MISSION DECISION-MAKING METHOD OF MULTI-AIRCRAFT COOPERATIVELY ATTACKING MULTI-TARGET BASED ON SITUATIONAL ASSESSMENT

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Abstract

Coordinated mission decision-making is one of the core steps to effectively exploit the capabilities of cooperative attack of multiple aircrafts. However, situational assessment is essential base for us to realize mission decision-making. Therefore, in this paper, we develop a mission decision-making method of multi-aircraft cooperative attack multi-target based on situational assessment. We have studied the situational assessment mathematical model based on the D-S evidence theory and the mission decision-making mathematical model based on the game theory, respectively. Finally, the mission decision-making method of antagonized airfight is validated by giving a simulated example of the swarm of UCAVs carrying out the mission of the Suppressing of Enemy Air Defenses (SEAD).

1 Introduction

The team of multiple aircrafts has stronger capability than single aircraft in the aspects of cooperative detect targets, pierce through defenses and carry out attack. Each one of the team can share in the information acquired by any other one and carry out mission of cooperative attack targets according to its position in air and resources of fighting for an uniform airfight intention. The team of multiple aircrafts is able to form easily all kind of vertiginous attack situation in airfight so that those opposed targets will be confronted with the defending difficulties. Thereby, the fashion of multiple aircrafts cooperative attack targets will be main pattern in future airfight.

In this paper, the phrases of attacking (fighting) effect consists of validity, invalidity and uncertainty. The validity and invalidity of fighting effect is defined as the advantage acquired by our aircraft or foe's target and the cost paid for achieving intention by our aircraft or foe's target in antagonized airfight, respectively. Sensors aboard aircraft affect the attacking effect of aircraft due to the capability of sensors detecting, tracking and identifying target, while weapons aboard aircraft affect the attacking effect due to the capability of weapons hitting and destroying target. However, the above capabilities of sensors and weapons all rest with the distance, azimuth and pitching between one our aircraft and one foe's target. Accordingly, the fuzzy mapping function of the fighting effect of sensor and weapon is respectively constructed by selecting the three position parameters of distance, azimuth and pitching as variables for establishing the correspondence between the position parameter and the ability of sensor and weapon. In this paper, the Dempster-Shafer synthesize rules are used for formulating the situational assessment method.

When multiple our aircrafts are antagonizing some foe's targets simultaneously, one of multiple our aircrafts is able to either detect and identify the foe's targets by sensors aboard the aircraft or receive the information about the same or other foe's targets by wireless data link. Therefore, in this paper, we suppose that our team of multiple aircrafts has known the position and identity of all foe's targets and is able to acquire the important reasoning from the position and identity of foe's target to the capability of sensors' detecting and weapons' attacking, the defending strategies and the advantage(showed by numerical value) acquired by selecting a certain defending strategy. If the above situation of antagonizing airfight is analyzed by quoting the game theory model, the situation means that our team has known the opponents and the opponents' strategies and cost function. Considering our opponent is powerful, we think that our opponent also has known the equivalent information about our team of multiple aircrafts at least. We suppose that our team and our opponent simultaneously carry out action for equality because they all try to be the first actor. In this paper, the static non-cooperative and non-zero Nash games are used for formulating the mission decision-making method.

An effective team composition and tasking mechanism and an optimal team dynamics and tactics algorithm for mission planning under a hierarchical game theoretic framework are presented in [1]. A strategic decision model based on game theory-enabled analysis for a systematic exploration of both engineering and business decisions is proposed in [2]. A greedy heuristic approach to enable a swarm of UCAVs to execute a SEAD mission is presented in[3]. A way to propagate risk assessments and incorporate the offensive capabilities is proposed in [4].

This paper will treat the development of a mission decision-making algorithm based on the game model and a situational assessment algorithm based on the D-S evidence synthesize rules for a swarm of UCAVs in SEAD mission. In section II, a situational assessment algorithm of coordinated airfight is presented in detail and the D-S evidence theory is introduced simply for sustaining the mentioned situational assessment algorithm above. In section III, a mission decision-making algorithm is designed by formulating the strategies and cost function in the game model. Section III is based on section II. Section IV will show an simulation example of a typical mission performed by a swarm of UCAVs. In Section V, the simulating results in section IV are analyzed deeply. Section VI summarizes the conclusions.

2 Formulating Situational Assessment Based on Evidence Theory

2.1 Preliminaries

For sustaining the situational assessment algorithms mentioned in latter chapter, the basic concepts of the D-S evidence theory are firstly introduced in follows:

Let Θ be a set consisting of the all values that X might be and a element of set Θ is not consistent with the other elements, then Θ is called as the discernment frame of X .

Definition 1: Let Θ be a frame of discernment, if the function $m:2^{\Theta}\rightarrow[0,1]$ fulfills following conditions:

1. $m(\phi)=0$
2. $\sum\limits_{A\subset\Theta} m(A)=1$

Then m is called as the basic probability assignment on the frame of discernment Θ and $m(A)$ is called as the basic probability number of A. Where $m(A)$ denotes the believed degree of A oneself.

Definition 2: Let Θ be a frame of discernment, if the function $m:2^{\Theta}\rightarrow[0,1]$ is the basic probability assignment on Θ, then the function $Bel:2^{\Theta}\rightarrow[0,1]$ is called the belief function and is defined by

$$Bel(A)=\sum_{B\subset A} m(B)(\forall A\subset\Theta).$$

Where $Bel(A)$ denotes the believed degree of A including all its subsets.

