A HYBRID GENETIC/INTERIOR-POINT ALGORITHM FOR MULTI-DISCIPLINARY OPTIMIZATION (MDO) PROBLEMS IN AIRCRAFT PRELIMINARY DESIGN

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Abstract

A hybrid optimization algorithm is proposed to solve MDO problems encountered in aircraft design at the preliminary stage. It combines the global behavior of an evolutionary algorithm with the efficient but local convergence of a deterministic algorithm.

Numerical analytical tests show the advantages of the approach as well as its limits. Results for a more realistic case, namely the range maximization for a generic supersonic business jet (SSBJ), are also given.

1 Introduction

We are interested in using the Multi-Disciplinary design optimization (MDO) approach, as described for example in [5]-[6], to formalize our preliminary aircraft design process. In a slight difference from the above mentioned references, we have put the emphasis on "D" as discipline (instead of design) as we believe that to achieve this MDO goal, relevant and accurate simulation feedbacks are needed, from specific disciplines such as aerodynamics or structural mechanics.

They are readily available but to include all these computations into a single optimization package is a huge endeavor and practically not desirable. To alleviate these code integration issues, we rely instead on response surface approximations, as proposed in [2].

These approximations are *locally* polynomial and thus are smoother than the original functions. However, since the modeling complexity varies from one discipline to another, the overall optimization problem, although smooth, can present a lot of local minima.

To sum up, we need at this stage, an optimization algorithm for *smooth* functions which does not get stuck in *local minima*. In this paper, a hybrid genetic/interior-point algorithm is proposed as a possible candidate for our MDO optimizer.

1.1 The mathematical MDO problem

Our one-objective MDO problem, supplied with response surface approximations, is cast into the following general non-linear programming framework :

$$\begin{cases} \text{Minimize} & f(x) \\ \text{subject to :} & \begin{cases} g_i(x) \le 0, i = 1,\ldots,m \\ h_i(x) = 0, i = 1,\ldots,p \end{cases} \end{cases} \quad (1)$$

where x is a n-dimensional design variable, f is the cost function and g_i (resp. h_i) are inequality (resp. equality) constraint functions. In accordance with our working hypothesis, these functions are supposed to be *smooth*, at least continuously differentiable. In practice, it would be the case if we limit ourselves to optimization problems of the "sizing" type.

2 The hybrid algorithm

Hybridization techniques combining genetic and deterministic algorithms to solve problem (1) have been extensively investigated : indeed, a Web search on this subject would produce over 4,000 hits ! Following a pragmatic approach, we will try to combine two optimizers which have been validated on a wide range of mono-disciplinary optimization problems at Dassault-Aviation.

2.1 AGORA : an evolutionary algorithm [8]

It is a binary-coded genetic algorithm with tournament based selection, crossover and mutation. Constraints are treated via the superiority of feasible points method, which consists in adding penalty terms. AGORA has been effective for optimization problems with large topological modifications. Given enough computer time, AGORA will go to the *global* optima. In some cases, clustering techniques have been shown to speed-up convergence.

2.2 IPA : a deterministic algorithm [3]-[4]

It is a gradient-based feasible direction interior-point algorithm, which will converge to a *Karush-Kuhn-Tucker* point of problem (1), nearest to the initial *feasible* point, at best a *local* optimum. IPA has been effective for optimization problems with non-linear constraints, as encountered in aerodynamics and structures. Convergence is almost quadratic and when constraints are active, Lagrange multipliers (i.e. sensitivities w.r.t constraints) are available.

2.3 A hybrid algorithm

The general idea behind our hybrid algorithm is to use IPA to create clusters for AGORA, thus greatly reducing its search space. The clusters are the clouds of *feasible* points generated by the convergence trajectories toward local minima found by calls to IPA : each cluster acts as an "attraction" basin for its corresponding local minimum and is stored in a database, namely **data(IPA)**. The generic algorithm would be the following :

We start with AGORA generating its populations of individuals. AGORA will continue its evolution strategy until it encounters a *feasible* individual **I** :

If I is close enough to a cluster in **data(IPA)**, AGORA gets the corresponding local minimum values as the function values and generates a new *random* individual.

Else, IPA is called up with **I** as an initial guess. The corresponding convergence trajectory is stored in **data(IPA)** and AGORA gets the local minimum values as the function values.

3 Analytical test cases

The hybrid algorithm combining AGORA and IPA has been validated on several analytical test cases of increasing complexity. For each IPA call, the convergence tolerance is set to 10^{-3} and the maximum number of function calls to 200. For validation purpose, the results "precision" are computed by re-running the hybrid algorithm 50 times for each test-case.

3.1 Six-hump back case

3.1.1 Description

We have : 2 bounded design variables, no non-linear constraint. The cost function is the following :

$$f(x_1,x_2) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4 \tag{2}$$

It has 6 local minima, the minimum of which is the global minimum (see fig. [1]).

