# MATRIX FORMULATION FOR CALCULATIONS OF WING LOADING DISTRIBUTION

Prof. Antonin Pistek
Brno University of Technology, Czech Republic, pistek@fme.vutbr.cz
**Keywords**: *aircraft, load distribution, matrix, automation*


## Abstract

*An aeroplane is influenced by wide spectrum of loads during operations. The spectrum of loads is given by flight and landing load cases, mass and operation configurations leading to a great number of load cases that all have to be taken into consideration for loading of an aeroplane. The automation of computation is necessary and cannot be done without modification of the computation methodology. The paper describes the process of wing loading computation using a matrix formulation. This formulation simplifies the solution of complicated and time consuming dynamic loading, caused by numerical methods, and enables the selection of the essential loads in a uniform data basis. Explanation is based on an example of a wing loading. The method, described in this paper, was successfully applied in the CAE system for aeroplane loading computation, named SAVLE (System for Automation of Load Calculation of Aeroplanes), which was used in Czech aircraft industry during the development of several aeroplanes.*


## 1 General Introduction

The wing is most frequently loaded by three types of aerodynamic loading, namely, by the effects of lift forces, drag forces and the moments. Loading in the direction of the lift force is given by the distribution of lift over the wing span for the basic loading up to the influence of deflected flaps, ailerons, fuselage influence etc. The loading in the direction of drag is defined by the drag coefficients of particular structural parts. Similarly, the torque moment loading is defined by appropriate moment coefficients. The principle of the described method is division of the wing into an arbitrary number of wing segments for which the angles of dihedral and sweepback are constant and the unit loading is computed independently for each wing segment along the wing span.

Fig. 1 shows the division into segments and the resulting loading components in wing cross-sections along the wing span. It is possible to compute 6 components of resulting loading for the defined wing cross-section *r*

$$\mathbf{Z}_r = \{ \ T_N \ T_T \ N \ M_N \ M_T \ M_K \ \}$$

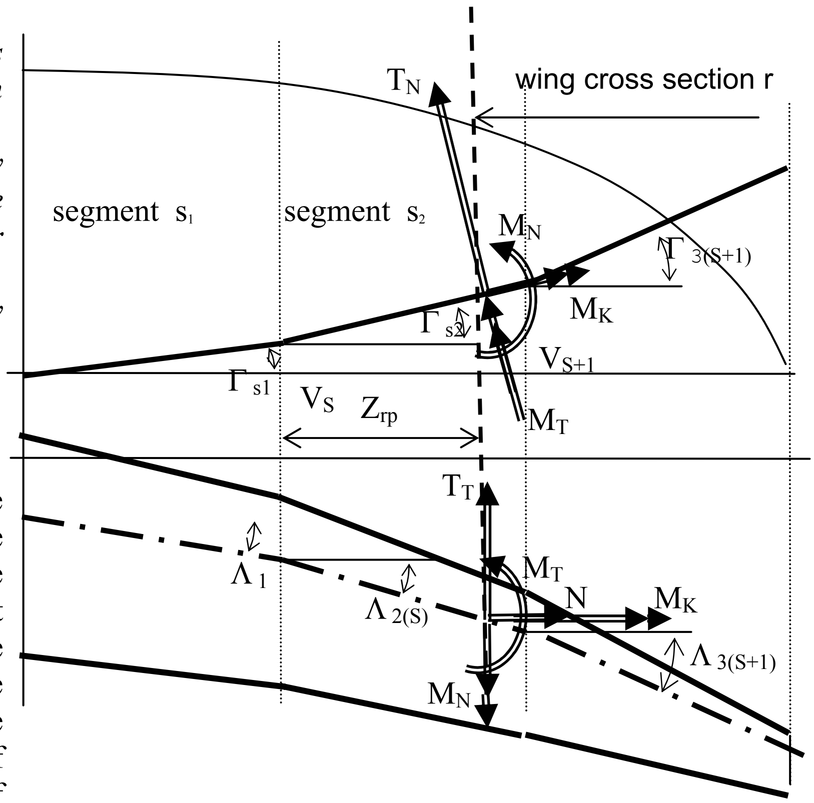


Fig.1 Geometry and the components of resulting wing loading

## 2. Load of aircraft structural parts

Aircraft loads estimation is generally divided into:

- calculation of wing load, tail units, fuselage etc. by air forces
- load calculation by weight and inertial forces,
- load calculation by landing load cases.