Dempster synthesizing rule:

Let $Bel_1,\cdots,Bel_n$ are the belief functions on the same frame of discernment Θ, $m_1,\cdots m_n$ are the basic probability assignments correspondingly. If $Bel_1\oplus\cdots\oplus Bel_n$ is existent and it's the basic probability assignment is m, then

$$\forall A\subset\Theta, A\neq\varnothing, A_1,\cdots,A_n\subset\Theta$$

$$m(A)=K\sum_{\substack{A_1,\cdots,A_n\subset\Theta\\ A_1,\cap\cdots\cap,A_n=A}} m_1(A_1)\cdots m_n(A_n)$$

$$K = \left(1 - \sum_{\substack{A_1,\cdots A_n \subset \Theta \\ A_1 \cap \cdots \cap A_n = \varnothing}} m_1(A_1) \cdots m_n(A_n) \right)^{-1}$$

The Dempster synthesizing rule is a rule that reflects the effect of combined operations made by many evidences.

2.2 Situational assessment algorithm of coordinated airfight

For achieving situational assessment of multi-aircraft cooperative attack multi-target, the relevant mathematical model is established based on the D-S evidence theory in follows:

Firstly, we establish the discernment frame of fighting effect Θ.

Let $\Theta = \{a, b, \theta\}$ is the discernment frame of fighting effect defined in section I. Where a, b and θ denotes respectively the fighting effect, ineffect and uncertainty in the three cases of one our aircraft attack one enemy target, multiple our aircrafts cooperative attack one enemy target and multiple our aircrafts cooperative attack multiple enemy targets.

Secondly, we design the basic probability assignment on the frame of discernment Θ for our aircraft.

The basic probability assignment of the sensor distance evidence is formulated in the expression (1), (2) and (3) and is depicted in Fig. 1.

$$m1_{RL}(a) = \begin{cases} m1rla_{zo} & (0 \le RL < m1rl_{zo}) \\ \dfrac{(m1rla_{js} - m1rla_{zo}) \cdot RL}{(m1rl_{js} - m1rl_{zo})} + m1rla_{zo} - m1rl_{zo} \cdot \dfrac{(m1rla_{js} - m1rla_{zo})}{(m1rl_{js} - m1rl_{zo})} & (m1rl_{zo} \le RL < m1rl_{js}) \\ \dfrac{(m1rla_{zj} - m1rla_{js}) \cdot RL}{(m1rl_{zj} - m1rl_{js})} + m1rla_{js} - m1rl_{js} \cdot \dfrac{(m1rla_{zj} - m1rla_{js})}{(m1rl_{zj} - m1rl_{js})} & (m1rl_{js} \le RL < m1rl_{zj}) \\ \dfrac{(m1rla_{ks} - m1rla_{zj}) \cdot RL}{(m1rl_{ks} - m1rl_{zj})} + m1rla_{zj} - m1rl_{zj} \cdot \dfrac{(m1rla_{ks} - m1rla_{zj})}{(m1rl_{ks} - m1rl_{zj})} & (m1rl_{zj} \le RL < m1rl_{ks}) \\ \dfrac{(m1rla_{zb} - m1rla_{ks}) \cdot RL}{(m1rl_{zb} - m1rl_{ks})} + m1rla_{ks} - m1rl_{ks} \cdot \dfrac{(m1rla_{zb} - m1rla_{ks})}{(m1rl_{zb} - m1rl_{ks})} & (m1rl_{ks} \le RL < m1rl_{zb}) \\ m1rla_{zb} & (m1rl_{zb} \le RL) \end{cases} \quad (1)$$

$$m1_{RL}(b) = \begin{cases} m1rlb_{zo} & (0 \le RL < m1rl_{zo}) \\ \dfrac{(m1rlb_{js} - m1rlb_{zo}) \cdot RL}{(m1rl_{js} - m1rl_{zo})} + m1rlb_{zo} - m1rl_{zo} \cdot \dfrac{(m1rlb_{js} - m1rlb_{zo})}{(m1rl_{js} - m1rl_{zo})} & (m1rl_{zo} \le RL < m1rl_{js}) \\ \dfrac{(m1rlb_{zj} - m1rlb_{js}) \cdot RL}{(m1rl_{zj} - m1rl_{js})} + m1rlb_{js} - m1rl_{js} \cdot \dfrac{(m1rlb_{zj} - m1rlb_{js})}{(m1rl_{zj} - m1rl_{js})} & (m1rl_{js} \le RL < m1rl_{zj}) \\ \dfrac{(m1rlb_{ks} - m1rlb_{zj}) \cdot RL}{(m1rl_{ks} - m1rl_{zj})} + m1rlb_{zj} - m1rl_{zj} \cdot \dfrac{(m1rlb_{ks} - m1rlb_{zj})}{(m1rl_{ks} - m1rl_{zj})} & (m1rl_{zj} \le RL < m1rl_{ks}) \\ \dfrac{(m1rlb_{zb} - m1rlb_{ks}) \cdot RL}{(m1rl_{zb} - m1rl_{ks})} + m1rlb_{ks} - m1rl_{ks} \cdot \dfrac{(m1rlb_{zb} - m1rlb_{ks})}{(m1rl_{zb} - m1rl_{ks})} & (m1rl_{ks} \le RL < m1rl_{zb}) \\ m1rlb_{zb} & (m1rl_{zb} \le RL) \end{cases} \quad (2)$$

$$m1_{RL}(\theta) = m1rl\theta \qquad RL \ge 0 \qquad (3)$$

The function formulated in the expression (1), (2) and (3) is actually the fuzzy mapping function of the fighting effect of the sensor distance evidence and establishes the correspondence between the distance from our aircraft to foe's target and the sensor detecting ability of our aircraft.

In the expression (1), (2) and (3), $m1_{RL}$ is the basic probability assignment of the sensor distance evidence, where $m1$ denotes that the function is used for describing our aircraft ($m2$ denotes that the function is used for describing enemy target correspondingly) , the subscript RL stands for sensor (Radar) distance(Length), the other mathematics symbols are depicted in Fig. 1.

