THE FEASIBILITY OF THE SCRAMJET; AN ANALYSIS BASED ON FIRST PRINCIPLES

Erik G. Prisell
FMV, Swedish Defence Materiel Administration

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Abstract

The 2004 successes with the NASA X-43A program have given a vitalization to the soon 50 year old scramjet principle. The challenge of achieving hypersonic velocities with an air-breathing engine has been the prospect since the late 1950's. High speed air-breathing propulsion has been researched both in the academia and in different national establishments around the world. Although break-through events have been discerned they have not yet been realized in an operational hypersonic air-breathing propulsion system.

The **S**upersonic **C**ombustion **RamJet, SCRJ**, the **Scramjet**, used as a hypersonic propulsion system in the upper atmosphere has some distinctive advantages in comparison with propulsion by rockets. For instance, already basic performance estimations point at very high thermal efficiencies for hypersonic ramjets. Furthermore, a typical rocket propellant is composed of more or less 80% oxidizer, whilst the ramjet uses atmospheric air as an oxidizer in the combustion process. Also, the maximum specific impulse for a rocket engine with chemical propellants is limited to around 4500 m/s.

Although the principles for the "ideal" hypersonic ramjet, which uses supersonic internal combustion, are known, its practical realization has been notoriously elusive, despite years of R&D work. It may be that there are still aspects of hypersonic aerodynamics or supersonic combustion that have to be discovered for engineering realizations. It may also be that combinations of different propulsion units should be applied to create hypersonic aerodynamic flight, for instance the combination of rocket and ramjet principles. It may even be that a hypersonic flight device would turn out to possess an extremely simple geometry, provided it can be accelerated to hypersonic speeds by some external means.

Furthermore, performance calculations of hypersonic ramjets with supersonic combustion are difficult to establish, because detailed knowledge of the gas dynamic processes involved is lacking and because the thrust performance appears as a difference between two very big numbers. As a ramjet cannot develop thrust at stand-still, some form of accelerator/ booster engine is needed. Also, the ramjet must interact with the aerodynamic environment in a stationary sense, while the rocket is by large independent of the atmosphere.

The problems with high speed aerodynamic flight originate in the physical properties of air: At low subsonic speeds (with Mach numbers M <<1) the air behaves as an ideal fluid and is unaffected by the flight speed. At supersonic speeds (Mach 2 ~ 4), compressible aerodynamics comes into play and the flow character changes radically as shocks, and pressure and expansion waves are generated by the aircraft and its engine. At lower hypersonic speeds (Mach 4 ~ 5) shock phenomena and kinetic aerodynamic heating create high thermal loads on the structure. At higher hypersonic speeds (Mach >5), real gas effects such as dissociation and ionization of the molecular constituents of the air are coming into play and the compressible aerodynamics of supersonic flight will now incorporate elements of plasma aerodynamics.

This paper is an attempt to demonstrate the feasibility of the scramjet by analysis based on first principles. The analysis will focus on the thermodynamic processes for some specific engine components.

1 Introduction; Parametric analysis based on first principles

One may ask why parametric analysis, based on first principles, in these days of readily available CFD-programs? Some aspects of this matter are presented below:

* The internal, supersonic flow of a hypersonic ramjet is composed of interacting subsonic, transonic and supersonic two-phase fluid dynamics: Assuming the fuel is a liquid, it is injected, maybe with a low velocity, subsequently disintegrating with still a rather low velocity at the spray core, then mixing with air, vaporizing, undergoing an induction time at elevated temperatures before igniting and burning and all of this more or less overlapping. These phenomena are mathematically represented by elliptic, parabolic and hyperbolic differential equations, together with two-phase formulations. It is questionable to what extent this kind of problem can be handled by concurrent CFD-codes.

* For instance, there exists no CFD-code that can predict the liquid fuel atomization and spray development performance given the boundary and initial conditions of the fuel injection.