The load calculation method is a part of the SALVE system, where each case of load is uniquely determined by a data sequences .The most important data are - weight, speed, control surfaces deflections, load factors, rotation accelerations, aerodynamic characteristics (angle of attack, lift, drag and _moments coefficients, mass characteristics, power unit loads, landing gear forces etc.) The data set is called “load case data record” and it includes about 180 data items. This article deals with wing load calculation method from aerodynamic forces.

### 2.1. Wing Design Loads

Design wing loads consist of the shearing forces, bending moments and torsions as a result of air pressures along the wing span. The main loads limits are given by V-n diagrams for maneuver, for gust envelope and other flight conditions which are associated with control surface deflection, landing condition etc. Aerodynamic load spanwise distribution consists of two parts: basic loading (normal) and additional loading (effects of fuselage, nacelle, flaps, deflected ailerons, damping etc.

Wing load by aerodynamic forces is represented by loads

- in lift direction,
- in drag direction,
- by moment effect.

Each of mentioned types has different manner of definition and also different way of calculation of wing load components.
Load in lift direction is given by local lift coefficient diagram $c_L^i$ and local chord length of wing $c_r$. Each type of load distribution depend on the angle of attack and characteristic value for example $c_{LW}$ for the normal load distribution etc. We get in the given cross-section r for each load distribution the value called the first integral - shearing forces and the second integral - bending moment, for unit aerodynamic coefficients. Load in lift direction is set by spreading of local coefficient of pressure along the $C_L^i$ and local depth of wing $C_r$. Each type of spreading is depended on the angle of attack and characteristic quantity for example $C_{LW}$ for the normal distribution etc. We get a value in the given cross-section for each distribution. This is called the first integral - moving force and the second integral - bending moment, for unit aerodynamic coefficients, for example $C_{LW} = 1$.
Load in drag direction is defined with drag coefficient related to wing area and with geometry structural parts along wing span. The load acting in drag direction and the value is usually proportional with wing chord. The load is the product of local drag coefficient and local wing chord ($c_{Dr}$ $c_r$). The first and the second integral calculation will be done again for individual drag coefficients and parts.
Load with moment effect – is set by value of local moment coefficient $c_m$ and local wing chord cr, where the unit load is calculated for product ($c_{mr}$ . $c_r^2$). The first integral value means the torsion moment.

### 2.2. The Basic Transformation

The basic transformational relationship distributes the loads in lift direction, drag direction and moment into the local coordinate system of wing cross-section using the following relations.

$$c_n = c_L \,.\, cos\ \alpha + c_D \,.\, sin\ \alpha$$

$$c_t = c_L \,.\, sin\ \alpha + c_D \,.\, cos\ \alpha$$

$$c_m = c_m$$

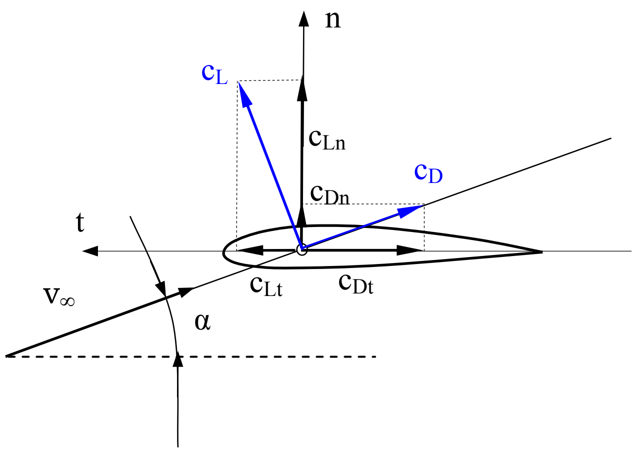


Fig. 2 The transformation from aerodynamic to wing co-ordinate system

We arrange the matrix of aerodynamic coefficients transformed to wing coordinate system.

$$\mathbf{C} = \begin{bmatrix} c_n^1 & c_n^2 \ldots & c_n^{ild} \ldots & c_n^{nld} \\ c_t^1 & c_t^2 \ldots & c_t^{ild} \ldots & c_t^{nld} \\ c_m^1 & c_m^2 \ldots & c_m^{ild} \ldots & c_m^{nld} \end{bmatrix}$$

This matrix is arranged just once for all solved load cases. The next matrixes are vectors of unit first and second integrals for each individual air loading distribution in cross-section r.

$$\mathbf{T_r^s} = \left\{ T_r^1 \quad T_r^2 \ldots \quad T_r^{ild} \ldots \quad T_r^{nld} \right\}$$

$$\mathbf{M_r^s} = \left\{ M_r^1 \quad M_r^2 \ldots \quad M_r^{ild} \ldots \quad M_r^{nld} \right\}$$

These matrixes are arranged for solved cross-section r on wing segment s by all type loads distribution computed in wing projection plane. Integration is carried out separately in individual wing segment s.