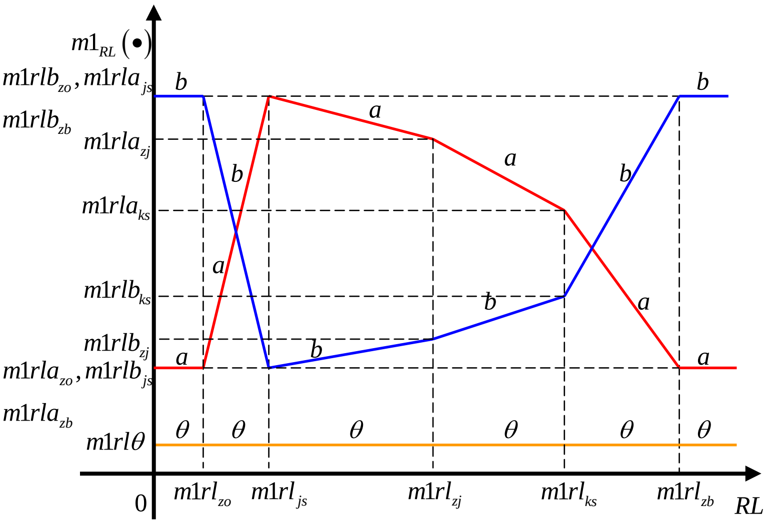

Fig.1 Basic Probability Assignment Function $m1_{RL}$

Figure 1 illustrates that the unitary fighting effect value $m1_{RL}(\cdot)$ (the vertical axis of coordinates, value is limited in $[0,1]$, no unit)of the synthesis capability achieved in the process of sensors aboard our aircraft detecting, tracking and identifying foe's target is depended on the distance RL (the horizontal axis of coordinates, unit is kilometer)between our aircraft and foe's target. $m1_{RL}(\cdot)$ implies how much the sensors ability being enslaved to the distance contributes to the whole attacking action. In Fig. 1, the cures with sign (a , b and θ) represents respectively the validity, invalidity and uncertainty of the fighting effect of the sensor distance evidence; The values of the vertical axis parameters (e.g., $m1rlb_{zo}$) and the horizontal axis parameters(e.g., $m1rl_{zo}$) will be embodied by the capability of the actual sensors.

Analogously, the basic probability assignments of the sensor azimuth and pitching evidences and the weapon distance, azimuth and pitching evidences can be also formulated to be $m1_{R\Theta}$, $m1_{R\Psi}$, $m1_{ML}$, $m1_{M\Theta}$,and $m1_{M\Psi}$, respectively.

For enemy target, by adopting the same method, the basic probability assignments of the sensor distance, azimuth and pitching evidences and the weapon distance, azimuth and pitching evidences can be formulated to be $m2_{RL}$, $m2_{R\Theta}$, $m2_{R\Psi}$, $m2_{ML}$, $m2_{M\Theta}$,and $m2_{M\Psi}$, respectively.

Finally, we achieve the evidences synthesizing of coordinated airfight.

It needs to be specially pointed out that the attacking effect will be different when our swarm of aircrafts employ the different strategy (the allocations of a set of our aircrafts to a number of foe's targets) to attack foe's group of targets in despite of the positions between any one of our aircrafts and each foe's target being the same.

Wherefore, in this paper, the situational assessment is equivalent to calculate the validity $m1^{\alpha_i}(a)$, invalidity $m1^{\alpha_i}(b)$ and uncertainty $m1^{\alpha_i}(\theta)$ of the certain our strategy α_i and the validity $m2^{\beta_j}(a)$, invalidity $m2^{\beta_j}(b)$ and uncertainty $m2^{\beta_j}(\theta)$ of the certain foe's strategy β_j .

The certain our strategy may consist of the three cases: a) One our aircraft attacks one foe's target; b) A set of our aircrafts cooperative attack the same foe's target; c) No our aircraft attack the certain foe's target. The certain foe's strategy is similar to the certain our strategy.
The validity of the fighting effect of k aircrafts cooperative attack l targets may be acquired by a set of evidences synthesizing processes based on the evidence theory, in follows:

$$m1_{R_{tp_n}}^{\alpha_i} = m1_{RL_{tp_n}}^{\alpha_i} \oplus m1_{R\Theta_{tp_n}}^{\alpha_i} \oplus m1_{R\Psi_{tp_n}}^{\alpha_i} \tag{4}$$

$$p_n = 0,1,2,\cdots,k \qquad n = 0,1,2,\cdots,k \qquad t = 1,2,\cdots,l$$

Where n denotes that the number of our aircrafts attending the action of cooperative attack the certain foe's target, p_n denotes that the the serial number of those aircrafts attending the action, and t denotes that the serial number of the certain foe's target meeting with attack.

The formula (4) denotes that the total fighting effect (including validity, invalidity and uncertainty) of sensor aboard on the aircraft with the serial number p_n (in the subscript) $m1_{R_{tp_n}}^{\alpha_i}$ is acquired by synthesizing the fighting effect of the sensor distance evidences $m1_{RL_{tp_n}}^{\alpha_i}$, the fighting effect of the sensor azimuth evidences $m1_{R\Theta_{tp_n}}^{\alpha_i}$ and the fighting effect of the sensor pitching evidences $m1_{R\Psi_{tp_n}}^{\alpha_i}$ based on the Dempster synthesizing rule in section II(A) when one certain our aircraft p_n (in the subscript) attack one certain foe's target t (in the subscript) according to the certain our strategy α_i (the superscript).

$$m1_{M_{tp_n}}^{\alpha_i} = m1_{ML_{tp_n}}^{\alpha_i} \oplus m1_{M\Theta_{tp_n}}^{\alpha_i} \oplus m1_{M\Psi_{tp_n}}^{\alpha_i} \tag{5}$$

The meaning of the formula (5) is similar to that of the formula (4). The distinguish is only that the weapon is described instead of the sensor.