* The fuel mixing and combustion phenomena can be described thermodynamically as open, compressible and dissipative flow processes. As will be shown later, such processes are often characterized by the existence of a "critical" Mach number (not necessarily equal to one), representing a condition where the entropy increase during the process has reached a maximum value that cannot be surpassed. In the language of air-breathing engine technology, such a condition is referred to as "thermal choking" and violating this condition by applying unsuitable initial conditions will (in practice) result in a transient change of state in the inlet conditions. It is questionable to what extent CFD-codes can handle this kind of problem.

* In parametric analysis by first principles, a length coordinate cannot be predicted. The analysis is based on the assumption that the length is "sufficient", the area is constant and the conditions are appropriate for the processes to go to completion/ equilibrium in a well mixed state at the outlet. Further, the gas composition is assumed to be constant and only constant averages of physical parameters over the processes are used. At some stage, evidently, CFD codes become necessary to utilize in order to proceed in the development of an engine.

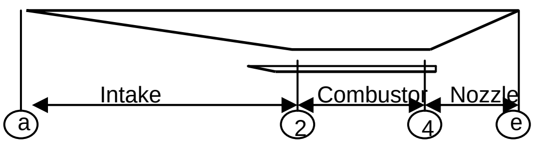

Fig. 1. Hypersonic ramjet with supersonic combustion, SCRJ, ”The Scramjet”

2 Aero-thermo dynamic boundary conditions of ramjet engines. “The problem of hypersonic air breathing propulsion”.

The velocity of the aircraft represents means to pre-compress the air. It was since long realized that, at sufficient velocities, a mechanical compressor (and turbine) were not needed in order to produce thrust. However, the corresponding engine, the ramjet, would not develop thrust at zero velocities, because there are no elevated pressures inside the engine at standstill. **Fig.1** shows a schematic sketch of a supersonic combustion ramjet. The following **Table 1**, indicates certain aspects of the extraordinary aero-thermo dynamics of hypersonic flight.

In the table the static temperature T_s and the static pressure p_s are calculated at the entrance of a **sub**sonic and **super**sonic combustor, respectively, for an ideal ramjet at 30 km altitude as a function of the flight Mach number M_a. The Mach number of the subsonic combustor is M_{sub} << 1.0 and of the supersonic combustor is set to $M_{\sup}$ = 3.5 in this example:

Subsonic combustor >< Supersonic combustor

M_a	T_s [deg.K]	p_s [bar]	$M_{\sup}$	T_s [deg.K]	p_s [bar]
0.0	231	0.0116	--	--	--
2.5	520	0.2	--	--	--
5.0	1400	6.1	3.5	405	0.08
10	4900	490	3.5	1400	6.4
15	10700	7700	3.5	3100	100

Table 1. Flight Mach number vs. static temperature and static pressure in a ramjet with a subsonic combustor or a supersonic combustor

Inspecting the numbers of the table, especially the relation between temperature and Mach number (for the subsonic combustion case), provides an illustration of the extremely high heat energies that are carried in the air flow at hypersonic Mach numbers. To make a physical interpretation, expressing the air kinetic energy as $1/2 \cdot u^2$, where u is the speed of the airflow in (m/s). At Mach number 0.5 the kinetic energy is ~ 11 kJ/kg (of air); at Mach number 2 ~ 180 kJ/kg and at Mach number 8 ~ 3000 kJ/kg.

Conventional combustion processes release the fuel chemical energy as heat into the air and the heat release is at most ~ 2000 to 3000 kJ/kg for typical hydrocarbon fuels. The reason for this limitation is that most fuels have rather constant energy densities, ~ 40 ~ 50 MJ/kg and also have maximum allowable fuel-air ratios around 0.06 ~ 0.07 (due to stoichiometry). As the numbers above indicate, the free stream air reaches "combustion-level" kinetic energies at Mach number ~ 8. That is, as the air is decelerated in the engine intake and the kinetic energy converted to heat energy, temperature levels rise in parity with what otherwise at most can take place in combustion. At this, some special complications come into play: Generally, at very high temperature levels, all gases tend to "dissociate" into simpler constituents. In practice, this means that the combustion process cannot continue according to chemistry that otherwise is valid at lower temperature levels. This matter is illustrated by the following example of hydrogen and oxygen combustion:

