FATIGUE ANALYSIS: THE SUPER-NEUBER TECHNIQUE FOR CORRECTION OF LINEAR ELASTIC FE RESULTS

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Abstract

Fatigue analysis with very long loading histories and considerable yielding poses big problems to the analyst. Considerable yielding calls for a FE analysis with non-linear material model. With a very long loading sequence and a FE-model with necessary resolution the computing time can raise to such unrealistic levels as several weeks or even years. The solution to this problem has so far been to make a linear elastic FE-analysis of the loading history and to perform elastic-plastic corrections afterwards, using the Neuber rule or the linear rule. The quality of such corrected stresses and strains can however be doubtful; using one or the other of the two principles can cause quite different results and there are no safe rules for how to select the better of the two.

A new technique is here suggested, the "Super Neuber technique". It is based on the Neuber rule, with the Neuber hyperbola: $\sigma = \frac{K}{\varepsilon}$ *. It also uses a non-linear FE-result from one "simple load cycle" extracted from the loading history: "minimum load" – "maximum load" – "minimum load". The result from the "simple load cycle" is used to calibrate a parameter q characteristic of the Super Neuber technique, with the Super Neuber hyperbola:* $\sigma = \frac{K_q}{\varepsilon^q}$. *When this parameter has been determined it is possible to calculate corrected stresses and strains for the complete loading sequence. The quality of this result will be comparable to the quality of a non-linear FE-result, as long as the loading can be characterized as proportional or close to proportional.*

INTRODUCTION

Volvo Aero (VAC) is developing jet engine and rocket engine components. Fatigue life prediction is of course of utmost importance in our design system, and best possible methods for fatigue analysis are therefore necessary. We have during several years been working with improvements of methods involved at fatigue life prediction. One result of this work [2] was dealing with the Fatemi and Socie critical plane model, and how to calculate the fatigue damage based on among other things the initially calculated states of stress and strain. Of all factors influencing the quality of calculated fatigue damage, the quality of calculated stress and strain has a dominating impact. This first and vitally important step in the chain of analyses is normally performed with a FE-code. The most appropriate way to handle cases with multiaxial stresses above the yield point is to perform the FE-analysis with a suitable material model. Better computing capacity and improved FE-programs make non-linear FE-analysis increasingly frequent. For shorter load sequences this should be the primary choice. Non-linear FE-analyses can however be very time-consuming for very long load sequences and convergence problems are difficult to avoid. A tendency, that we are experiencing, is that longer and longer load sequences are delivered by engine contractors to VAC as a subcontractor. If the amount of yielding is small to considerable (often/sometimes such "contained plasticity" exist at "notches") on such very long load sequences, a linear elastic analysis can be justifiable. Elastic-plastic corrections should then be performed directly after the FE-analysis.

The limitations of such elastic-plastic corrections must however be clear. They will inevitably be performed on a "node-by-node" basis; information about the situation for "the neighbouring nodes" is normally very difficult to utilize:

- Stiffness dependent geometric redistribution of stresses can not be handled
- Advanced yield models with visco-plastic effects and strain hardening are not meaningful
- Non-proportional loading should be treated with caution; the use on strongly non-proportional loading such as thermally dominated loading is not recommended.

The analyst is recommended to take the limitations under extra consideration:

- Is a non-linear FE-analysis (despite being arduous and time-consuming) the best approach ?; or
- Will elastic-plastic corrections after a linear elastic FE-analysis be sufficient ?

The limitations of the presumptions lead to the conclusion, that the elastic-plastic correction should be kept as simple as possible. It must however be able to follow the intended non-linear material model; the one that had been used, if a non-linear FE-analysis had been performed. As kinematic hardening is a good model for many of the materials we are using, this is a necessary feature to be included in our elastic-plastic correction procedure. Kinematic hardening behaviour is therefore assumed in the following description.

NEW PROCEDURE FOR ELASTIC-PLASTIC CORRECTION

Elastic-plastic corrections of linear elastic stress/strain have been generally accepted in industry and performed as standard procedure according to procedures described in for instance reference [1].

