

ENTROPY METHOD OF RELIABILITY ASSESSMENT FOR AIRCRAFT SYSTEMS

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Keywords: *complex system; reliability synthesis assessment; data conversion; information entropy method; research field*

Abstract

The significance and applicable value of reliability multi-level synthesis for large complex system are described, the studying status and developing trend of system reliability synthesis are analyzed, the traditional reliability synthesis method based on probability and statistics theory are indicated, the reliability multi-level synthesis for complex system based on information theory are discussed, the main research problems of reliability synthesis based on information entropy method are presented, and the basic thought of entropy method of reliability assessment is given. By the use of general expression of the information entropy for discrete variable and addible characteristics of information, the reliability assessment method is studied for the system composing of different and same distribution type components according to equivalence principle of information quantity. The basic conversion models of the entropy method for component data and the reliability assessment formulae of complex systems are presented.

1 General Introduction

The reliability synthesis of system is an important branch in the field of reliability engineering. Nowadays, with the high tech progress drastically, all kinds of large complex systems are coming out like bamboo shoots after a spring rain, for example aircraft, aero-engine, communication net, artificial satellite

and so on. These systems are not only expensively cost but also have a very high requirement for reliability. If the whole system is tested, much money will be consumed, or the period will be too long. The whole system is not permitted or is not able to be tested as what inexpensive parts or components are massively done. So the times of test are very small, or even the test at the level of whole system is never done at all. Even did, the failure is usually zero. Therefore before the reliability assessment, enough data information from tests or from practice operations cannot usually be acquired, that is to say the sample size of usable data information from system self is very small or even no at all. Under this situation it is very difficult or even impossible to assess the reliability parameters of the system only using the data information from system self. A great deal of information from tests or operation of those low levels of assembly, such as subsystems, whole machines, subassemblies, essential parts/components which compose the complex system must be fully exploited, and test or operation information in different conditions should be converted and synthesized to entirely or partly substitute experimental information of higher levels of assembly in order to do pyramidal multi-level synthesis evaluation (synthesize at every assembly level of complex system, assemble every level into the system, and assess reliability of the complex system). The main research contents of reliability synthesis are to resolve problems on

equivalence conversion of reliability data information from each assembly level and synthesis assessment of reliability parameters for a large complex system and its each assemble level. The main research objective of reliability synthesis is to offer theoretical base and information support for making full use of various reliability information from each assembly level in a complex system, for expanding information quantity and improving the precision of assessment on reliability parameters. This problem is the most important and complicated problem in reliability engineering theory-researching field; the research has great engineering applicable value in the development, manufacture and application management of large complex system or product: (1) This is an scientific and advanced reliability synthesis technique, which can provide theoretical base for making full use of testing information from each assembly level in complex system. It has important instructive meanings in reducing test fees, shortening development period, arranging test items reasonably and harmonize the test times of every assembly level in the system. (2) It can verify the reliability requirement and rationality of reliability design, reliability allocation for system and its each assembly level or constituent units by scientific and advanced reliability synthesis evaluation. (3) It can point the weakness of existing system, indicate direction for improving design and manufacture craftwork, and then accelerate the reliability increase process of system development. (4) From scientific and advanced reliability evaluation, the reliability level of related parts or components, materials, aircraft, subsystem and the whole complex system can be acquired, it can provide theoretical support for reliability plan of new product. Because of the importance of reliability evaluation, it has been attracting the attention of international academe and industry for many years. However, the theory and arithmetic of reliability precise limit are very complicated, especially for the great complex system containing many types of units, it is difficult even impossible to present precise limit of their reliability parameters. Therefore, it

is important to find a scientific and advanced method of reliability precise limit for complex system in engineering application, and the research in this field is very active.