At 1800 deg K, the (isothermal) reaction between hydrogen and oxygen proceeds as:

$H_2 + 0.5O_2 \rightarrow H_2O$ Δh= -242 kJ, (enthalpy of reaction)

However, at 5200 deg.K the same reaction proceeds as:

$H_2 + 0.5O_2 \rightarrow 0.86H_2 + 0.43O_2 + 0.14H_2O$ Δh = -34 kJ , (enthalpy of reaction)

The combustion products at the right hand side of the equations correspond to an equilibrium composition and this is specified by a temperature dependent thermodynamic function of state (Gibbs function). In the equations above the value of the corresponding reaction enthalpies Δh are also given. The reaction enthalpy is in principle equivalent to the energy density of the fuel. Paradoxically, at very high temperatures, a mixture of hydrogen and oxygen will only react to a small extent. In engineering terms, one can thus say that at a combustion temperature of 5200 deg.K, hydrogen releases only ~ 14% (34/242) of its ordinary energy density to heat.

This means that at sufficiently high temperature levels, the concept "combustion" ceases to have the standard meaning, something that would constitute an upper speed limit, another internal "heat barrier", for air-breathing engines with *subsonic* combustion processes. The very high temperatures encountered here would also put an excessively high thermal load on the engine structures. However, the high-temperature problem can in principle be circumvented by keeping the air speed supersonic through the engine, that is, to realize *supersonic* combustion processes. At supersonic air speeds the static

temperature can be kept sufficiently low as shown in **Table 1**.
But then a new problem arises: Supersonic combustors must necessarily have an inlet temperature that with some margin exceeds the auto-ignition temperature of the fuel used. And as of the rather "low" temperature values in the table, this circumstance may constitute a problem in the lower hypersonic flight range. A still further problem arises due to compressibility effects in high speed flow: The subsonic combustor case typically has a low inlet Mach number comfortably distanced from the (constant-area) choking limit M=1.0. The supersonic case has an inlet Mach number (3.5 in this example) that may be uncomfortably near the choking limit M=1.0; a phenomena that will be more detailed later on in the text.

All in all, it seems reasonable to denote the Mach number range of about 6 ~ 8 as a "problematic" regime for ramjets as based on the calculation in the **Table 1**, using first principles: The Mach numbers here are too high for subsonic combustion- and too low for supersonic combustion to be realized. A qualitative characterization of the phenomena in **sub**sonic- and **super**sonic combustion in this "unsuitable" regime is summarized in the **Table 2**:

Subsonic Combustion	Supersonic Combustion
* Any standard liquid hydrocarbon fuel can be used. Fuel-air mixing and flame stabilization is enhanced by turbulence generators (flame-holders). Spontaneous ignition is easily achieved in the high temperature air flow	* The fuel selected must have a low auto-ignition temperature. How could fuel and air be mixed in a supersonic flow without turbulence generators? How can the combustion process be ignited and stabilized without flame-holders? Compounded problems arise because of the high flow velocities involved with little time available
* Low pressure losses exist in mixing and combustion but high (shock) pressure losses in the air intake	* Very high pressure losses occur in mixing and combustion. The constant-area choking condition (M=1) may impose gas dynamic limits to the fuel-air ratio. Without normal shock losses the efficiency of the air intake is high
* The very high temperatures in the combustor results in substantial dissociation. At this, the effective heat value of the fuel can be considerably reduced	The very low temperatures in the combustor inlet can jeopardize the spontaneous ignition and stable combustion of the fuel
* Very high mechanical and thermal loads impact the engine structure	* Limited technology is available for engine design

Table 2: Comparison between subsonic- and supersonic combustion in the "problematic" flight Mach number regime M~ 6-8

The "barriers" and "problematic" flight regions mentioned in the text above are not bound to be absolute or fixed forever. They could as well be considered as representations of the state-of-the-art of aerospace technology. Further, by using the principle of hybrid propulsion, that is, combining different engines for propelling the aircraft, the barriers can be bridged over. For instance, the rocket-ramjet combination is a classical way to deal with the ramjet's poor performance at low speeds. Dual-mode combustion, using partly subsonic and partly supersonic combustion in the same combustor is another way to make the transition between the two operationally very different modes more flexible however the hybrid alternatives give penalties of increased complications.