One basis for such elastic-plastic correction ***is the yield criterion of von Mises***, stating that ***yield occurs when the effective stress reaches the uniaxial yield stress value*** of the material. This is one of the assumptions we also make in our elastic-plastic correction procedure. Calculation of von Mises effective stress and strain, based upon stress and strain components are according to:

$$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2 - \sigma_x \cdot \sigma_y - \sigma_y \cdot \sigma_z - \sigma_x \cdot \sigma_z + 3 \cdot (\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)} =$$

$$= \frac{1}{\sqrt{2}} \cdot \sqrt{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_x - \sigma_z)^2 + 6 \cdot \left[\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right]}$$

$$\varepsilon_e = \frac{1}{(1+\nu) \cdot \sqrt{2}} \cdot \sqrt{(\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_x - \varepsilon_z)^2 + 6 \cdot \left[\left(\frac{\gamma_{xy}}{2}\right)^2 + \left(\frac{\gamma_{yz}}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2\right]}$$

(from ref. [1] pages 245 and 246)

A second basis is material behaviour as described by the stress-strain curve, with a link between linear elastic data and corrected elastic-plastic data.

Traditionally this link has been the Neuber rule, with the linear rule as alternative.

The Neuber rule is applicable at states of "contained plasticity"="plasticity limited to areas surrounded by and ruled by elastic conditions". Normally this condition exists at "notches", a situation for which the method was developed (actually torsion of a notched shaft):

$$\sigma_e^{elastic} \cdot \varepsilon_e^{elastic} = \sigma_{corr} \cdot \varepsilon_{corr} = constant = K$$

where

σ_{corr} and ε_{corr} are the intersection point for the material data curve and the Neuber hyperbola,

$\sigma_e^{elastic}$ and $\varepsilon_e^{elastic}$ are elastic effective stresses according to formulae above. The only existing alternative has up to now been "the Linear rule", see figure 1.

Two major problems have however been:

- Which one to select
- Bad agreement between corrected stresses/strains and stresses/strains achieved with a non-linear material model.

A modification of the Neuber rule is suggested, the Super Neuber Rule:

The Neuber equation

$\sigma_e^{elastic} \cdot \varepsilon_e^{elastic} = \sigma_{corr} \cdot \varepsilon_{corr} = K$ is often presented in a more general form, $\sigma = \dfrac{K}{\varepsilon}$, representing the Neuber hyperbola.

We are suggesting a modification, which reads as follows: $\sigma = \frac{K_q}{\varepsilon^q}$

It can be concluded, that putting q =1 results in the normal Neuber hyperbola, and putting $q = \infty$ results in the so called "Linear rule", which is represented by a vertical line through $(\sigma_e^{elastic}, \varepsilon_e^{elastic})$, intersecting the material data curve at $(\sigma_{corr}, \varepsilon_{corr})$, see fig. 1.

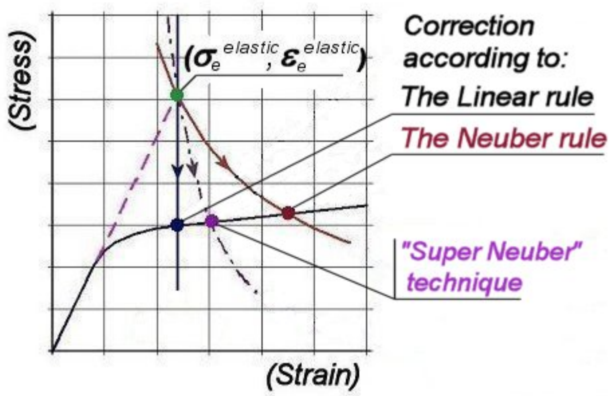

Fig. 1. The Neuber rule, the Linear rule and the "Super-Neuber" technique.

The modification implies that, if a loading cycle is analyzed both linearly and with a non-linear material model, it is by changing q possible to match corrected results from the linear analysis, with results from a non-linear FE-analysis (fig. 1). If the loading cycle represent one of the largest cycles in a specific loading sequence, it is now possible, with the acquired value of q, to produce corrected stress and strain values not only for the analyzed loading cycle, but for the complete loading sequence.

This is essentially what the new technique is all about. We call it the "Super Neuber technique". There are two reasons for using "super" in the name:

1) the expression $\sigma = \frac{K_q}{\varepsilon^q}$ could be called a super hyperbola (in analogy with the definition of a super ellipse)
2) the technique is with respect to accuracy clearly superior to the Neuber rule, but we still consider it as being just a modification to the well reputed and acknowledged Neuber rule.

The formulation $\sigma = \frac{K_q}{\varepsilon^q}$ is elegant, but it has one drawback when it comes to the numerical part. It leads to an iterative procedure to find the intersection with the stress-strain curve. A more efficient approach is to make a linear interpolation between results achieved using the "Linear rule" and results achieved using the "Neuber rule". This modification is illustrated in fig. 2.