Transformational relationship for loads in the normal direction, $c_{Ln}$ and $c_{Dn}$

$$T_N^r = T_R^r / \cos\Gamma_s$$
$$T_T^r = 0$$
$$M_N^r = M_R^r . z_{RP} / \cos^2\Gamma_s$$
$$M_T^r = 0$$
$$M_K^r = T_R^r . z_{RP} / \cos^2\Gamma_s . \sin\Lambda_s$$
$$N^r = 0$$

Transformational relationship for loads in tangent direction, $c_{Lt}$ and $c_{Dt}$

$$T_N^r = 0$$
$$T_T^r = T_R^r$$
$$M_N^r = 0$$
$$M_T^r = -z_{RP} / \cos\Gamma_s$$
$$M_K^r = 0$$
$$N_r = 0$$

Transformational relationship for loads from $c_m$

$$T_N^r = 0$$
$$T_T^r = 0$$
$$M_N^r = 0$$
$$M_T^r = 0$$
$$M_K^r = T_R^s / \cos\Gamma_s$$

The given relationships we include into transformational matrixes

$$\mathbf{T_G^s} = \begin{bmatrix} 1/\cos\Gamma_s & 0 & 0 \\ 0 & 1 & 0 \\ -1/\cos^2\Gamma_s & 0 & 0 \\ 0 & -1/\cos\Gamma_s & 0 \\ \sin\Lambda_s/\cos^2\Gamma_s & 0 & 1/\cos\Gamma_s \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{M}_G^s = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1/\cos^2\Gamma_s & 0 & 0 \\ 0 & 1/\cos\Gamma_s & 0 \\ -\sin\Lambda_s/\cos^2\Gamma_s & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The upper matrixes $\mathbf{T}_G^s$ and $\mathbf{M}_G^s$ are valid for all cross-sections of wing segment s.
The next matrix defines the position of cross-section r measured from left margin of wing segment s

$$\mathbf{T}_G^r = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ z_{rp} & 1 & 1 \\ 1 & z_{rp} & 1 \\ z_{rp} & 1 & 1 \end{bmatrix}$$

This simple matrix is assembled for solved cross-section r given by co-ordinate zrp.

Using the matrix operation

$$\mathbf{Z}_{rs} = (\mathbf{T}_G^r * \mathbf{T}_G^s \,.\, \mathbf{C} \,.\, \mathbf{T}_r^s + \mathbf{M}_G^s \,.\, \mathbf{M}_r^s \,.\, \mathbf{C})\, q$$

we receive all loads components in cross-section r which is

$$\mathbf{Z}_r = \{\ T_N\ T_T\ N\ M_N\ M_T\ M_K\ \}$$

where,

$\mathbf{T}_G^r$ matrix of wing cross section r position

$\mathbf{T}_G^s$ transformation matrix of forces for wing segment geometry *s* in cross section *r*

$\mathbf{T}_r^s$ matrix of the unit forces (the first integral) on section *s* for wing cross section *r* for all types of aerodynamic load distributions,

$\mathbf{M}_G^s$ transformation matrix of moments for wing section geometry *s* in cross section *r*

$\mathbf{M}_r^s$ matrix of the unit moments (the second integral) on section *s* for wing cross section *r* for all types of aerodynamic load distributions,

**C** matrix of aerodynamic coefficients for individual types of aerodynamic distributions transformed into structural co-ordinate system,

$\mathbf{T}_v^s$ transformation matrix incorporating the influence of section *s* geometry and foregoing section *s+1*

$\mathbf{Z}_{V(s+1)}$ matrix of the resulting loading at the point of modified section *s* geometry

*q* dynamic pressure

*r* index of wing cross section

*s* index of wing segment

Remind that operator * represents a scalar multiplication of each elements of both matrixes. Each wing segment s is defined at position, where the wing dihedral angle and (or) wing sweep angle changes. The influence of wing segment s to s-1 we use the loads components in point $V_s$. The transformation relations depend on change of wing sweep angle.