3. Parametric analysis of ramjet performance

3.1 First level: The ideal ramjet without speed limitation

The purpose of analyzing an "ideal" propulsive engine is to establish the "best" available performance of the engine principle in question. Further, the ideal engine provides a reference set for definitions of various efficiencies. The ideal ramjet process is conveniently defined graphically in an enthalpy-entropy (h-s) diagram as shown in **Fig.2**: The speed of the engine, "ramming" the ambient air represents a kinetic energy, which provides the pressure increase inside the engine, pos.1->pos.2, at which the stagnation pressure of the air is isentropic recovered. The combustion is represented by a heat addition at constant pressure to a gas (air) with constant composition and constant specific heats, pos.2->pos.4. Thereafter follows an isentropic expansion to the ambient pressure, generating the kinetic energy of the exhaust gas, pos.4->pos.e. Finally, the cycle is closed figuratively by a "cooling" process, pos.e->pos.1.

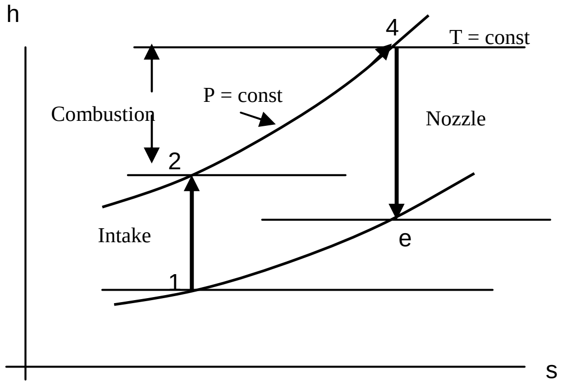

Fig.2. The ideal ramjet in an enthalpy (h) – entropy (s) diagram

3.1.1. The thermal efficiency of an ideal ramjet

The thermal efficiency η_T is a measure of what fraction of the heat supplied to the engine combustor is converted to net mechanical work (such as kinetic energy of the gas). This is a basic parameter in comparing different engines. By definition and with reference to the h-s diagram of **Fig.2**, the following relation is obtained:

$$\eta_T = \frac{netmechanicalwork}{heat\_added} =$$

$$= \frac{c_p(T_4 - T_e) - c_p(T_2 - T_1)}{c_p(T_4 - T_2)}$$

where c_p is the specific heat at constant pressure for air. Using the standard isentropic pressure-temperature relations, the relation between stagnation pressure, static pressure and the Mach number, the thermal efficiency can readily be expressed as a function of the flight Mach number M_a:

$$\eta_T = \frac{\kappa - 1}{2} \cdot \frac{M_a^2}{1 + \frac{\kappa - 1}{2} \cdot M_a^2}$$

where $\kappa = c_p / c_v$ =1.4 for air. The following table gives some numerical values of the thermal efficiency as a function of the flight Mach number:

M_a	η_T
0.0	0.0
0.5	0.05
1.0	0.17
2.5	0.55
5.0	0.83
10	0.95
15	0.98

Table 3.: The thermal efficiency of an ideal ramjet vs. flight Mach number

Thus hypersonic propulsion by ramjets, conceptually being the most simple heat engine imaginable, just a "straight tube without moving parts", has the potential to achieve very high efficiencies.

At a first glance, it may seem surprising that the ideal thermal efficiency is independent of the temperatures involved in the ramjet process, but this fact follows directly from the process definition as represented by the h-s diagram. However, it should be noted that this efficiency tells nothing about the thrust level or the kinetic energy produced.