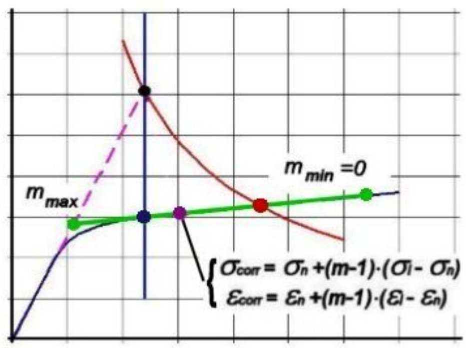

Fig. 2. Correction achieved by an approximation with a line through the intersection points of the Neuber rule and the Linear rule.

The corrected stresses and strains will then be according to:

$$\sigma_{corr} = \sigma_n + (m-1)\cdot(\sigma_l - \sigma_n)$$
$$\varepsilon_{corr} = \varepsilon_n + (m-1)\cdot(\varepsilon_l - \varepsilon_n)$$

Here

σ_n and ε_n are the corrected values using the Neuber rule

and

σ_l and ε_l are the corrected values using the Linear rule.

Then

m=1 will represent the Neuber rule and

m=2 will represent the Linear rule.

1<m<2 represents a linear interpolation between the Neuber rule and the Linear rule.

m<1 is a possible situation. It represents an extrapolation with respect to the "line" between the Neuber rule and the Linear rule, but the value is controlled by a non-linear FE-result (see below), without the usual risks with extrapolation. We are today suggesting m=0 to represent the minimum value of m.

m>2 represents also a possible situation; we are suggesting a maximum value to be given by the intersection with the E-modulus line, see fig. 2.

m (as well as q) is thus not known á priori. A best value of m is achieved by running a FE-analysis with a non-linear material model (showing kinematic hardening behavior) and with the component loaded by a simple load sequence consisting of minimum load and maximum load in the load sequence of interest. The same simple load sequence is run with linear elastic data in the FE-model. The parameter m is then varied to get best match between corrected stresses/strains and non-linear stress-strain FE-results.

The introduction of this new method, "the Super Neuber Rule", represents a significant improvement with respect to the quality of corrected stresses and strains. The quality is comparable to FE-results achieved with a non-linear material model.

This method then implies that a FE-analysis with a non-linear material model must be performed. As it is only necessary to analyze a simple load sequence containing "minimum load" and "maximum load", this will be a reasonable effort, compared to analyzing the complete sequence containing numerous load steps. The method works very well on cases with proportional loading and on cases with non-proportional loading, if the load steps with yielding are mainly proportional. If the loading is mainly thermal (thermal loading is to its character non-proportional) the method will not work very well. Neither will the Neuber rule nor will the Linear rule work well on thermal loading. Having said this, it could be added, that the "Super Neuber rule" will unarguably work better than the Neuber rule or the Linear rule because of the matching procedure.

It must also be pointed out, that acquired m-values are different for different nodes, and to be able to perform the described procedures consistently and rationally we have developed a computer program, *elasplasgen.f90*.

Now that we have calculated $(\sigma_{corr}\ ,\ \varepsilon_{corr})$, it is possible to make corrections on the component stresses and strains. This is done by first calculating the ratios

$SR_{ep}=\sigma_{corr}\ /\ \sigma_e^{elastic}$ and

$ER_{ep}=\varepsilon_{corr}\ /\ \varepsilon_e^{elastic}$.

The component stresses and strains are then calculated as

$\sigma_{x\ corr} = \sigma_x \cdot SR_{ep}$,

$\sigma_{y\ corr} = \sigma_y \cdot SR_{ep}$, …,

$\tau_{xz\ corr} = \tau_{xz} \cdot SR_{ep}$

$\varepsilon_{x\ corr} = \varepsilon_x \cdot ER_{ep}$,

$\varepsilon_{y\ corr} = \varepsilon_y \cdot ER_{ep}$,…,

$\gamma_{xz\ corr} = \gamma_{xz} \cdot ER_{ep}$

This means that the ***corrected component stresses and strains (state after yielding) remains relative to each other as the elastic component stresses and strains (state before yielding)***, which often is a deviation from actual behaviour. This deviation is however small for proportional loading.

Now we have performed a best possible elastic-plastic correction during a load cycle with yielding. To make the process work for a whole load sequence, more definitions and procedures have to be added.