$$m1_{tp_n}^{\alpha_i} = m1_{R_{tp_n}}^{\alpha_i} \oplus m1_{M_{tp_n}}^{\alpha_i} \tag{6}$$

The formula (6) denotes that the total fighting effect (including validity, invalidity and uncertainty) of one our aircraft with the serial number p_n (in the subscript) $m1_{tp_n}^{\alpha_i}$ is acquired by synthesizing the total fighting effect of the sensor evidences $m1_{R_{tp_n}}^{\alpha_i}$, and the total fighting effect of the weapon evidences $m1_{M_{tp_n}}^{\alpha_i}$ based on the Dempster synthesizing rule in section II(A) when one certain our aircraft p_n (in the subscript) attack one certain foe's target t (in the subscript) according to the certain our strategy α_i (the superscript).

$$m1_t^{\alpha_i} = m1_{tp_1}^{\alpha_i} \oplus m1_{tp_2}^{\alpha_i} \oplus \cdots \oplus m1_{tp_n}^{\alpha_i} \qquad (7)$$

The formula (7) denotes that the total fighting effect (including validity, invalidity and uncertainty) of n our aircrafts with the serial number $\left(p_1, p_2, \cdots p_n\right)$ (in the subscript) $m1_t^{\alpha_i}$ is acquired by synthesizing the total fighting effect of all our aircraft with the serial number $\left(p_1, p_2, \cdots p_n\right)$ (in the subscript) cooperative attack the same foe's target t (in the subscript) $\left(m1_{tp_1}^{\alpha_i}, m1_{tp_2}^{\alpha_i}, \cdots, m1_{tp_n}^{\alpha_i}\right)$ based on the Dempster synthesizing rule in section II(A) when multiple our aircrafts cooperative attack one certain foe's target according to the certain our strategy α_i (the superscript).

Considering there is the case that no our aircraft attack the certain foe's target, a special rule is made in follows:

$$m_{t0}^{\alpha_i}\left(a\right) = 0.299, m1_{t0}^{\alpha_i}\left(b\right) = 0.689, m1_{t0}^{\alpha_i}\left(\theta\right) = 0.012$$

$$m1_t^{\alpha_i} = m1_{tp_0}^{\alpha_i} = m1_{t0}^{\alpha_i} \qquad (8)$$

The formula (8) denotes that the total fighting effect (validity, invalidity and uncertainty) of our aircraft team is set to be 0.299, 0.689 and 0.012 respectively, when no our aircraft attack the certain foe's target t (in the subscript) according to the certain our strategy α_i (the superscript).

$$m1^{\alpha_i} = m1_1^{\alpha_i} \oplus m1_2^{\alpha_i} \oplus \cdots \oplus m1_t^{\alpha_i} \oplus \cdots \oplus m1_l^{\alpha_i} \qquad (9)$$

The formula (9) denotes that the total fighting effect (including validity, invalidity and uncertainty) of the whole our aircrafts team $m1^{\alpha_i}$ is acquired by synthesizing the fighting effect of all our aircrafts cooperative attack all l foe's target with the serial number $\left(1, 2, \cdots, t, \cdots l\right)$ (in the subscript) $\left(m1_1^{\alpha_i}, m1_2^{\alpha_i}, \cdots, m1_t^{\alpha_i}, \cdots m1_l^{\alpha_i}\right)$ based on the Dempster synthesizing rule in section II(A) when multiple our aircrafts cooperative attack multiple foe's targets according to the certain our strategy α_i (the superscript).

For enemy target, by adopting the same method , the total fighting effect (including validity, invalidity and uncertainty) of the whole foe's targets group $m2^{\beta_j}$ may be also acquired when multiple foe's targets cooperative defend multiple our aircrafts according to the certain foe's strategy β_j (the superscript).

3 Formulating Mission Decision-Making Based on Game Theory

For achieving mission decision-making of multi-aircraft cooperative attack multi-target, the relevant mathematical model is established based on the game theory in follows:

The game model is set as follows:

$$G = \left\langle N, S_1, S_2, u_1, u_2 \right\rangle, \quad N = \left\{1, 2\right\},$$

Number 1 and Number 2 represents respectively our team consisting of k aircrafts and foe's group consisting of l targets.

$$S_1 = \left(\alpha_1, \alpha_2, \cdots, \alpha_m\right), S_2 = \left(\beta_1, \beta_2, \cdots, \beta_n\right)$$

$$\alpha_i = \left(\left(W_1, d_1\right), \left(W_2, d_2\right), \cdots, \left(W_l, d_l\right)\right)$$

Where $W_1, W_2, \cdots W_l \subset W = \left(w_1, w_2, \cdots w_k, \phi\right)$ and

$$W_1 \cup W_2 \cup \cdots \cup W_l = W \qquad W_1 \cap W_2 \cap \cdots \cap W_l = \Phi$$

$$\beta_j = \left(\left(D_1, w_1\right), \left(D_2, w_2\right), \cdots, \left(D_k, w_k\right)\right)$$

Where $D_1, D_2, \cdots D_k \subset D = \left(d_1, d_2, \cdots d_l, \phi\right)$ and

$$D_1 \cup D_2 \cup \cdots \cup D_k = D \qquad D_1 \cap D_2 \cap \cdots \cap D_k = \Phi$$

Where $\alpha_i (i = 1, 2, \cdots, m)$ represents the m strategies of k our aircrafts coorperative attack l foe's targets, while $\beta_j (j = 1, 2, \cdots, n)$ represents the n strategies of l foe's targets coorperative defend k our aircrafts. One among of the parameters ($d_1, d_2, \cdots, d_l$) represents respectively the serial number of l foe's targets;

One among of the parameters ($w_1, w_2, \cdots, w_k$) represents respectively the serial number of k our aircrafts; ϕ represents nobody.