3.1.2. Thrust, specific impulse and range of an ideal ramjet

By definition, the thrust Th of an ideal ramjet can be expressed as:

$$Th = \dot{m}_a \cdot (u_e - u_a)$$

where $\dot{m}_a$ is the air mass flow through the engine and $(u_e - u_a)$ is the air velocity increase through the engine. Defining the air specific impulse I_{spa} as thrust per air mass flow, after some manipulation as of the h-s diagram in **Fig.2**, the air specific impulse becomes:

$$I_{spa} = u_a(\sqrt{\tau} - 1)$$

Here, the stagnation temperature ratio $\tau = \dfrac{T_{04}}{T_{0a}}$ in the heat supply process is given by an approximate heat flow balance, resulting in:

$$\tau = 1 + \frac{f \cdot \mathrm{H}_f}{c_p \cdot T_{0a}}$$

where H_f is the fuel energy density, ~ 45 kJ/kg for typical hydrocarbon fuels, f is the fuel-air ratio, that can at most be ~ 0.07 for most hydrocarbon fuels due to stoichiometry and T_{0a} is the stagnation temperature of the air flow into the engine.
To compare an air-breathing engine with a rocket engine, the fuel amount carried on-board the craft will be used as a reference. Defining the fuel specific impulse I_{spf} as the thrust per fuel mass flow, then:

$$I_{spf} = \frac{Th}{\dot{m}_f} = \frac{I_{spa}}{f}$$

As the fuel-air ratio f must be less than ~ 0.07 for most hydrocarbons in order to keep the combustion process less than stoichiometric, the I_{spf} will become very large for air-breathing engines as compared to rocket engines. (Note that for the rocket, the "fuel " flow $\dot{m}_f$ is actually the sum of the oxidizer- and the "genuine" fuel flows. Typically, a unit mass of chemical fuel requires around 6 ~ 8 units of chemical oxidizer for combustion). This aspect of ramjet-rocket comparison is illustrated with a simple numerical example: The flight condition is set as Mach number 5, altitude 30 km and fuel-air ratio 0.07. Using the equations above, these data furnish τ = 3.27, I_{spa} = 1230 m/s and I_{spf} = 17600 m/s. For an ideal rocket, this last parameter I_{spf} would be much less, ~ 4000 m/s. By inspection of the classical aircraft range equation, the superiority of air-breathing engines regarding range R becomes evident as the range is directly proportional to the fuel specific impulse I_{spf}:

$$R = u_a \cdot \frac{I_{spf}}{g} \cdot \frac{L}{D} \cdot \ln \frac{m_0}{m_0 - m_f}$$

where $\frac{L}{D}$ is the lift-drag ratio for the craft, m_0 is the initial mass of the craft, m_b is the total fuel mass and $g = 9.81$ m/s2.

3.2 Second level: Analysis of the thermodynamic processes of some components

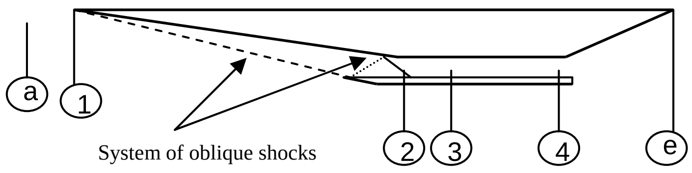

Fig.3 A principal sketch of a Scramjet

3.2.1 Intake system

One of the most important subsystems of a ramjet is the intake system. Its function and efficiency has the most significant impact on the performance. This system will therefore in principle be analyzed to some deeper extent.