2 Traditional Reliability Synthesis Methods

The analysis and disposal of reliability data information is the basis of the assessment of reliability parameters. In 1957 Buehler^[1] discussed the assessment problem of reliability parameters of parallel system consisting of binominal units firstly. Since then, scientists from every country made wide researches on the problem of assessment of reliability parameters utilizing the theory and methods of statistics and adopting the ways of precise limit and approximate confidence limit^[2-9], a large number of achievements were obtained and three schools, namely Classical, Bayes and Fiducial had been formed gradually. Each school develops lots of methods. To taking classical school as an example, there are no less than 50 assessment methods of reliability parameters from series system consisting of binomial units, in which those frequently used are MML method^[2], SR method^[3], L-M method, CMSR method and etc. The MML method is based on the theory of the maximum likelihood estimation, which is simple to calculating, but the estimation result is too radical (too high) to be used when there is no failure of units. For the disadvantage of MML, the article [3] presents a SR method, which successively compresses test data according to the principle of invariable point estimation. As a result, it loses information and enlarges estimation deviation, so the estimation result of reliability parameters obtained is prone to be conservative. For a large complex system, successive compression would make reliability confidence limit of whole system more conservative. Similar to the SR method, L-M method compresses test data according to the principle of invariable point estimation. However it compresses more, so the result is worse. Some articles suggest that the L-M method should not be used. The CMSR method is the combination of MML and SR,

which compresses only once when the least testing unit has no failure; and afterwards it applies MML to get equivalent data and makes estimation of reliability parameters. From practical usage, the CMSR method is better than MML method and SR method. After all the CMSR method compresses the test data of units, information would have a loss, so the result of CMSR method is also prone to be conservative. The research way of the school of Bayes and Fiducial is different from that of the classical school. The theory basis of the Bayes method and Fiducial method is that the distribution function of bounded random variable is uniquely determined by endless sequence of each rank moment (first moment, second moment, ..., k-th moment, ...), and making the moment of approximate distribution equal to that of precise distribution as more as possible to simulate distribution and deal with data. In practical application the Bayes method usually involves the choosing problem of the prior distribution. The posterior distribution is obtained according to prior distribution and the acquired test information, from which the assessment of reliability parameters is made. Because the Bayes method utilizes the prior distribution of concerned products (the classical statistics method ignores these information), the Bayes should be attractive for the assessment of reliability parameters of complex system usually with small samples. However, how to reflect exactly and reasonably historical experience in engineering practice to the prior distribution has not been resolved. For example, using the Bayes method to estimate the reliability parameters of success-failure products, we usually choose the Beta function, (a, b) , as the conjugated prior density. But before obtaining the operation data, it is hard to determine the practical parameters of a and b . For a large complicated system, we face usually the circumstance of small samples. If the $(1,1)$ is chosen as the prior density, the assessment result would be conservative; if $(0,1)$ is chosen, the parameter a disobeys the definition domain of Beta distribution. Because it is assumed that the product quality before test is very bad, the assessment result is certainly most conservative; if the $(0,1)$ is chosen, the

parameter b disobeys the definition domain of Beta distribution. Because it is assumed that the product quality before test is very good, the assessment result is certainly radical (too high). Therefore two ways mentioned above that treat absolutely the prior information of the product quality are not helpful to objectively estimate the quality of products. If the $(0,0)$ is chosen, the parameter a and b disobeys the definition domain of Beta distribution. And it means that the product quality is either very bad or very good. Though the result of assessment is better than the two ways mentioned above, it lacks the definite explanation in physics, which is very necessary in choosing the prior distribution for making the Bayes prediction. At the same time, the approximate method of Beta only makes use of the front two rank moments (first and second moment) of reliability and supposes artificially that the density function of reliability is Beta distribution. In this way, not only has the information not been fully made use of, but also subjective factor has been included, which would inevitably cause disputation. In the aspect of converting the reliability information of units into that of system as the prior information of system reliability, in addition to the Beta approximate method, other methods such as the Mellin method^[8], the Chebyshev method^[9] are in common use. But these methods have their limitation. The Mellin method is too complicated along with the system scale enlargement; the accuracy of Chebyshev method cannot meet the requirement of engineering under the circumstance of lower rank, and its calculation workload increases rapidly with the order increasing. As to Fiducial method, it has been proved as an improper method in system reliability assessment whether in theory or practice. The universal methods in engineering are classical method and Bayes method. Since 1964, with the suggestion of comrade Qian Xueshen, our country started the theory and practical application research of reliability assessment, which has developed from few institution in the beginning to many university, academy in nowadays, and has got a great deal of achievements. Reference [10] is the only academic literature of reliability in our

country; it absorbs large quantities of domestic and overseas researching achievements of reliability assessment and it introduces the moment technique detailed, by utilizing which the author developed the classical method and bayes method.