$(W_t, d_t), t = 1, 2, \cdots, l$ represents the a set of our aircrafts marked by W_t cooperative attack the same foe's target marked by d_t , while (D_t, w_t), $t = 1, 2, \cdots, k$ represents the a number of foe's targets marked by D_t cooperative defend the same our aircraft marked by w_t .

When our strategy of α_i and foe's strategy of β_j are selected simultaneously, our payment function is set as follows:

$$u_1(\alpha_i, \beta_j) = a_{ij} = \frac{m1^{\alpha_i}(a) \cdot m2^{\beta_j}(b)}{m1^{\alpha_i}(b) \cdot m2^{\beta_j}(a)}, \tag{10}$$

Similarly, foe's payment function is set as follows:

$$u_2(\alpha_i, \beta_j) = b_{ij} = \frac{m1^{\alpha_i}(b) \cdot m2^{\beta_j}(a)}{m1^{\alpha_i}(a) \cdot m2^{\beta_j}(b)}, \tag{11}$$

The meaning of $m1^{\alpha_i}(a), m1^{\alpha_i}(b), m2^{\beta_j}(a)$ and $m2^{\beta_j}(b)$ has been defined in section II(B).

The $A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$ is called respectively our payment matrix and the foe's payment matrix.

The Nash equilibrium state of the game model G may be acquired by the certain computing method based on the matrix A and B .

Our strategy of α_i and foe's strategy of β_j denoted by the Nash equilibrium state will be used to decide the fighting action of our aircrafts team and guess the defending action of foe's targets group, respectively.

4 Simulation

To illustrate how our algorithm works, we developed a software package, which can accomplish the computing of the whole our algorithm designed in section II and section III.

4.1 Scenario

A typical scenario shown in Fig. 2 with several experiments is simulated on our software to evaluate the performance of our proposed Mission Decision-making algorithm for Multi-aircraft Cooperative Attack Multi-target.

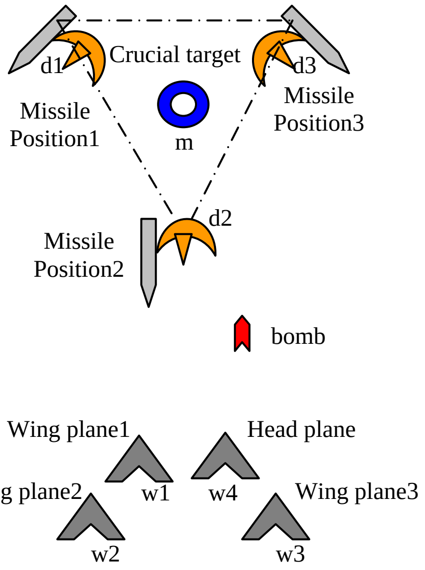

Fig.2 A Typical Scenario of SEAD

In the scenario, one our team consists of four UCAVs with the serial number (w_1, w_2, w_3, w_4) in the air and one enemy group consists of three missile positions with the serial number (d_1, d_2, d_3) and a crucial target with the serial number m on the ground.

In our team, each UCAV has the same sensor detecting and weapon attacking ability, flight altitude (10000 meters) and flight velocity (240 meters per second). The four UCAVs take the form of trapezium shown in Fig. 2. In the trapezium team, the distance between the frontal two UCAVs is 5 kilometres and the distance between the latter two UCAVs is 10 kilometres and the distance between the frontal two UCAVs and the latter two UCAVs is 2.5 kilometres.

In enemy group, the three missile positions take the form of equilateral triangle shown in Fig. 2. In the equilateral triangle team, the distance between the any two missile positions is 10 kilometres and the crucial target is located on the center of equilateral triangle. The distance between the center of the our team and the center of the enemy group is a variable that may be value from 5 to 25 kilometres (the value is 10 kilometres in this paper).

The fighting ability of our UCAV is lied on the suppositional sensor (SAR radar) and weapon (JDAM bomb) and the ability of enemy missile position is restricted to the suppositional sensor (track homing radar) and weapon (short ground to air missile).

4.2 Model

The mathematics model used for developing simulation software package is established based on the scenario in section IV(A) and the algorithm designed in section III as follows:

$$G=\langle N,S_1,S_2,u_1,u_2\rangle,\quad N=\{1,2\}$$

Number 1 represents our team consisting of four UCAVs and Number 2 represents enemy group consisting of three missile positions and a crucial target.

$S_1=(\alpha_1,\alpha_2,\cdots,\alpha_{12})$ represents the strategy set of four our UCAVs cooperative attack four foe's targets. Where

$$\alpha_1=((w_1,d_1),(w_2,d_2),(w_3,m),(w_4,d_3))$$

$$\alpha_2=(((w_1,w_2),d_1),(w_3,d_2),(w_4,m),(\phi,d_3))$$

$$\alpha_3=(((w_1,w_2,w_3),d_1),(\phi,d_2),(w_4,m),(\phi,d_3))$$

$$\alpha_4=((w_1,d_1),((w_2,w_3),d_2),(w_4,m),(\phi,d_3))$$

$$\alpha_5=((\phi,d_1),((w_1,w_2,w_3),d_2),(w_4,m),(\phi,d_3))$$

$$\alpha_6=((w_1,d_1),((w_2,w_3),d_2),(\phi,m),(w_4,d_3))$$

$$\alpha_7=(((w_1,w_2),d_1),(w_3,d_2),(\phi,m),(w_4,d_3))$$

$$\alpha_8=(((w_1,w_2,w_3),d_1),(w_4,d_2),(\phi,m),(\phi,d_3))$$

$$\alpha_9=(((w_1,w_2,w_3,w_4),d_1),(\phi,d_2),(\phi,m),(\phi,d_3))$$

$$\alpha_{10}=((\phi,d_1),((w_1,w_2),d_2),(\phi,m),((w_3,w_4),d_3))$$

$$\alpha_{11}=((\phi,d_1),((w_1,w_2,w_3),d_2),(\phi,m),(w_4,d_3))$$

$$\alpha_{12}=((\phi,d_1),((w_1,w_2,w_3,w_4),d_2),(\phi,m),(\phi,d_3))$$

Where $w_i(i=1,2,3,4)$ represent four our UCAVs , $d_j(j=1,2,3)$ represent three foe's missile positions , m represents a foe's crucial target, $((a,b),c)$ represents that a and b cooperative attack c, and (ϕ,m) represents no our aircraft attack the foe's target m.

