The dynamic response of laminate composite plate under low-velocity impact

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Abstract

In this investigation, the fiber-reinforced composite plates, without damage, subject to low velocity impact are studied by the use of finite element analysis (FE). To analyze the response of the plate, a Newmark time integration algorithm is used. In the present work the FE approach based on the modified Hertz contact law and Mindlin's plate theory tacking account of the transverse shear stress is developed. The contact force is calculated in conjunction with the loading and unloading processes. The time history of the impact process such as the acceleration, velocity of impactor and the target fiber-reinforced composite plate deflection due to an impact force acting at the center has been calculated. The effects of various parameters, such as impactor velocity, clamped or simple supported boundary conditions and lamination stacking sequence are examined.

1 Introduction

Fiber-reinforced composite plates have been used in many applications such as in aircraft, automobile industries and sport equipments. The Fiber-reinforced composite plates are very susceptible to low velocity transverse impact which can caused significant damage, such as matrix cracks, delamination and fiber breakage [1,2,3,4,5]. The dynamic response of composite structures subjected to transient dynamic loading has been studied in terms of analytical, numerical [1,2] and experimental works[5,6]. Theoretically, many works have been developed with an aim of studying the behaviour of composite targets under low-velocity impact. The result of these works was compared to the experimental data obtained using different types of impact machines. The analytical model presented by Pierson and Vaziri [7] based on the combined effects of shear deformation, rotary inertia and the nonlinear Hertzian contact law with an aim of studying the low-velocity impact response of simply supported laminated composite plates. C. T. Sun applied the modified Hertzian contact law in low-velocity impact response analysis of composite laminates [4]. Yang and Sun presented the experimental indentation law through static indentation tests on composite laminates [8]. Tan and Sun developed their own finite element program to analyze impact response of composite laminates and they performed impact tests using pendulum type low-velocity impact test system [9]. Ik Hyeon Choi and Cheol Ho Lim proposed a linearized contact law to studying low-velocity impact analysis of composite laminates and compared its to modified Hertzian contact law [10].
In the present study, a Mindlin's plate theory alongwith a Hertzian contact law is used to describe the behavior of Carbon/Epoxy

laminated composite plates subjected to the central impact of a spherical rigid projectile. P parametric study in conducted by variying some parameters of the impactor and the composite plate such as: velocity and radius of the impactor and the boundary condition.

2. Impact analysis

The dynamic response of the impact is investigated with the consideration of the following assumptions:

1. frictionless between the impactor and composite plate;
2. neglecting the damping affect in the composite plate;
3. the impactor is a rigid body with isotropic properties.

In the present work. A nine-node isoparametric element based on the Mindlin plate [1,2,3], which takes into account the effect of transverse shear deformation is used. Each node has five degrees of freedom $(u_i, v_i, w_i, \theta_{xi}, \theta_{yi})$.

3.Constitutive relations

Introducing the conventional stress and moments resultants (N_x, N_y, N_{xy}), (M_x, M_y, M_{xy}) and (Q_x, Q_y), the laminate constitutive equation is as follow:

$$\begin{Bmatrix} N \\ M \\ Q \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B^T & D & 0 \\ 0 & 0 & F \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \\ \gamma \end{Bmatrix} \tag{1}$$

Where, $[A]$ is the extensional stiffness matrix, $[B]$ is the bending-extensional coupling matrix, $[D]$ is the bending stiffness matrix and $[F]$ is the transverse shear stiffness.

4. Finite element equations of motion

The dynamic response of a plate is governed by the following equation [18]:

$$[M]\{\ddot{u}\} + [K]\{u\} = \{F\} \tag{2}$$

Where $[M]$ *and* $[K]$ are respectively, the mass matrix and stiffness matrix of composite plate. Are given by:

$$[M] = \sum \int_V [N]^T [\overline{m}][N] dV \tag{3}$$

Where,

$$[\overline{m}] = \begin{bmatrix} I_1 & 0 & 0 & I_2 & 0 \\ 0 & I_1 & 0 & 0 & I_2 \\ 0 & 0 & I_1 & 0 & 0 \\ I_2 & 0 & 0 & I_3 & 0 \\ 0 & I_2 & 0 & 0 & I_3 \end{bmatrix} \tag{4}$$

$$(I_1, I_2, I_3) = \int_{-h/2}^{h/2} \rho \left(1, z, z^2\right) dz \tag{5}$$

$$[K_L] = \int_V B_L^T D B_L dV \tag{6}$$

$\{u\}$ and $\{\ddot{u}\}$ are respectively the displacement and acceleration vector. $\{F\}$ is the equivalent of external load, which include the impact force. The dynamic equation of a rigid ball is given by the use of the Newton's second law:

$$m_i \ddot{w}_i = -F_c \tag{7}$$

Where m_i is the mass of the ball (impactor) and F_c is the contact force.