In a word, those methods (three schools) mentioned above are base on the theory of probability and statistics. Indubitably, these methods play the important roles in the work of assessment of reliability parameters. Whereas from the above analysis we can know that the traditional methods based on probability and statistics have obvious weakness and shortage: (1) The result of the classical method which compressing test data according to the principle of invariable point estimation is prone to be conservative because of the information lose resulted from information compression. For a large complex system, successive compression would make reliability confidence limit of whole system more conservative. (2) The Bayes method has a difficult problem on how to accurately determine the prior distribution. To suppose artificially the density function of reliability is easy to add the subjective factors; for the problem of small samples for the large complex system (aero-plane, aero-engine, and so on), how to use various information to acquire the final prior distribution now does not have a scientific theory foundation, which leads to confusion of ideas and methods; only making use of front few ranks of moment of reliability to simulate the distribution and dealing with data will also lose information, and cause the truncation error. In addition, the method of dealing with data usually becomes very complicated with the system scale enlargement and the addition of rank, and the precision of assessment of reliability parameters decrease with the increase of system complexity. (3) When we use the Fical method to convert reliability data, the case that the converted failure number is negative will come out, and the result of assessment will have negative deviation error; when we make a multi-level synthesis for a large complex system, the results acquired from different flows of synthesis have huge differences.

Nevertheless, these problems mentioned above are inherent in the traditional methods that cannot be satisfactorily resolved only using statistical methods and moment techniques. Thereby, seeking an advanced method of data equivalence conversion and reliability synthesis that can make full use of kinds of reliability information, which the result of reliability synthesis is close to true value and neither conservative (too low) nor radical (too high), is a difficult problem that has puzzled the field of reliability engineering for many years. Nowadays, with the rapid development of science and technology, the quality and reliability of all kinds of large complex systems or products have become significant basis for the acceleration development of civil economy, so the solution to this problem are in the most urgent requirement and at the same time promote people to find its new solution.

3 Reliability Synthesis Assessment Based on Information Theory

In order to make full use of reliability information of each unit or subsystem and not to include human factors, articles [11-13] present a method that uses the maximal entropy method to acquire system reliability prior distribution. Virtually, this method is to convert the problem of determination of density function to the problem of nonlinear programming with constraint conditions. Theoretically, the maximal entropy method is a relatively reasonable one in various Bayes approximate methods, which can comparatively make full use of reliability information of each unit. But it is rather difficult to calculate and needs the technique of optimization, and also based on moment technique. The articles [14-15] utilize the principle that the separate test information of units is equivalent to the information of the units that tested in the system to obtain the mean test numbers of equivalence conversion of each unit in the system test and then get the failure probability of each unit, and from this assess the reliability of the system. Nevertheless, the result

of reliability synthesis of this method is even more conservative than that of classical method and even more radical (higher) than that of Bayes method. The article [16] gives the relationship between reliability parameters and the entropy of state variable based on the minimal expectation value of cost function by utilizing Markov's inequation and Jaynes maximal entropy theorem. But in this method the state variable and state equation of the system have to be known, which is impossible for a large complex system, besides it did not study the problems of data conversion in the paper, hence this method is only fit for the assessment of reliability parameters for single unit.

Carrying on the equivalence conversion of reliability data information and the multi-level synthesis of reliability parameters of a large complex system, we must pay special attention to the revision problem of data information coming from different information sources and different assembly levels which is used in different operation conditions. Supposing the data are standardized which can directly be used for data conversion and reliability synthesis and ignoring the equivalence research of all kinds of raw information is not complete for the theory of data conversion and reliability synthesis. In order to expand the information quantity, except for the little information of system itself, it is necessary to make use of the information of each assembly level of the system, which probably is the reliability information of each assemble level itself from different environment (operational condition), or the reliability information of similar products and specialists' experience. How to transform these original information to engineering information that can be used for data conversion and reliability synthesis is vitally important, which determine the rationality and validity of the results of the data conversion and reliability synthesis. The traditional data equivalence method considering environment influence is the environment factor method, usually different distribution functions adopt the difference environment factors, which