Memory rules have to be added for how the yielding in different load steps is related to the stress-strain data of the material. The memory rules described below are modelling "kinematic hardening" and "Bauschinger effect".

"Kinematic hardening" and "Bauschinger effect" can be seen in fig. 3, where the "single cyclic stress-strain curve" is exchanged by the "double cyclic stress-strain curve" upon reversing the load after reaching maximum load.

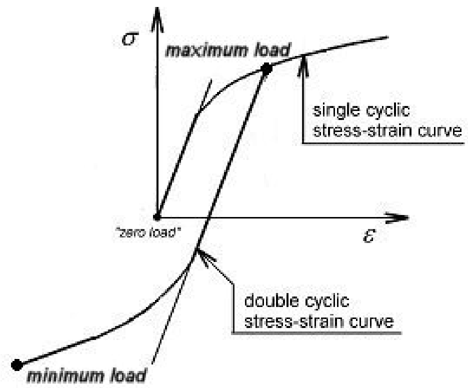

Fig. 3. Single and double cyclic stress-strain curve, also depicting kinematic hardening.

It is however possible to also have other types of behaviour, such as “isotropic hardening”, by modifying the procedure.
Here is the description of “the memory rules”, see fig. 4.

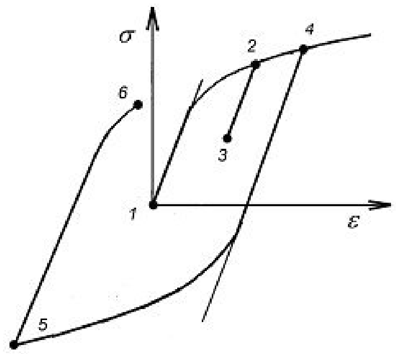

Fig. 4. Stress-strain for 6 load steps, here used in the explanation of “memory rules”

Before first “turning point” (point 2 in fig. 4) (turning is assumed to take place, when the largest of the normal stress component ranges changes sign), all load steps will be corrected using the “single cyclic stress-strain curve”. As the temperature in the general case is changing, the procedure will be to interpolate the “single cyclic stress-strain curve” to the mean temperature value of first load step and the “turning point” load step.
The “single cyclic stress-strain curve” is the normal cyclic stress-strain curve, which can be obtained at uniaxial material testing. The “single cyclic stress-strain curve” should not be mixed up with the “monotonic stress-strain curve”, which is obtained at a tension test.

For fatigue analyses, the “single cyclic stress-strain curve” should normally be used. It can be provided as a table of data points with stress versus strain (to be preferred) or on the Ramberg-Osgood form. The “double cyclic stress-strain curve” is differing from the single (or normal) cyclic stress-strain curve with a factor of 2 . Having the single cyclic stress-strain curve as a table of data points, the “double cyclic stress-strain curve” is obtained by multiplying each of the stress versus strain data points with 2, see fig. 3.
Upon turning (after point 2 in fig. 4), the “double cyclic stress-strain curve” (interpolated to the mean temperature value of first “turning point” load step and the second “turning point” load step) will be used until the second turning point (point 3 in fig. 4). If next turning takes place without yielding, the former curve (in this case the “single cyclic stress-strain curve”) will be used again, but with the yield point shifted to the stress and strain of the first turning point (point 2 in fig. 4). If, however, next turning takes place after yielding (point 4 in fig. 4), a new “double cyclic stress-strain curve” will be used. This principle is followed and repeated through the whole load sequence: turning after yielding – use new “double cyclic stress-strain curve”, turning without yielding – remember the former curve and use it, but with shifted yield point. The values of those “turning points” having yielding are especially important (as for instance point 5 in figure 4). Each “turning point” with yielding represents the start of a new stress-strain curve. It also represents a reference value for increments in stress-strain of succeeding load steps, until next yielding takes place. As these “turning points” represent such reference points, it is important to make the calculation of the corrected value of these “turning points” as numerically accurate as possible, to minimize accumulation of truncation and interpolation errors. Again, the procedures require a computer program.
Fig. 3 is showing a typical behaviour at load cycling: during the first phase, before the load is turning, the behaviour follows the “single cyclic stress-strain curve” (or just “cyclic stress-strain curve”), during next phase, after the load has

turned, the behaviour follows the "double cyclic stress-strain curve". The data points of the "double cyclic stress-strain curve" are achieved as the data points of the "single cyclic stress-strain curve" multiplied by 2. The Bauschinger effect is also clearly illustrated: A material subjected to tension above the yield stress point will normally yield in compression at a considerably lower stress value than original yield stress.