Consider the contact between a spherical ball made of an isotropic material and a target laminated composite plate containing N transversely thin layers. The contact is located at the center of the plate. Fig. 1 describes the impact procedure of the two structures (rigid body and the composite plate).

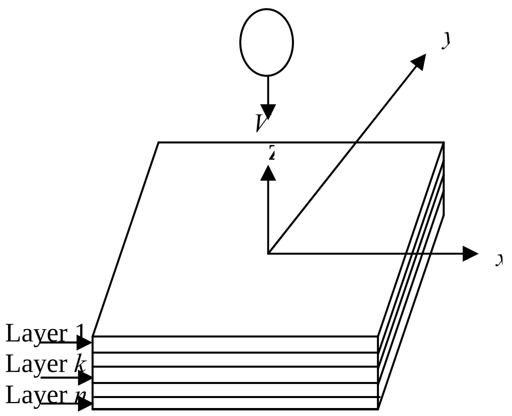


Fig.1.Composite laminates and impactor before impact.

3. Contact force

Contact force between the impactor and the plate can be calculated using a modified Hertz indentation law proposed by Tan and Sun [9].

$$F=\begin{cases} k\alpha^{3/2} & \textit{For loading} \\ F_m\left(\dfrac{\alpha-\alpha_0}{\alpha_m-\alpha_0}\right)^q & \textit{For unloading}\end{cases} \qquad (8)$$

$$\alpha_0=\begin{cases}\beta(\alpha_m-\alpha_{cr})\ if(\alpha_m \succ \alpha_{cr}) \\ 0\ if(\alpha_m \le \alpha_{cr})\end{cases} \qquad (9)$$

Where q, β are experimental constants, k is the Hertzian contact constant. Coefficient F_m and α_m means, respectively, maximum impact force, and maximum indentation depth, corresponding to F_m, before the unloading phase. α_0 is the permanent indentation depth. The contact deformation α, defined as the difference between the displacement of the impactor and that of the composite laminates, is given by the following:

$$\alpha(t)=w_i(t)-w_s(t) \qquad (10)$$

$w_i(t)$ and $w_s(t)$ are the displacement of impactor and displacement of the impact point on the mid surface of the plate. The solution of nonlinear equation obtained from equations1,2,3 and 4, is carried out by an iterative procedure using Newton-Raphson method. Fig.2 describes the impact procedure.

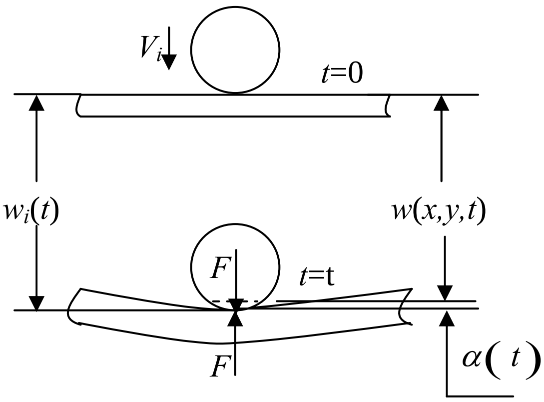


Fig. 2. Description of impact procedure

The Hertzian contact constant k for plate is given by the following equation [12]:

$$k=\frac{4}{3}\frac{R^{1/2}}{\dfrac{1-\nu_i^2}{E_i}+\dfrac{1}{E_2}} \qquad (11)$$

Where R, E and ν are the radius, the Young's modulus and the Poisson's ratio of the impactor and E_2 is the transversal Young's modulus of the plate.

In order to solve equations (1) and (2) we adopt a Newmark algorithm [11].
Newmark's integration scheme is employed to solve the dynamic equations of the plate and the impactor for each time step. The algorithm is giving by the following equation:

$$(w_i)_{n+1}=(w_i)_n+\Delta t(\dot{w}_i)+\left(\frac{\Delta t^2}{4}\right)(\ddot{w}_i)_n-\left(\frac{\Delta t^2}{4m_i}\right)(F_c)_{n+1} \qquad (12)$$

For contact force at the time step $n+1$ is determined by the use of the same algorithm given in [11].