The meaning of the strategy $\alpha_i(i=1,2,\cdots,12)$ is explained by describing the strategy α_4 as follows:

$$\alpha_4=((w_1,d_1),((w_2,w_3),d_2),(w_4,m),(\phi,d_3))$$

α_4 represents that our UCAV w_1 attacks foe's missile position d_1, our two UCAVs w_2 and w_3 cooperative attack foe's missile position d_2, our UCAV w_4 attacks foe's crucial target m, and no our aircraft attack foe's missile position d_3.

The character of the strategies $\alpha_1\sim\alpha_5$ is that foe's missile positions and crucial target are attacked comparably, while the character of the strategies $\alpha_6\sim\alpha_{12}$ is that the action of attacking foe's missile positions is prior to the action of attacking foe's crucial target.

$S_2=(\beta_1,\beta_2,\cdots,\beta_6)$ represents the strategy set of four foe's targets cooperative defend four our UCAVs. Where

$$\beta_1=((d_1,w_1),(d_2,w_2),(d_3,w_4),(\phi,w_3))$$

$$\beta_2=(((d_1,d_2),w_1),(d_3,w_4),(\phi,w_2),(\phi,w_3))$$

$$\beta_3=(((d_1,d_2,d_3),w_1),(\phi,w_2),(\phi,w_3),(\phi,w_4))$$

$$\beta_4=((d_1,w_1),(d_2,w_3),(d_3,w_4),(\phi,w_2))$$

$$\beta_5=((d_1,w_1),((d_2,d_3),w_4),(\phi,w_2),(\phi,w_3))$$

$$\beta_6=(((d_1,d_2,d_3),w_4),(\phi,w_1),(\phi,w_2),(\phi,w_3))$$

The meaning of the strategy $\beta_j(i=1,2,\cdots,6)$ can be explained similarly according to the meaning of the strategy $\alpha_i(i=1,2,\cdots,12)$.

When our strategy α_i and foe's strategy β_j are selected simultaneously, our payment function is set as follows:

$$u_1(\alpha_i,\beta_j)=a_{ij}=\frac{m1^{\alpha_i}(a)\cdot m2^{\beta_j}(b)}{m1^{\alpha_i}(b)\cdot m2^{\beta_j}(a)},$$
$$i=1,2,\cdots,12;\ j=1,2,\cdots6 \qquad (12)$$

Similarly, foe's payment function is set as follows:

$$u_2\left(\alpha_i,\beta_j\right)=b_{ij}=\frac{m1^{\alpha_i}\left(b\right)\cdot m2^{\beta_j}\left(a\right)}{m1^{\alpha_i}\left(a\right)\cdot m2^{\beta_j}\left(b\right)},$$
$$i=1,2,\cdots,12;\ j=1,2,\cdots 6 \qquad (13)$$

Actually, the formula (12) and (13) are embodiment of the formula (10) and (11) after our strategies α_i and foe's strategies β_j are confirmed.

4.3 Result

The simulation consists of a set of computing process as follows:

Firstly, when α_i and β_j mentioned in section IV(B) are selected simultaneously, the actual capability of the suppositional sensor (SAR radar) and weapon (JDAM bomb) aboard on the UCAV and the suppositional sensor (track homing radar) and weapon (short ground to air missile) in the missile position mentioned in section IV(A) may be transformed respectively into the unitary fighting effect values ($m1_{RL}^{\alpha_i}\left(\bullet\right)$, $m1_{R\Theta}^{\alpha_i}\left(\bullet\right)$, $m1_{R\Psi}^{\alpha_i}\left(\bullet\right)$, $m1_{ML}^{\alpha_i}\left(\bullet\right)$, $m1_{M\Theta}^{\alpha_i}\left(\bullet\right)$,and $m1_{M\Psi}^{\alpha_i}\left(\bullet\right)$; $m2_{RL}^{\beta_j}\left(\bullet\right)$, $m2_{R\Theta}^{\beta_j}\left(\bullet\right)$, $m2_{R\Psi}^{\beta_j}\left(\bullet\right)$, $m2_{ML}^{\beta_j}\left(\bullet\right)$, $m2_{M\Theta}^{\beta_j}\left(\bullet\right)$,and $m2_{M\Psi}^{\beta_j}\left(\bullet\right)$) by adopting the algorithm designed in section II.

Secondly, the total fighting effect (including validity, invalidity and uncertainty) of the whole our aircrafts team $m1^{\alpha_i}\left(\bullet\right)$ and the total fighting effect of the whole foe's targets group $m2^{\beta_j}\left(\bullet\right)$ mentioned in the formula (12) and (13) in the section IV(B) can be acquired by a set of computing processes based on the algorithms designed in section II and the unitary fighting effect values acquired by the above computing.