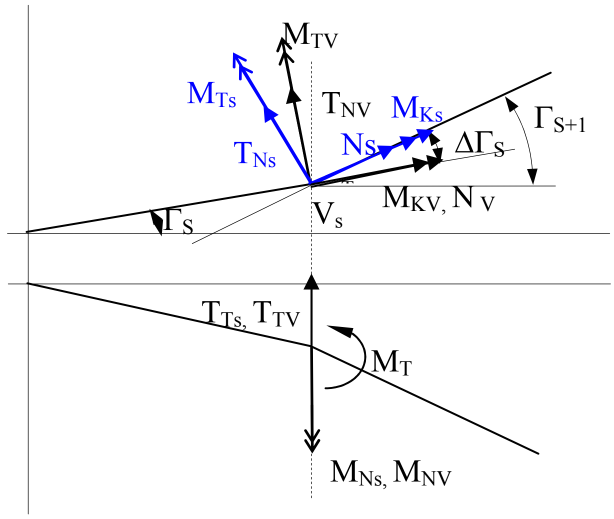


Fig. 3 The geometry of transformation loads in point Vs

$T_{NV} = T_{Ns} \,.\, \cos \Delta\Gamma_s + N_s \,.\, \sin \Delta\Gamma_s$
$T_{TV} = T_{Ts}$
$M_{NV} = M_{Ns}$
$M_{TV} = M_{Ts} \,.\, \cos \Delta\Gamma_s + M_{Ks} \,.\, \sin \Delta\Gamma_s$
$M_{KV} = M_{Ks} \,.\, \cos \Delta\Gamma_s - M_{Ts} \,.\, \sin \Delta\Gamma_s$
$N_V = -T_{Ns} \,.\, \sin \Delta\Gamma_s + N_s \,.\, \cos \Delta\Gamma_s$

The upper relations are in matrix form as follows:

$$\mathbf{T}_{\mathbf{v}}^{\mathbf{s}} = \begin{bmatrix} \cos\Delta\Gamma_s & 0 & 0 & 0 & 0 & \sin\Delta\Gamma_s \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\Delta\Gamma_s & \sin\Delta\Gamma_s & 0 \\ 0 & 0 & 0 & -\sin\Delta\Gamma_s & \cos\Delta\Gamma_s & 0 \\ -\sin\Delta\Gamma_s & 0 & 0 & 0 & 0 & \cos\Delta\Gamma_s \end{bmatrix}$$

This matrix is arranged for each origin of wing segment. The loads in origin of segment s are calculated by matrix operation

$$\mathbf{Z}_{vs} = \mathbf{T}_{\mathbf{v}}^{\mathbf{s}} \,.\, \mathbf{Z}_{rs+1}$$

Where $\mathbf{Z}_{rs+1}$ are loads in origin of wing segment s+1 for $z_{rp} = 0$

The resultant loads is given by sum of loads in cross-section r and loads acting in origin of previous wing segment s+1, using the following transformation:

$T_{Ns} = T_{NVS}$
$T_{TS} = T_{TVS}$
$M_{NS} = M_{NVS} + T_{NVS} \,.\, \Delta z_{rs}$
$M_{Ts} = M_{Tvs} + N_{vs} \,.\, \Delta z_{rs}$
$M_{Ks} = M_{Kvs} - T_{NVs} \,.\, \Delta z_{rs} .\, \sin \Lambda_s$
$Ns = N_{VS}$

The upper relationships can be written to the matrix form

$$\mathbf{T}_{\mathbf{v}r}^{\mathbf{s}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \Delta z_{rs} & 0 & 1 & 0 & 0 & 0 \\ 0 & \Delta z_{rs} & 0 & 1 & 0 & \Delta z_{rs}.\sin\Lambda_s \\ -\Delta z_{rs}.\sin\Lambda_s & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The resulting load in cross-section z is given by matrix operation

$$\mathbf{Z}_r = \mathbf{Z}_{rs} + \mathbf{T}_{\mathbf{v}r}^{\mathbf{s}} \,.\, \mathbf{Z}_{vs}$$

## Conclusion

The main advantages of the described solution are as follows: the most work – difficult and time-consuming computations are done only once and the majority of matrixes is independent on the loading case and include simple geometrical parameters. The method can also be applied on other parts of an aeroplane, especially, on tail units, fuselage and other types of loading. The method, described in this paper, was successfully applied in the CAE system for aeroplane loading computation, named SAVLE, which was used in Czech aircraft industry during the development of several aeroplanes (L-410, L- 610 commuters for 19 and 40 passengers, L-159 light combat aircraft, Ae 270 commuter for 9 passengers and VUT 100 Cobra new generation GA aircraft). The basic conceptual scheme of the system SAVLE is shown in the Fig. 5. and the example of resulting shear forces diagram on L610 commuter wing is on fig 4.