4. Numerical investigation

a. Numerical model

AFE analysis based on a 9 node quadrilateral and isoparametric element is conducted for the study of the dynamic response of laminated composite plates due to low velocity impact.

In order to investigate the effect of the low velocity impact on the dynamic behavior of fiber-reinforced composite plates a numerical model is developed. The stacking sequence of the composite plate is $[90/45/90/-45/90]_{2s}$. The fibers are made of carbon and supposed to be continuous. The thickness of each layer is 1.35 $x10^{-4}$ m, and the properties of the laminated plate are those of Carbon-Epoxy composites:

$E_1 = 120\ GPa$, $E_2 = 7.9\ GPa$, $\nu_{12} = 0.3$, $G_{12} = 5.5\ GPa$, $\rho = 1582\ Kg/m^3$

The parameters of impactor are:

$r = 12x10^{-3}\ m$, $E = 207\ GPa$ $\nu_{12} = .3$, $\rho = 7800 Kg/m^3$

The parameters used in the indentation law between the rigid ball and plates are: $q = 2$, $\beta = 0.094$, $_{cr} = 1.7\times10^{-4}\,m$. Two kinds of boundary conditions are studied in the present work, i.e. the simply–supported and clamped.

4. Numerical results and discussion

In order to determine the effect of the impactor velocity on the impact behavior of a plate without damage, the impact force is calculated at increasing impactor velocity of 0.51, 0.85 and 1.6 *m/s*. The central deflection is also calculated at impactor velocities of .85 and 1.6 *m/s*. The impact force and the central deflection are seen to be proportional to the impact velocity during the loading phase of a simply supported plate [Figs 3 and 4]. After the first contact, the plate and the impactor contacted again after the plate reversed its direction of motion. In the present work, only the first impact is considered.

Fig.3. Impact force of clamped plate with a lenght of 0.2m and impactor radius of 12e-3m

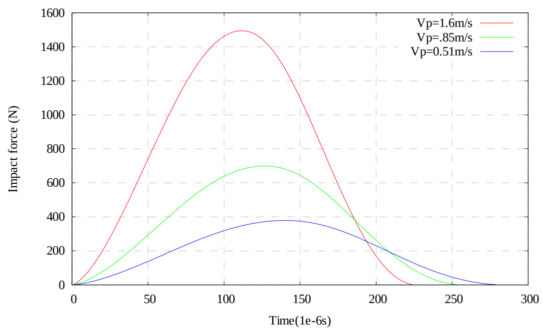
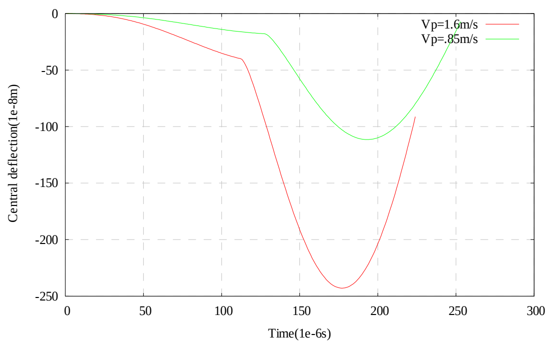


Fig.4. Impact deflection history with differents impactor velocity



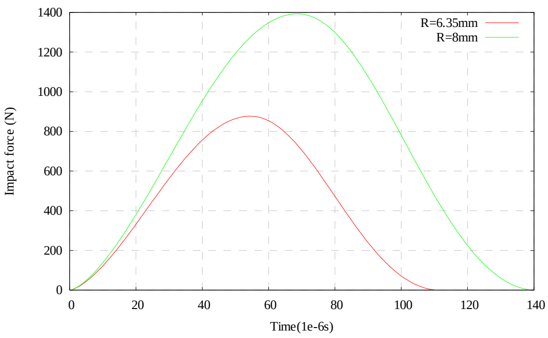
In order to demonstrate the effect of the impactor radius on the dynamic behavior of the plate, two impactors with radius of 6.35 and 8 *mm* are used. The impactor velocity is 3.1 *m/s*. In this case, the stacking sequence of laminate is $[90/45/90/-45/90]_{2s}$ and the material properties and parameters of the impactor are:

$E_1 = 30.5\ GPa$, $E_2 = 6.9\ GPa$, $\nu_{12} = 0.3$, $G_{12} = 4.65\ GPa$, $\rho = 1678\ Kg/m^3$

$E = 210\ GPa$ $\nu_{12} = .3$, $\rho = 7800 Kg/m^3$. with the thickness of the single layer of 1.8 x 10^{-3} *m*.