As a result, our payment matrix $A=\left(a_{ij}\right)_{m\times n}$ and foe's payment matrix $B=\left(b_{ij}\right)_{m\times n}$ mentioned in section III can be acquired by computing the formula (12) and (13) and shown as follows:

$$A=\left(a_{ij}\right)_{12\times 6}=$$
$$\begin{array}{c|cccccc|}
 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 \\
\alpha_1 & 1.120 & 3.692 & 9.216 & 1.115 & 3.690 & 8.687 \\
\alpha_2 & 2.314 & 7.631 & 19.05 & 2.305 & 7.626 & 17.96 \\
\alpha_3 & 0.126 & 0.416 & 1.038 & 0.126 & 0.416 & 0.978 \\
\alpha_4 & 12.56 & 41.41 & 103.4 & 12.51 & 41.38 & 97.43 \\
\alpha_5 & 2.724 & 8.984 & 22.43 & 2.714 & 8.979 & 21.14 \\
\alpha_6 & 2.408 & 7.940 & 19.82 & 2.399 & 7.934 & 18.68 \\
\alpha_7 & 0.444 & 1.463 & 3.653 & 0.442 & 1.462 & 3.443 \\
\alpha_8 & 0.012 & 0.039 & 0.098 & 0.012 & 0.039 & 0.092 \\
\alpha_9 & 0.009 & 0.029 & 0.071 & 0.009 & 0.029 & 0.067 \\
\alpha_{10} & 0.096 & 0.318 & 0.793 & 0.096 & 0.317 & 0.747 \\
\alpha_{11} & 0.522 & 1.723 & 4.301 & 0.521 & 1.722 & 4.054 \\
\alpha_{12} & 0.113 & 0.374 & 0.933 & 0.113 & 0.374 & 0.880
\end{array}$$

$$B=\left(b_{ij}\right)_{12\times 6}=$$
$$\begin{array}{c|cccccc|}
 & \beta_1 & \beta_2 & \beta_3 & \beta_4 & \beta_5 & \beta_6 \\
\alpha_1 & 0.893 & 0.271 & 0.109 & 0.897 & 0.271 & 0.115 \\
\alpha_2 & 0.432 & 0.131 & 0.052 & 0.434 & 0.131 & 0.056 \\
\alpha_3 & 7.931 & 2.405 & 0.963 & 7.960 & 2.406 & 1.022 \\
\alpha_4 & 0.080 & 0.024 & 0.010 & 0.080 & 0.024 & 0.010 \\
\alpha_5 & 0.367 & 0.111 & 0.045 & 0.368 & 0.111 & 0.047 \\
\alpha_6 & 0.415 & 0.126 & 0.050 & 0.417 & 0.126 & 0.054 \\
\alpha_7 & 2.254 & 0.683 & 0.274 & 2.262 & 0.684 & 0.290 \\
\alpha_8 & 84.39 & 25.59 & 10.25 & 84.71 & 25.61 & 10.88 \\
\alpha_9 & 115.4 & 34.99 & 14.02 & 115.8 & 35.01 & 14.87 \\
\alpha_{10} & 10.38 & 3.149 & 1.262 & 10.42 & 3.151 & 1.338 \\
\alpha_{11} & 1.914 & 0.580 & 0.233 & 1.921 & 0.581 & 0.247 \\
\alpha_{12} & 8.820 & 2.675 & 1.071 & 8.853 & 2.676 & 1.137
\end{array}$$

Finally, the Nash equilibrium state of the game model G mentioned in section IV(B) may be acquired by solving the matrix A and B based on the Scarf arithmetic in the Gambit software package.

In the situation of Multi-aircraft Cooperative Attack Multi-target enacted section IV(A), the Nash equilibrium state of the game model G is that our strategy α_4 and foe's strategy β_4 are selected simultaneously.

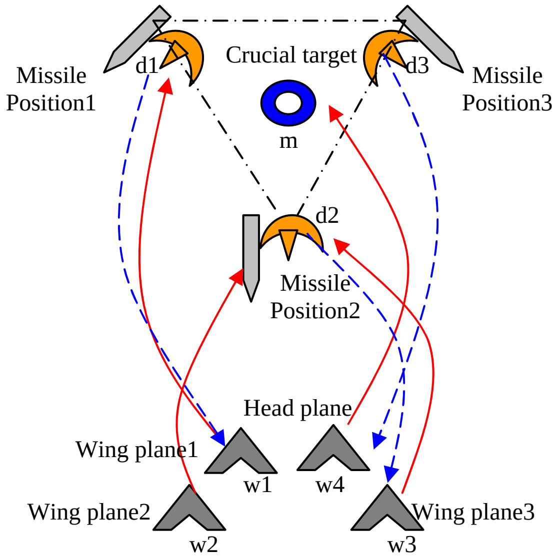

Fig.3 Our No.4 Strategy VS Foe's No.4 Strategy

Our strategy of α_4 and foe's strategy of β_4 will be regarded as the mission decision-making of our team antagonizing foe's group (shown in Fig. 3) and be used to decide the fighting action of our aircrafts team and guess the defending action of foe's targets group, respectively.

5 Result Analysis

For validating the rationality of the studying results in section IV, on the one hand, the other two strategies α_5 and α_6 that the fighting effect difference between the two strategies and strategy α_4 is least are selected to antagonize foe's strategy β_4 (shown in Fig.4 and Fig.5); on the other hand, the other two strategies β_2 and β_6 that the fighting effect difference between the two strategies and strategy β_4 is least are selected to antagonize our strategy α_4 (shown in Fig.6 and Fig.7).

The result of comparing the fighting effect of our strategies (α_4 , α_5 , and α_6) and the fighting effect of foe's strategy β_4 is shown in Table 1, while the result of comparing the fighting effect of foe's strategies (β_2 , β_4 and β_6) and the fighting effect of our strategy α_4 is shown in Table 2.