As the ramjet engine will be utilized in supersonic and also hypersonic flight, is seems reasonable to suspect that considerable "losses" can take place in the air-intake of the engine in decelerating the high velocity incoming air to lower values suitable for sustaining combustion. Although early investigators of ramjets conceptually suggested isentropic inlet diffusers in the form of the well-known Laval exhaust nozzle, now run in the "reverse" direction, the practical realization turned out to be quite complicated. Basically, the air intake must provide the following functions (see **Fig.3** for definitions):

* The stagnation pressure ratio p_{02} / p_{0a} must be high (maximum value is 1.0)
* The air (mass-flow) capture coefficient C_A must be high (maximum value is 1.0)

The reason for this is that the stagnation pressure loss in the intake is also a part-measure of the air kinetic energy loss in the intake. The stagnation pressure loss is mainly due to the shocks existent in the intake. The air mass flow is directly proportional to the thrust produced. The maximum air flow corresponds to a flow situation where the streamlines are parallel up to the intake edge, denoted in shorthand as C_A=1.0. Further, any flow reduction will, indirectly, cause an increased aerodynamic resistance on the engine. The air flow through the intake is mainly decided by the various processes following the intake in so far they constitute a "blockage" of the flow. Typically, an intake configuration is characterized by a stagnation pressure ratio vs. capture coefficient relation. This relation is established empirically in wind tunnel experiments, at which trade-off among several influencing factors is achieved. A severe limiting phenomenon of supersonic intake performances is the shock-boundary layer interaction. In unfavorable cases, such interaction can cause flow separation and flow instabilities with concurrent low values of pressure ratio and capture coefficient.

Theoretically, an intake with a large number of oblique (weak) shocks would be preferable as regards pressure ratio. But again, due to shock-boundary layer interaction phenomena, the best that could be practically realized in supersonic flow were air-intakes characterized by two or three oblique shocks (and a final normal shock) to bring the air velocity down to subsonic values and the air (static) pressure up to levels that can sustain combustion. However in the hypersonic intake, the final normal shock must be eliminated, something that theoretically and beneficially would result in higher pressure ratio. But observations from experimental tests show that the shock-boundary layer interaction phenomena will become ever more severe at hypersonic velocities. These circumstances have contributed to focus the interest of hypersonic intakes to geometries that depend mainly on external compression, arranged by protruding, axi-symmetric "spikes" or 2-dimensional "wedge" designs as shown in the earlier **Fig.3**. However, as hypersonic test facilities are rare and the experimental registration of hypersonic intake characteristics is both a very difficult task and a very expensive endeavour, the open documentation of high performance intakes is quite limited.

But some basic theoretical considerations can nevertheless be made on the special functionality of hypersonic intakes. In the supersonic case, the performance criterion is by traditions closely tied to the values of the stagnation pressure ratio. Typically, a *supersonic* intake should have something like 0.5 ~0.7 in the stagnation pressure ratio. The question is then, what would be a reasonable value for a *hypersonic* intake? But strangely enough, the stagnation pressure ratio is becoming less and less important as the air velocity increases. As shown in the h-s diagram of **Fig.4**, the relevant property is instead the kinetic energy remaining after the intake process, rather than the pressure ratio. Defining now a kinetic efficiency for the intake η_K as:

η_K = (kinetic energy of air after intake) / (kinetic energy of air in free stream)

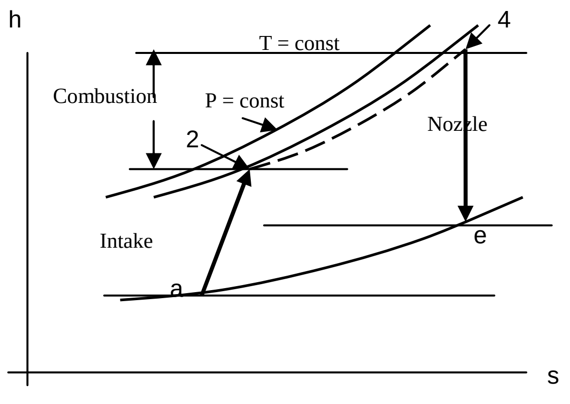

Fig.4. Introduction of losses in the ideal cycle, the "Semi-ideal"scramjet in an enthalpy (h) – entropy (s) diagram

After some manipulation of the relations stated in the h-s diagram results in:

$$\eta_K = 1 - \frac{\left(\frac{p_{02}}{p_{0a}}\right)^{-\frac{\kappa-1}{\kappa}} - 1}{\frac{\kappa-1}{2} \cdot M_a^2}$$