3.1.2 Results

The results are showed in the following table where we can see an important *reduction* in function evaluations and a *narrower* results dispersion when IPA is present.

	Nb. Function evaluations	Precision
AGORA	600	10^{-3}
AGORA+IPA	100	10^{-8}

3.2 New Branin test-case

3.2.1 Description

It is a test-case taken from reference [7]. We have 2 bounded design variables, a simple polynomial cost function and a non-linear constraint which is a combination of polynomials and cosines. The feasible region has 3 disconnected components (see fig. [2]).

3.2.2 Results

Once again, the results in the following table show the importance of having IPA coupled with AGORA, although in this case, the *reduction* in the number of function evaluations is not as great as for the 6-hump back case.

	Nb. Function evaluations	Precision
AGORA	600	10^{-2}
AGORA+IPA	200	10^{-8}

3.3 Gomez test-case

3.3.1 Description

This is another test-case taken from reference [7]. We still have 2 bounded design variables and 1 non-linear constraint which is a much more *wavy* combination of sines. The feasible region has 20 disconnected components and represents only 19% of the total space (see fig. [3]).

3.3.2 Results

We are now at the *limit* of our hybrid algorithm since our problem is more an *exploration* problem than a cost *reduction* one. The *reduction* in the number of function evaluations is minimal and the results dispersion did not improve.

	Nb. Function evaluations	Precision
AGORA	600	10^{-2}
AGORA+IPA	400	10^{-2}

4 Range maximization for a generic SSBJ

We move now to a more *realistic* test case, described in ref.[1] : the SSBJ model includes response surface approximations from 3 specific disciplines: aerodynamics, structures and power plant. We have run the following specific-case :

Cost function : maximum range.

Design variables : 16 variables, mainly shape and engine parameters.

Constraints : maximum weight and maximum landing speed.

Results shown in the following table proves that our hybrid algorithm can bring :

- a *better* solution than with AGORA alone
- an important *reduction* in the number of function evaluations
- a *lesser* dispersion of results.

	Nb. Function evaluations	Range(km)
AGORA	3500	5800±100
AGORA+IPA	1500	6400±75

5 Conclusions

A hybrid algorithm combining a genetic algorithm and a gradient-based interior point algorithm has been shown to be effective for MDO problems, formulated with response surface approximations. There is still room for improvement for the following points:

<u>*IPA database*</u> : we presently define closeness to this database via a discrete l2-norm. each cluster is in fact a convergence trajectory which can be defined as a curve in a search space.

<u>*IPA Lagrange multipliers*</u> : it is not yet clear how this data can bring benefit to the genetic algorithm AGORA. This information may have more meaning in a multi-objective MDO framework.

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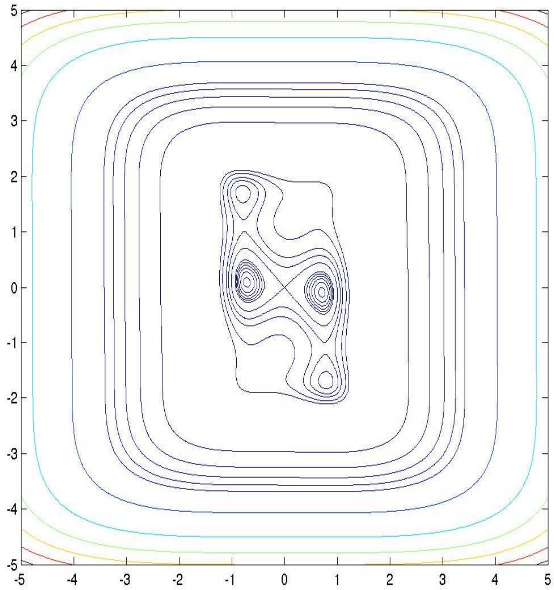

Fig. 1 : Six-hump back function

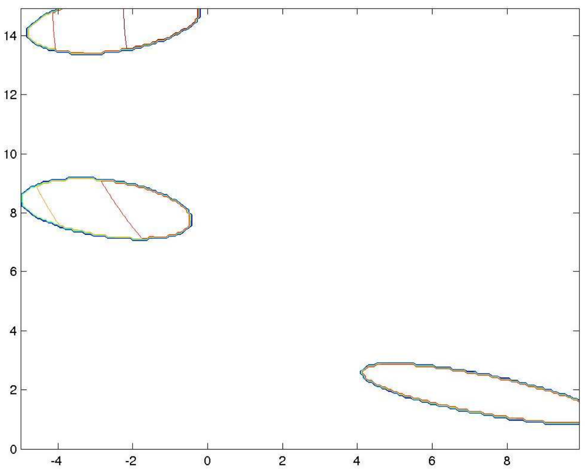

Fig. 2 : New Branin feasible region

Fig. 3 : Gomez feasible region