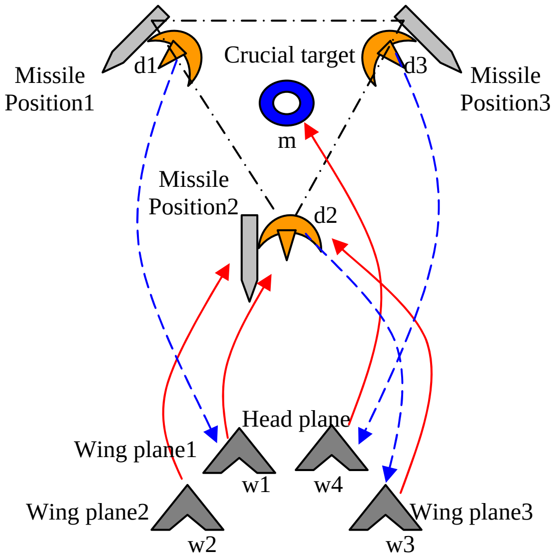

Fig.4 Our No.5 Strategy VS Foe's No.4 Strategy

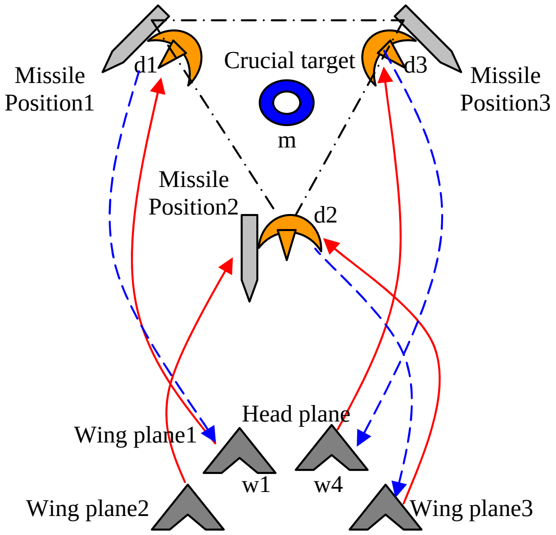

Fig.5 Our No.6 Strategy VS Foe's No.4 Strategy

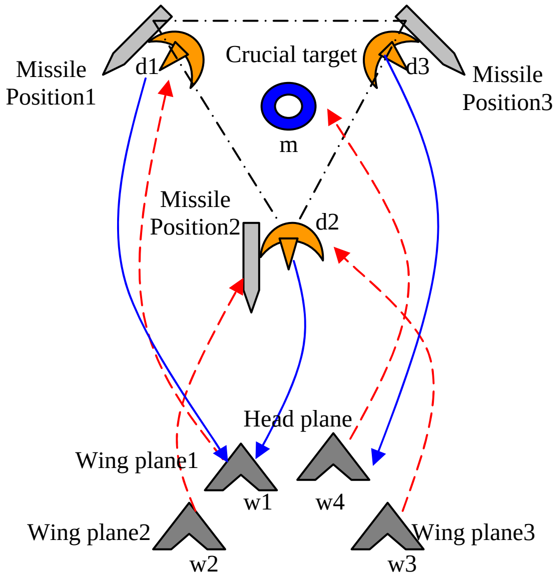

Fig.6 Foe's No.2 Strategy VS Our No.4 Strategy

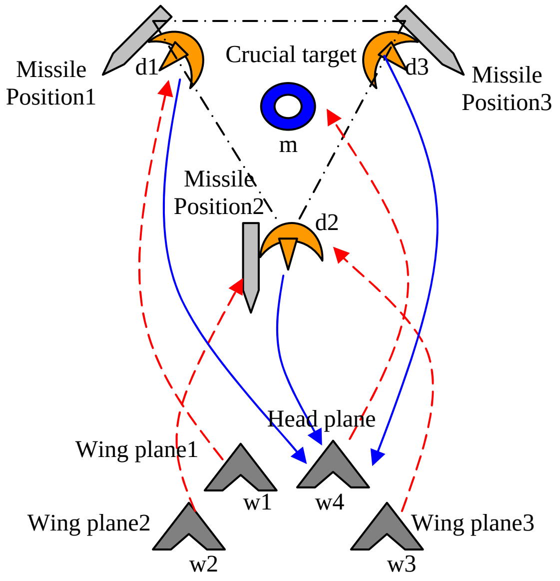

Fig.7 Foe's No.6 Strategy VS Our No.4 Strategy

Table 1 Comparing Result of Our Effect

	a	b	θ
$m2^{\beta_4}(\bullet)$	0.869	0.130	0.001
$m1^{\alpha_4}(\bullet)$	0.988	0.011	0.001
$m1^{\alpha_5}(\bullet)$	0.947	0.052	0.001
$m1^{\alpha_6}(\bullet)$	0.940	0.059	0.001

Table 2 Comparing Result of Foe's Effect

	a	b	θ
$m1^{\alpha_4}(\bullet)$	0.988	0.011	0.001
$m2^{\beta_2}(\bullet)$	0.669	0.330	0.001
$m2^{\beta_4}(\bullet)$	0.869	0.130	0.001
$m2^{\beta_6}(\bullet)$	0.467	0.532	0.001

Table 1 indicates that the fighting effect (validity) of our strategies α_4, α_5, and α_6 is respectively $m1^{\alpha_4}(a)=0.988$, $m1^{\alpha_5}(a)=0.947$, and $m1^{\alpha_6}(a)=0.940$. As a result, it can be explained why our strategy α_4 is regarded as the fighting action that our aircrafts team should carry out when our team is antagonizing foe's group in the case described in section IV(A).

Similarly, table 2 indicates that the fighting effect (validity) of foe's strategies β_2, β_4, and β_6 is respectively $m2^{\beta_2}(a)=0.669$, $m2^{\beta_4}(a)=0.869$, and $m2^{\beta_6}(a)=0.467$. As a result, it can be also explained why foe's strategy β_4 is regarded as the defending action that foe's targets group may carry out in the same case.

6 Conclusion

In airfight of multi-aircraft cooperative attack multi-target, mission decision-making is great importance for acquiring the highest fighting effect. In this paper, a mission decision-making algorithm is developed under a game theoretic framework and a situational assessment algorithm is designed based on the D-S evidence theory, the latter algorithm provides the foundation for the front algorithm. The simulation shows that the game theoretic algorithm is capable of solving the coordinated mission decision-making problem.

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