is very useful for reliability information conversion or equivalence if the distribution function and distribution parameters have been given. However, This kind of method does not take fully into account fuzziness, uncertainty, similarity and inheritance of the reliability information from different data sources (from different environment or similar product etc.), while the fuzziness, uncertainty, similarity and inheritance universally reflected in large complex system make traditional reliability data processing theory based on probability statistics face tremendous challenge. With the development and application of various kinds of large complex systems and implementation of project items, the conflict between traditional reliability synthesis theory and engineering practice will increasingly become severe.

According to the characteristic of large complex system and the deficiencies of the theory and methods of data information conversion and reliability synthesis mentioned above, combining the information theory with reliability engineering theory, reference [17-18] proposed a new set of theory and method system for data information equivalence, conversion and reliability synthesis assessment. This theory and method system mainly consists of several modules, namely "the theory and method of equivalence of data information from different information sources", "the theory and method of data information conversion based on information theory", "the theory and method of reliability synthesis of large complex system". Through the research of these modules, a set of completed theory and method system for reliability data information equivalence, conversion and multi-level reliability synthesis will be established. At present, references [17-18] apply the fundamentals of information theory, utilize the basic characteristic of information quantity and the fundamental principle of information equality, establish the basic models of data conversion and reliability synthesis for multi-level complex coherent systems consisting of success-failure units and exponential units initially. Compared with the

traditional method based on probability statistics, the initial research works have demonstrated that the reliability data information equivalence conversion theory based on information theory can use all kinds of reliability information from different assemble levels of the complex system more sufficiently and reasonably, and can conquer the deficiencies and shortages of traditional methods. On the other hand, this method is easy to apply in engineering, many engineering example indicated that its calculation results are similar to the traditional moment method; in the common experiment times (number of samples) and confidence of reliability engineering, especially in the situation of small sample and high confidence, the result is true and credible, so this method is especially suitable for large complex system with small sample. However, the research in the field is at the beginning, in order to solve all the problems in this field comprehensively, a great deal of deep and hard research works need to be finished.

4 Main Research Problems of Information Entropy Method Reliability Synthesis

A large complex system we will study can be described as a multi-level complex framework model of “essential parts/components—subassemblies—equipments/whole machines—subsystems—systems—complex system”, furthermore, each level have the reliability information of test or operation, which complies with certain distributions (such as binominal distribution, exponential distribution, normal distribution, logarithm normal distribution, Weibull distribution and any other distribution or unknown distribution that we could face in practice). The most general problems that we will study in this field are the equivalence conversion of reliability information and the multi-level synthesis of reliability parameters, and establish the basic theory and method system of the equivalence conversion of data information and the multi-level synthesis of

reliability parameters based on information theory for a multi-level large complex system.

4.1 Equivalent problems of data information from different information sources

The research on the equivalence problem of data information from different sources is a key to make rationally full use of all kinds of information that we have. In this part, we will use basic principles and methods of information theory to study how to convert headstream information (original information—direct or indirect) into engineering information that can be used in the information conversion and multi-level pyramid synthesis of reliability parameters. It mainly includes the following contents: how to convert the data information from different conditions to a given condition (such as how to convert the data information between different conditions; how to convert the data information from lower assemble level operation conditions to higher assemble level operation conditions); how to exploit, utilize and transform the indirect information from similar products and experts’ experience (that is, how to describe the information with exactly mathematics forms, how to evaluate the value of the information and etc.) and how to ascertain the information structure, and how to evaluate the value, sensitivity and accuracy of reliability information. The research of this part will put forward the theoretical base for reasonably using all kinds of reliability information and will directly provide support for data information conversion and multi-level synthesis of reliability parameters.