THE PRIMARY REASON FOR THE DEVELOPMENT OF THE NEW METHOD

Problems to achieve reliable stresses and strains are often encountered during product development. To us it was carried to an extreme during the development of a rocket engine component. The load specification was changed considerably with respect to the magnitude of a vibration load. It was then found, that one part of the component could become critical with respect to fatigue life, as considerable yielding took place. This critical part can be seen in fig. 5. Also seen there is a von Mises stress plot for a large amplitude value and accompanying deformation plot. As the loading was complicated, it was decided to run a component test, to get the correlation between fatigue life prediction and actual fatigue life. The result of this correlation is however another story.

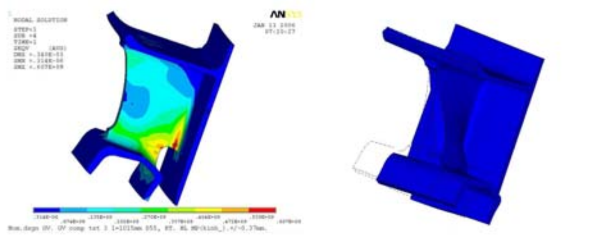

Fig. 5. von Mises effective stress for non-linear FE (left) and displacement plot (right) for load step "displacement=0.369 mm". Maximum von Mises value is 607 MPa, a linear analysis resulted in a maximum von Mises value of 2389 MPa.

The specified load, which had received a raised magnitude, was a random vibration. For the test however, it was transformed to a deterministic vibration load, see fig. 6, with very strong variation in amplitude and long duration. At the fatigue analysis stage it was found, that the load had around 27500 load steps and a large part of the steps resulted in considerable yielding. This indicated that a non-linear FE-analysis was needed. An estimation of computer time gave the unrealistic figure of 3.5 years (on a Silicon Graphics Octane workstation)! With the Super Neuber technique, the computer time was reduced to a couple of hours. For this specific problem, where the loading resulted in an eigenmode displacement pattern, there was at least one other way to go to solve the problem: non-linear FE-analysis of a couple of load cycles with increasing amplitude up to the maximum amplitude value of the sequence, followed by interpolation in these results to get stress and strain-values for all the cycles of the complete load sequence. With respect to resulting fatigue life, it gave exactly the same figure as the Super Neuber technique. It is however, in contrast to the Super Neuber technique, not applicable in more general cases of loading. It must be added here, that sequences with around 30000 load steps are very rare to us; more general are lengths of around 1000 load steps. 3.5 years divided by 30 is however 43 days, and that is, modestly expressed, a considerable amount of time, especially for a project leader!

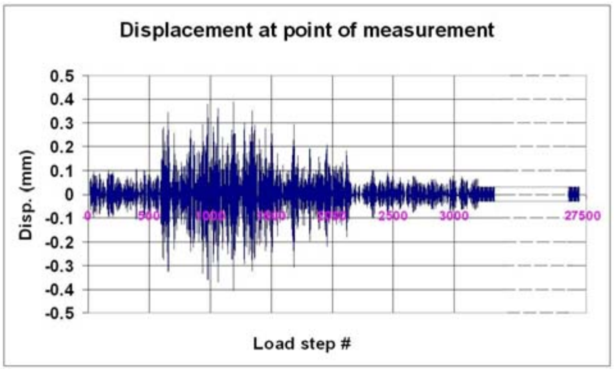

Fig. 6. The very long load sequence defined as a test specific displacement. Yield occurred at displacement values above 0.1 mm.

VALIDATION

The just related example shows the potential of the technique with respect to saving of time. It is however not showing clearly the accuracy. Two more examples are therefore presented to show more in detail what results the Super Neuber technique is giving. The validation is done by comparison with ANSYS results. ANSYS is here naturally the reference as the purpose with the new method was to replace the time-consuming non-linear FE-analyses performed with ANSYS.

Example 1:

Tension load sequence applied on a hollow test specimen with notch, see fig. 7.