It was found, as show in figure 5 that the impact forces start with small variation and ends in a great variation during the loading phase. From this figure we can say that the contact force increased with the impactor radius.

Fig.5. Impact force with differents impactor radius and impactor velocity of 3.1m/s



Two types of materials constant of composites are used, (*a*) $\frac{E_1}{E_2}=15$, (*b*) $\frac{E_1}{E_2}=20$. These two kinds of materials are considered in the order to show their effects on the impact force of the target plate of layer thickness *t*=0.18 mm and stacking sequences of $[0_4/90_2/0_4]$. The impact velocity is taking as 4.85m/s. Figure 6 show these effects. As can be seen from this Figure, for the two cases during the loading phase the first figure is above the second and during the unloading phase, the situation is reversed.

Fig.6.Impact force with two different ratio E1/E2

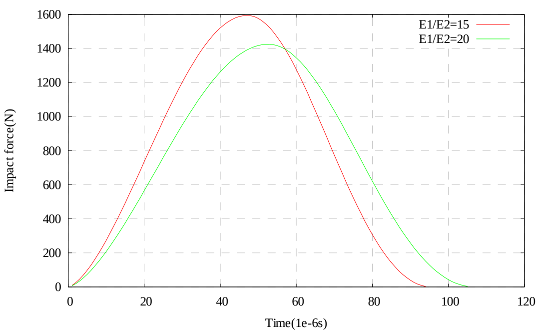
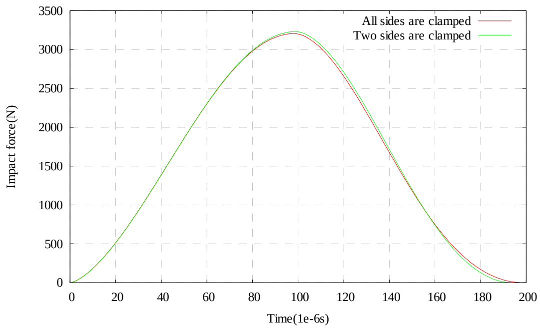


Figure 7 show the impact force calculated at impactor velocity 3m/s with two differents conditions boundary, in the first case the plate is clamped on edges a long the x axis, and in the second case the plate is clamped on al edges. The material constants of composites are the same used in the first application.

It was found that the two boundary conditions showed similar impact force history.

Fig.7. Impact force with two differents conditions boundary



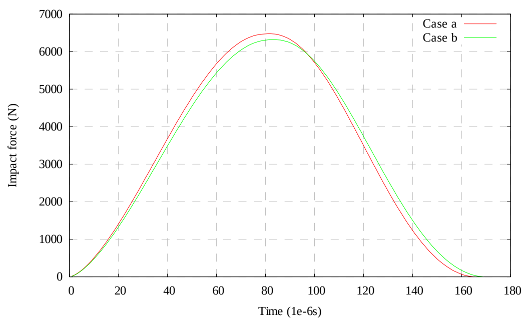
In this section, the effect of Hertzian contact constant *K* in contact law on the impact force is studied in this paper. The Hertzian contact constant *K* is varied as:

$$case(a)\ K = 1.15\times 10^{9}\quad N/m^{3/2}\quad and$$

$$case(b)\ K = 1.413\times 10^{9}\quad N/m^{3/2}$$

The stacking sequence of laminate is $[0_3/90_4/0_3]$. From Fig.8, which show force-time histories, we can see that the curve for the second case is above of the curve of the first case during the loading phase, but on the unloading phase we have found that the first case is above of the second case.

Fig.8. Impact force with different values of Hertzian contact constant K



6. Conclusion

The composite laminated plate structure subjected to low velocity impact has been analyzed by a program in finite element method. The contact force and central deflection with various parameters are studied.

From the numerical results found in this paper, the following conclusions are obtained:

1. The contact force is proportion to impactor velocity and impactor radius.

2. There is smaller difference effect between the impact force of a clamped and a simply supported plate.

3. When the stacking sequence [0/90_2/0] of the laminate was changed to [45/-45/-45/45], the impact force and central deflection are similar in shape and magnitude, as found in this paper.

The present work will be beneficial to our further research about the damage initiation and extension in laminated composites with different parameters under low velocity impact.

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