4.2 Estimation problems of information quantity or information entropy when the distribution function is unknown

Scientists increasingly attach importance to the combination of information theory and statistics theory. Analyzing reliability statistical data from the viewpoint of information extraction is a new direction of research and has large development potential. The main difficulty of

information's application in practice is how to calculate the information quantity, especially how to calculate the information quantity of continuous distribution. In this part, we will mainly study the following questions: discrimination arithmetic method of distribution function type of random variable based on the information theory; the estimation methods of information entropy when the distribution function is unknown; the calculation methods of information quantity of all kinds of continuous distributions. The research in this part will play an important guidance role in gathering, assorting and dealing with the reliability information. However, theoretic and application researches in this aspect are very insufficient nowadays.

4.3 Conversion problems of data information based on information theory

Conversion of data information is the foundation for pyramid multi-level synthesis of reliability parameters. In this part, we study how to combine effectively information theory with reliability engineering theory and use the basic principles and characteristics of information quantity or information entropy (Shannon entropy, Awad entropy) to deal with the data or information used for reliability synthesis, including how to convert the data or information from lower levels of assembly to higher ones, how to convert mutually the data or information between different distributions, how to ascertain the equivalent information (equivalent failure numbers) when considering the different influence degree of the failures of a lower assemble level (such as parts and subsystems) to the system reliability (or the reliability of a higher assemble level) and how to transform and deal with correlation (un-independent) data information derived from different units. The research in this part will be of vital importance to make good use of the data information of lower levels of assembly, magnify the information quantity of higher levels of

assembly and improve the evaluation precision of system reliability parameters.

4.4 Synthesis assessment of reliability parameters based on information theory for the large complex system

First, we will study how different flows of synthesis of reliability data information affect the result of synthesis of reliability parameters for a large complex system. Secondly, we will study the theory and methods of multi-level synthesis of reliability parameters for all kinds of reliability structure models (series model, parallel model, series-parallel model, parallel-series model, K/N(G) model, inactive standby model, network model and some other coherent and noncoherent structure models which have more complex logic) which consists of all kinds of distribution units (binominal distribution, exponential distribution, normal distribution, logarithm normal distribution, Weibull distribution, Γ distribution, extremum distribution and unknown distribution that we could face in practical engineering). Where, we will establish the synthesis models of reliability parameters of all kinds of reliability structure models with all kinds of distribution type units, do the comparative research with existing assessment methods and models in order to testify the applicability and correctness of the synthesis models. Lastly, we will study the evaluation problem of system reliability parameters when the distribution type or distribution function of units or system is unknown and the reliability block diagram of system cannot be expressed clearly, from this we will resolve the problems of evaluation of reliability parameters of the large complex system which has any structure and any distribution units.

5 The Basic Thought of Entropy Method of Reliability Assessment^[18]

5.1 Entropy and Information Quantity

5.1.1 General expression of the entropy

The average information quantity of each message from the information source is called entropy. The general expression of the entropy for discrete variable is

$$H = -\sum_{i=1}^K P_i \ln P_i \quad (1)$$

Where P_i is the probability of message i , K is the number of messages.

5.1.2 Real test information

Assumptions: A system consists of N independent different binomial components. For component i , the test numbers are n_i , in which the failure numbers are f_i , while success numbers are s_i . Meantime it is assumed that the success probability is p_i and the failure probability is q_i in each test for component i .

From eqn (1), the entropy in each real test for component i is $H_i = -(p_i \ln p_i + q_i \ln q_i)$. The information quantity I_i provided by component i in n_i tests is $H_i n_i$, according to the “addiable” characteristic, The total information quantity I provided by N components in all real tests is

$$I = \sum_{i=1}^N I_i = \sum_{i=1}^N H_i n_i = -\sum_{i=1}^N (p_i \ln p_i + q_i \ln q_i) n_i \quad (2)$$

5.2 The Equivalent “Test” Information of a System

5.2.1 The “test” information of a “success-failure” system

Assumptions: for the system, the equivalent “test” numbers are n , in which the failure numbers are f , while success numbers are s . Meantime it is assumed that the success probability (reliability) is P and the failure probability is Q in each equivalent “test”. The total information quantity provided by the system in n equivalent tests is $\tilde{I} = -(P \ln P + Q \ln Q)n$. According to the equivalent condition of information, $\tilde{I} = I$, and replaced p_i and P by the maximum likelihood estimators (MLE) \hat{p}_i and \hat{P} , we can obtain

$$n = \frac{\sum_{i=1}^N (s_i \ln s_i + f_i \ln f_i - n_i \ln n_i)}{P \ln P + (1 - P) \ln(1 - P)} \quad (3a)$$

$$f = n(1 - P), \quad s = nP \quad (3b)$$

Where, the P is determined according to the real reliability model.