The material of the specimen is Ti 6-4, inner diameter is around 10.85 mm, minimum outer diameter at notch is around 14.98 mm, and notch radius is around 2 mm. The applied load sequence has a character of "flight mission", but only the first 17 load steps of this typical "flight mission" have been used for this validation test. The complete flight mission load sequence can bee seen in fig. 8. The truncation at 17 load steps can also be seen. Fig. 9 is showing a plot of 1:st principal stress (linear elastic analysis) for a "maximum load" load step. Fig. 10 (next page) is showing plots of stress versus strain for a critical node for the 17 load steps: σ_x-ε_x , σ_y-ε_y and σ_z-ε_z as calculated by ANSYS and *elasplasgen.f90* with best possible m-value (1.67). Also shown are σ_y-ε_y as calculated by *elasplasgen.f90* having Neuber-value of m (=1) and Linear-value of m (=2). Fig. 10 (next page) clearly shows how well the *elasplasgen.f90* results follow the ANSYS results, and also that both the Neuber rule and the Linear rule cause rather bad corrections.

Fig. 7. Hollow test specimen with notch, which was subjected to tension. Fatigue tests were performed, hence the crack, these were however not the subject of this article.

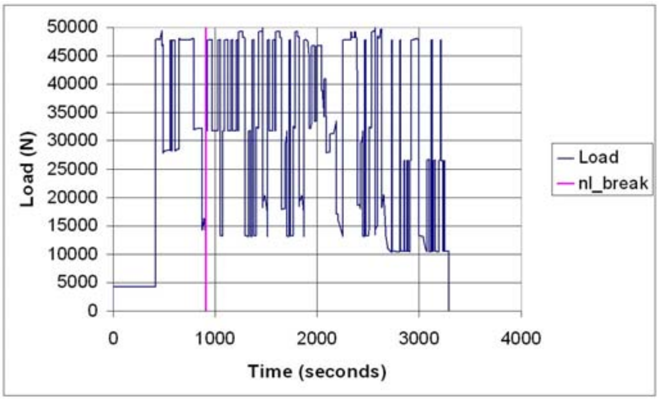

Fig. 8. The complete load sequence. The pink line shows where the sequence was truncated.

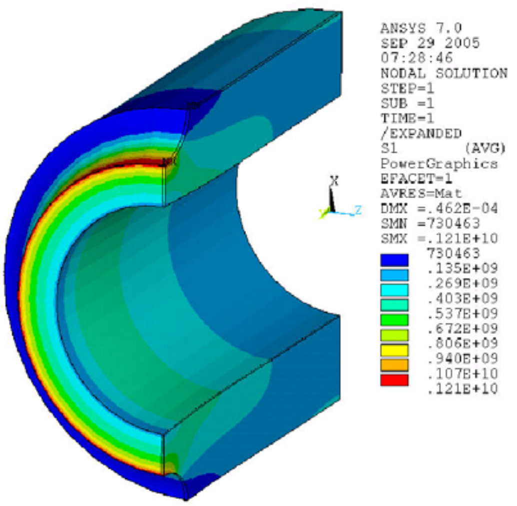

Fig. 9. Maximum principal stress for tension load case.