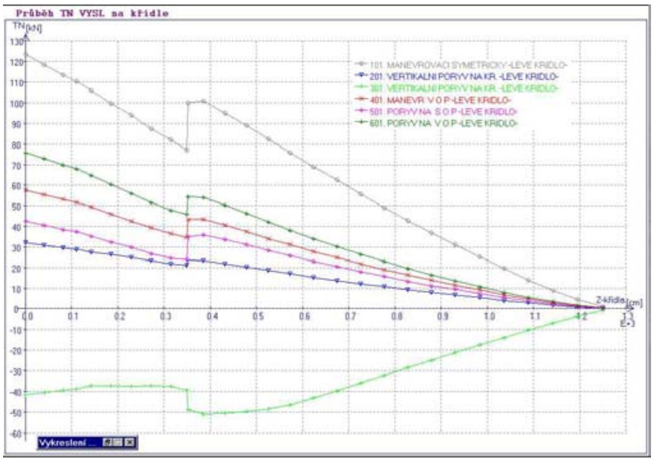


Fig. 4 The resulting shear forces diagram for L610 commuter

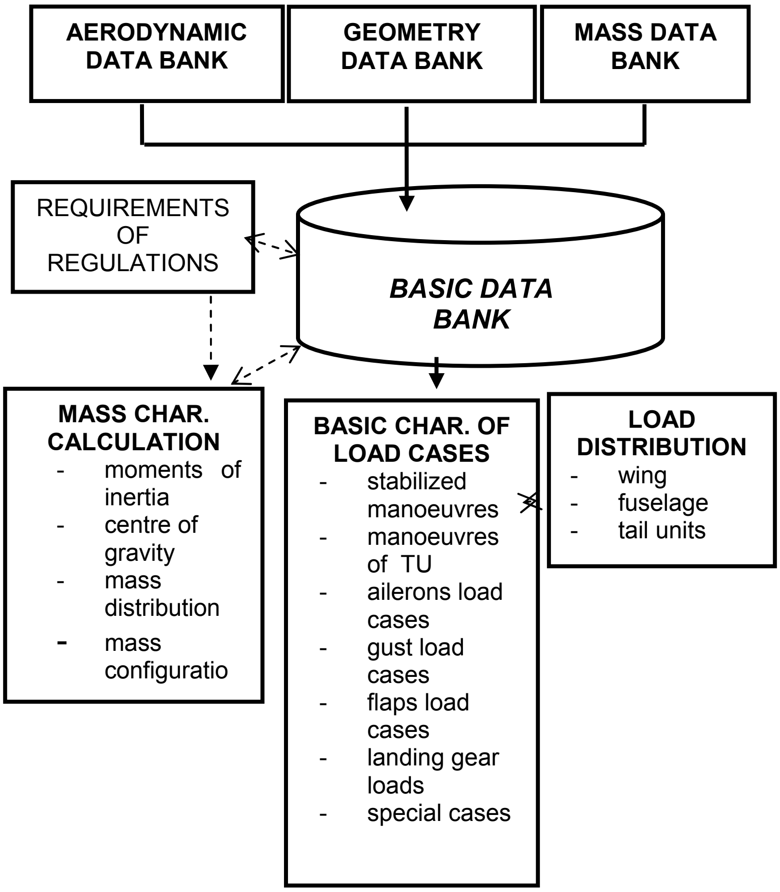


Fig.5 The conceptual scheme of the system SAVLE

## References


[1] Chun-Yung Niu,M.. Airframe *Structural Design*. Lockheed Aeronautical Systems Company,Conmilit Press Ltd., Los Angeles, California, 1988.

[2] Pistek A.,Bohm R, Gregr O, Kahanek V. *Pevnost a zivotnost letadel I,* VUT v Brne, Brno 1988.

[3] Pistek A. *Automation of load calculation,* VUT v Brne, Brno 1993.

[4] Pistek A. *Optimalizace leteckych konstrukci s vyuzitim MKP,* Doctoral thesis, Brno,1980.

[5] Hlinka J, Vanek F. *Glauert III,* Institute of aerospace Eng., Brno, 2000