5.2.2 The “test” information of a “exponential distribution” system

Assumptions: for the system in a equivalent conversion “test”, the failure numbers are z , task time is t_0 , total test time is τ , failure rate is λ , equivalent task numbers are $\eta = \tau / t_0$.

Using the non-memory characteristic of exponential distribution, the reliability P and unreliability Q are fixed in each η , the “discrete” P and Q is regarded as probability of “discrete” events, then the entropy \bar{H} provided by the system in each η is

$$\bar{H} = -(P \ln P + Q \ln Q) \quad (4)$$

The total information quantity provided by the system in η equivalent task numbers is $\bar{H}\eta$. Using equivalent condition of information quantity, and the λ is replaced by MLE $\hat{\lambda}$: $\lambda = \hat{\lambda} = z / \tau$, and $P = R(t_0) = \exp(-z / \eta)$, we obtain

$$\eta = n \quad (5a)$$

$$z = -\eta \ln P \quad (5b)$$

5.3 Systems Consisting of Same Components

Assumptions: A system consists of N independent same binomial components. For single component, the test numbers are n_1 , in which the failure numbers are f_1 , while success numbers are s_1 . Meanwhile it is assumed that the success probability is p_1 and the failure probability is q_1 in each test for single component.

In this case, only one random sample taking from N same components provides information (other $(N - 1)$ components do not make any test), therefore the total information quantity provided by the components in all real tests

should equal to the information quantity I_1 provided by single component in n_1 tests

$$I_1 = -(p_1 \ln p_1 + q_1 \ln q_1)n_1 \quad (6)$$

In the equivalent “test”: for a “success-failure” system, the calculating formulae of failure numbers f and success numbers s are same as eqn (3b), for a “exponential distribution” system, the calculation formulae of the equivalent task numbers η and failure numbers z are same as formulae (5a) and (5b) respectively, the equivalent “test” numbers n of the “success-failure” system is

$$n = \frac{s_1 \ln s_1 + f_1 \ln f_1 - n_1 \ln n_1}{P \ln P + (1 - P) \ln(1 - P)} \quad (7)$$

5.4 Entropy Method Approximate Limits

After obtaining the n and f or the η and z , we can gain the entropy method first or second approximate lower limit of system reliability

$$\tilde{R}_{L,s} = (1 + \frac{f+1}{s} F_{2f+2,2s;\gamma})^{-1} \quad (8a)$$

$$\bar{R}_{L,s} = \exp[-\chi_{2z+2,\gamma}^2 / (2\eta)] \quad (8b)$$

where, $F_{2f+2,2s;\gamma}$ is the γ fractile for F distribution with $(2f+2, 2s)$ degrees of freedom, $\chi_{2z+2,\gamma}^2$ is the γ fractile for χ^2 distribution with $(2z+2)$ degrees of freedom, γ is confidence level.

6 Conclusions

It is seldom to do reliability test for the large complex system because of its high cost and long period, so how to make full use of all kinds of reliability information from each lower level of assembly to assess the reliability parameters of the whole complex system is an important subject that scientists are devote themselves to study for many years. But the traditional data conversion methods based on statistics cannot either get suitable prior distribution or utilize reliability information of each level of assembly sufficiently, which inevitably bring great error to the evaluation results. Combining the information theory with reliability engineering

theory, using the fundamental principles and methods of the information theory to solve the problem of equivalence, conversion of reliability information and multi-level pyramid synthesis of reliability parameters is an efficient way, which will be an important developing direction in the field of reliability engineering. For various large complex systems, making use of the basic principles of modern information theory, studying multi-level synthesis problems of reliability parameters based on information theory, the research contents presented are standing on the academic front in the field of reliability engineering theory and application, the entropy method are important development for the reliability engineering theory, and have a wide application prospect. The assessment results of entropy method by examples are neither conservative nor radical, the accuracy is not decreased with the increase of complexity of systems.

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