This expression shows that the thermodynamically relevant property, the kinetic efficiency, becomes increasingly dominated by the flight Mach number as the speed increases. **Table 4,** shows a computational example of the kinetic efficiency of an intake as a function of the stagnation pressure ratios at a flight Mach number M_a = 7:

η_K	p_{02} / p_{0a}
0.37	0.001
0.77	0.0154
0.86	0.05
0.91	0.1
0.94	0.2

Table 4.: Kinetic efficiency of an intake at Ma = 7 vs. stagnation pressure ratio

The value 0.0154 is the stagnation pressure ratio over a normal shock at Mach number 7. A normal shock would occur in a pitot intake, which is the most simple intake configuration available. Because of poor supersonic performance it has only been used in subsonic and transonic ramjets. But obviously the performance becomes different at hypersonic speeds, displaying a kinetic efficiency of 77%. The question is would this value be enough in a hypersonic ramjet using subsonic combustion implying the possibility to use a most simple intake design?
The kinetic efficiency of the intake also enters the gross performance parameters as defined above in the case of the ideal ramjet. Assuming a non-ideal ramjet, where all adiabatic pressure losses are incorporated in the intake kinetic efficiency, the thermal efficiency η_{Tr} of this ramjet becomes:

$$\eta_{Tr} = \frac{\kappa-1}{2} \cdot \frac{M_a^2}{1+\frac{\kappa-1}{2} \cdot M_a^2} \cdot \frac{\tau \cdot \eta_K - 1}{\tau - 1}$$

In comparison with the ideal case formulated above, there is now a correction factor $\frac{\tau \cdot \eta_K - 1}{\tau - 1}$, always less than one, and thus the combustion stagnation temperature ratio τ becomes part of the issue.

3.2.2 The supersonic combustor

3.2.2.1 "Ideal" Fuel-air preparation

In supersonic ramjets, the air velocity is diminished in the air intake to low subsonic values. The subsonic ramjet combustor design is characterized by variously shaped baffles, inserted into the flow. The purpose of those area-blocking devices is to increase turbulence and thus fuel-air mixing and subsequent combustion rate and stabilization. However, in a hypersonic ramjet with supersonic internal flow, such baffles cannot be allowed because of shock generation and possible collapse of the supersonic flow to a subsonic flow with concurrent extreme mechanical and thermal loads on the engine structure. The question is then how to realize an efficient mixing and combustion in the supersonic flow. At this, one approach could be to provide a plain injection of a liquid fuel jet directly into the supersonic stream. The subsequent disintegration of this liquid due to aerodynamic forces, could then generate the necessary amount of turbulence for mixing enhancement. This scheme would constitute a basis for a simple and purposeful engineering design of a supersonic combustor. But obviously, the injection of a low velocity fuel mass-flow into a high velocity air stream would result in pressure losses. The possible amount of pressure losses and accompanying other effects are analyzed below:

The interactions between a high velocity gas and a low velocity liquid are very complicated, which is illustrated by the fact that no consistent theory for the liquid disintegration exists.

However, by first principles of mechanics, it is possible to enclose the entire atomization, spray development and mixing processes in a large control volume, as illustrated in **Fig.5**. This sketch shows a constant area case with a sufficiently long length coordinate between pos 2 and 3, so that complete mixing takes place. The complicated phenomena of fuel –air preparation, then become internal forces and the only remaining external forces on the control volume are the pressure forces, (assuming no wall friction). This arrangement is denoted as the "ideal" fuel-air preparation part of the supersonic combustor. By this way, both a qualitative classification and a quantitative calculation of flow variable changes can be achieved:

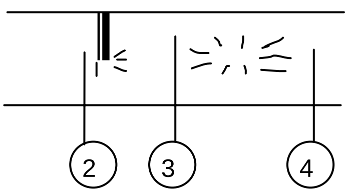

Fig. 5. Principal mixing and combustion zones

The momentum equation for the mixing zone is, with reference to **Fig.5**:

$p_2 + \rho_2 u_2^2 = p_3 + \rho_3 u_3^2$, (Ideal fuel-air preparation is between pos 2 and pos3).