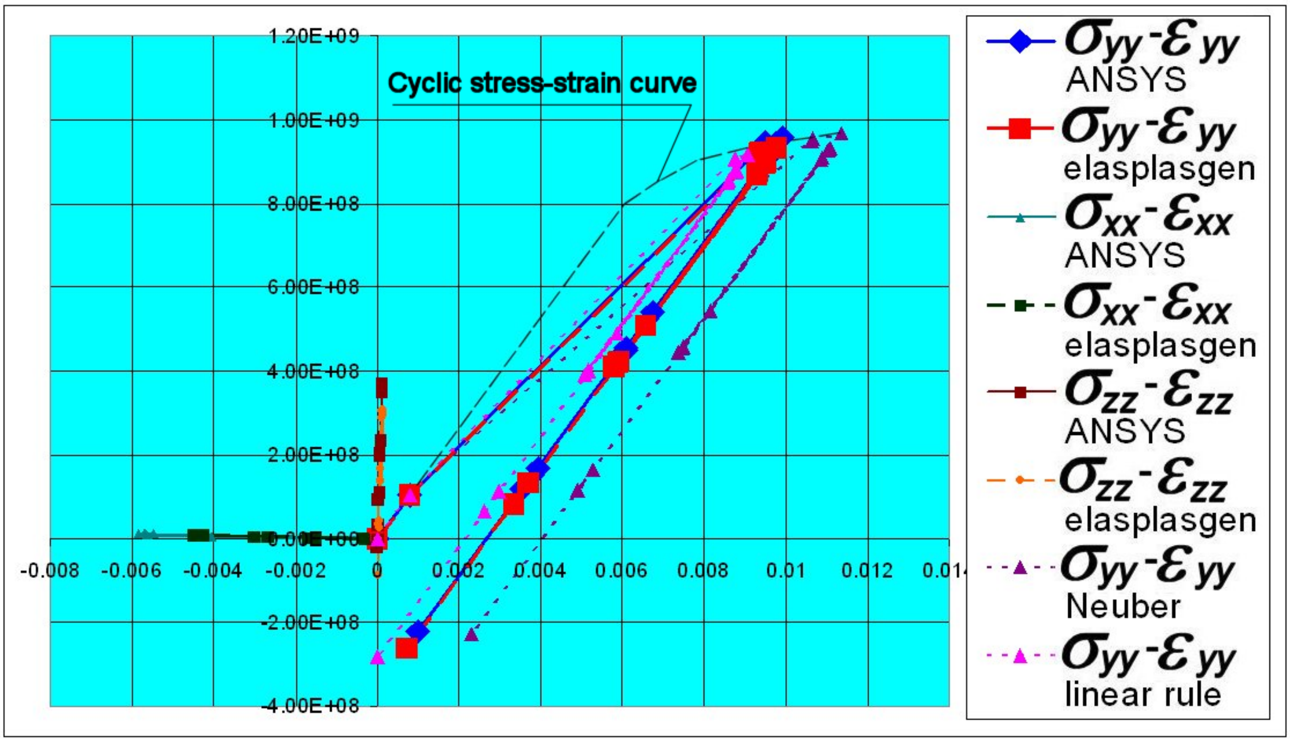

Fig. 10. Acquired stress-strain behavior of a critical node for ANSYS and for *elasplasgen.f90*: run with Neuber rule (m=1), linear rule (m=2) and with best correlated m=1.67. The points on the curves represent 17 load steps of the tension load sequence. Only the most significant stress- and strain components are plotted.

