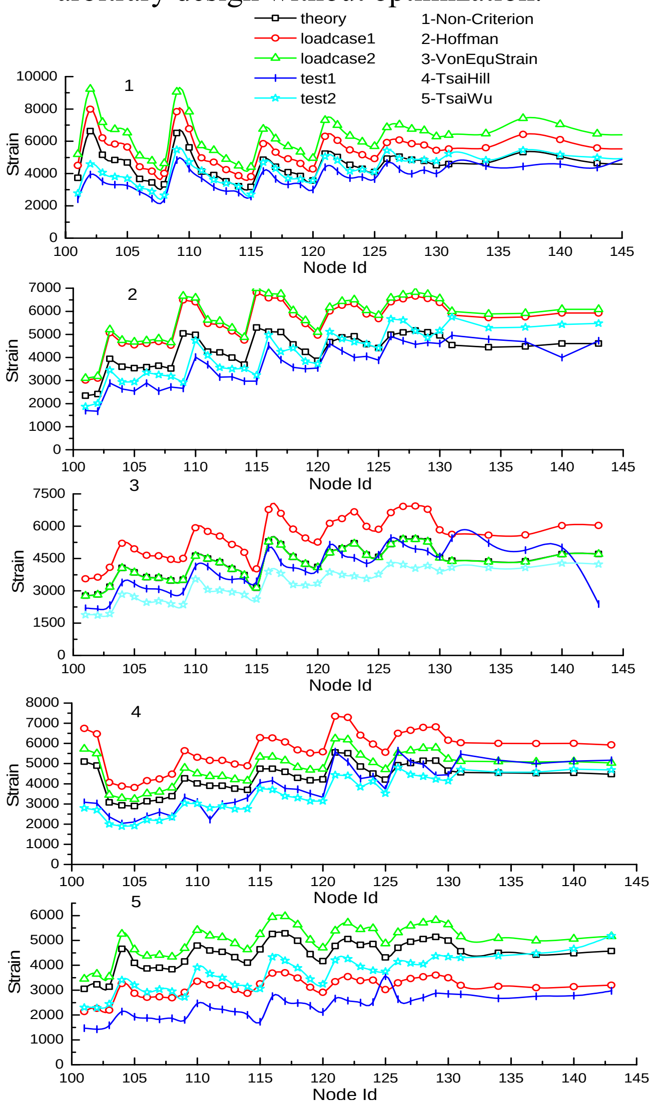

Fig 10 The test strain value in samples

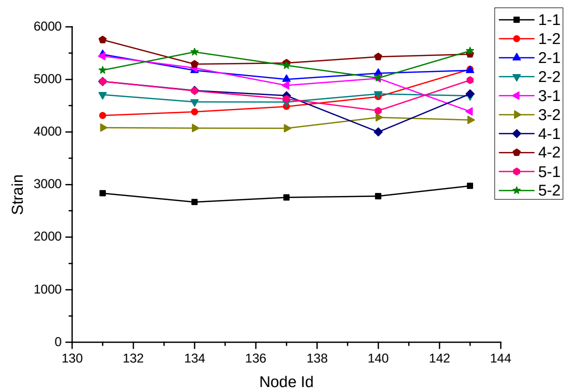

Fig 11 The test strain in transverse section

- The wave in TsaiWu criterion is the best and the distribution of strain is the most even, which means the effectiveness is best. Hoffman criterion is better than TsaiHill criterion, and the Equivalent Strain criterion is worst.
- The strain distribution in root position is even and the wave in loading section is sharp. It shows that the equal strain design is very successful in root position.

5. Conclusion

The results optimized by RMGOS have been verified through the experiment. It shows that using the distribution computing platform integrates mature FEA software and grid technology with GA, the complicated aircraft structure design problem can be solved efficiently, especially in solving the composite structure optimization with multiple constrains. From the computation and experiment results, TsaiWu criterion is best in effectiveness and the Equivalent Strain criterion may be as one evaluation method only as reference.

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