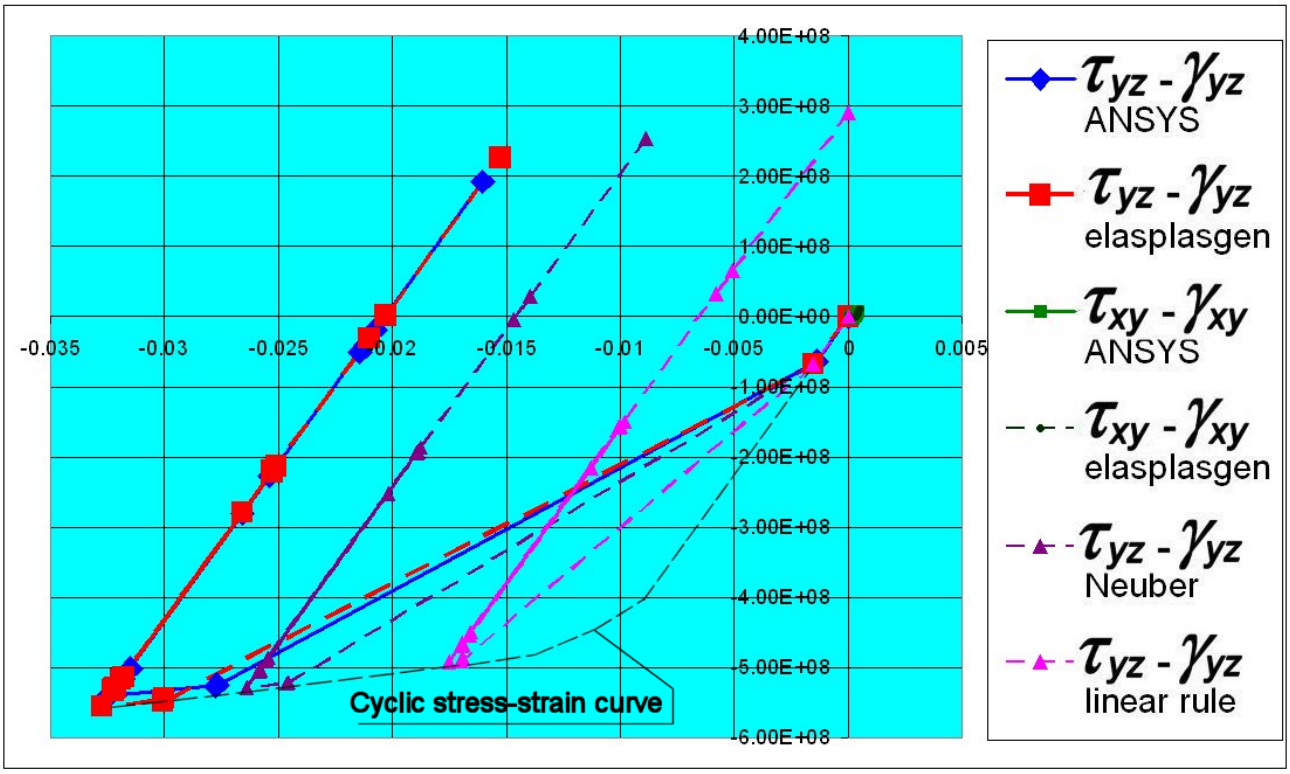

Fig. 14. Acquired stress-strain behavior of a critical node for ANSYS and for *elasplasgen.f90*: run with Neuber rule (m=1), linear rule (m=2) and with best correlated m=0.29. The points on the curves represent 17 load steps of the torsion load sequence. Only the most significant stress- and strain components are plotted.

Example 2:
Torsion load sequence applied on a hollow test specimen with notch, see fig. 11.
Here it is the same test specimen as in example 1, but now with a load sequence with torsion only. The applied load sequence has the same character as for example 1, similar to a "flight mission", but only the first 17 load steps have been used for the validation test. The complete flight mission load sequence can bee seen in fig. 12. The truncation at 17 load steps can also be seen. Fig. 13 is showing a plot of von Mises effective stress (linear elastic analysis) for a "maximum load" load step. Fig. 14 (former page) is showing plots of stress versus strain for a critical node for the 17 load steps: τ_{xy}-γ_{xy} and τ_{yz}-γ_{yz} as calculated by ANSYS and *elasplasgen.f90* with best possible m-value (0.29). Also shown are τ_{yz}-γ_{yz} as calculated by *elasplasgen.f90* having Neuber-value of m (=1) and Linear-value of m (=2). Fig. 14 (former page) clearly shows how well the *elasplasgen.f90* results follow the ANSYS results, and also that both the Neuber rule and the Linear rule cause really bad corrections.

Fig. 11. Hollow test specimen with notch, which was subjected to torsion. Fatigue tests were performed, hence the crack, this is however another story.

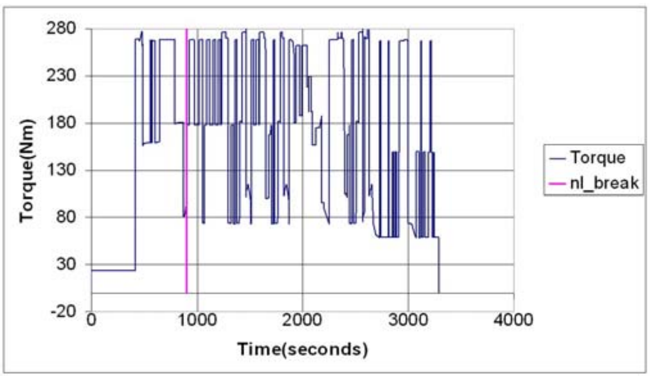

Fig. 12. The complete load sequence. The pink line shows where the non-linear FE-analysis was interrupted.

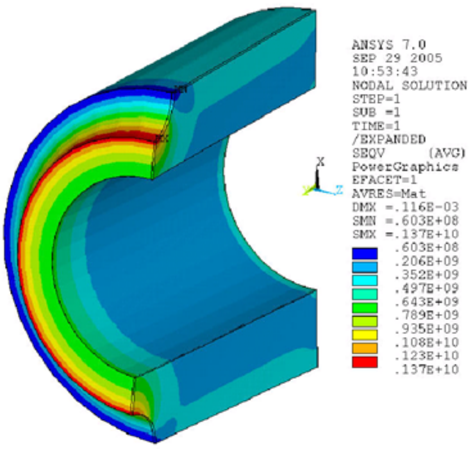

Fig. 13. von Mises effective stress for torsion load case

CONCLUSION

A new method, called the Super Neuber Technique, to make elastic-plastic corrections for a load sequence calculated with a linear elastic FE-analysis, has been developed. It has been shown in two examples, that the quality of the performed corrections is far better than what can be achieved with the Neuber rule or the Linear rule, and comparable to non-linear FE-results. It has also been shown in one example, that the purpose of speeding up the analyses, compared to non-linear FE-analysis of the complete load sequence, has been fulfilled.

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