The continuity equation becomes:

$(1+f) \cdot \rho_2 u_2 = \rho_3 u_3$

where f = (mass flow of disintegrating and dispersing injected medium) / (mass flow of air); in the corresponding practical case, the fuel air ratio. In the high velocity flow, compressibility effects must be taken into account: Using the definition of the Mach number:

$$M = \frac{u}{a}$$

Where a is the velocity of sound; $a = \sqrt{\kappa \cdot R \cdot T}$

The relations between local stagnation and static pressure and -temperature and Mach number are:

$$\frac{p_0}{p} = \left(1 + \frac{\kappa - 1}{2} M^2\right)^{\frac{\kappa}{\kappa-1}}$$

and

$$\frac{T_0}{T} = \left(1 + \frac{\kappa - 1}{2} M^2\right)$$

The liquid "fuel" dispersion process is assumed to take place at a constant stagnation temperature. That is, energy turnovers for evaporation and surface formation are neglected. (Actually, they can be taken into account but at the cost of some loss of the central overview). The equations and assumptions above will be termed as representing the ideal fuel atomization and mixing process, in a shorthand notation the "Atomixing case". Introducing now the following shorthand Mach number function:

$$Y(M) = \frac{M\sqrt{1 + \frac{\kappa - 1}{2} M^2}}{1 + \kappa \cdot M^2}$$

And after some manipulations, the basic equations yield a very simple relation between the fuel/ air ratio and the ensuing Mach number change:

$$1 + f = \frac{Y(M_3)}{Y(M_2)}$$

That is, the acceleration and dispersion of a secondary medium in a flow process will always change the Mach number. The graphic appearance of the Y-function vs. the Mach number is sketched in **Fig.6**. It starts from zero at Mach number zero, rises to a maximum at Mach number one, then decreases at increasing Mach number and eventually approaches an asymptotic value at higher Mach numbers. Some qualitative conclusions can now be drawn with the help of f vs. $Y(M)$ relation above and the sketch of the Y- function:

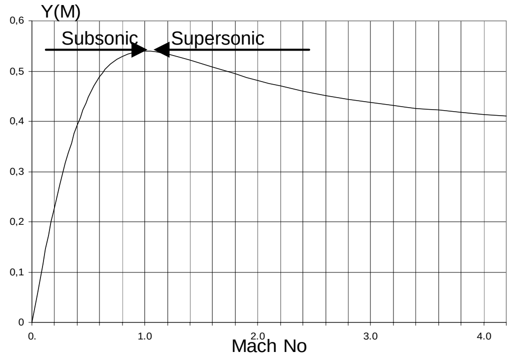

Fig.6. The shorthand Y(M) as a function of the Mach No.

"Atomixing" in subsonic flows will increase the Mach number. Atomixing in supersonic flows will decrease the Mach number. The limiting end Mach number in both cases is one. It can be shown by introducing an entropy function (for instance by plotting the equations in a h-s (enthalpy-entropy) diagram, that this limit (M=1) cannot be passed from either (subsonic or supersonic) side in a continuous change of state. However, starting in the supersonic regime, it is possible for the flow to "jump" to subsonic Mach numbers by a shock wave and thereafter follow the subsonic branch.

It follows from this discussion that there exists four main regions of the Atomixing flow process:

* A case starting with subsonic Mach numbers and ending with subsonic, higher values. This could be termed "Weak Subsonic Atomixing".

* A case starting with subsonic Mach numbers and ending with supersonic Mach numbers. This could be termed "Strong Subsonic Atomixing" and is probably non-existent because entropy laws would be violated.

* A case starting with supersonic Mach numbers and ending with supersonic, lower values. This could be termed "Weak Supersonic Atomixing".

* A case starting with supersonic Mach numbers and ending with subsonic values (involving shocks). This could be termed "Strong Supersonic Atomixing".

Introducing now another shorthand Mach number function:

$$Z(M) = \frac{1+\kappa \cdot M^2}{\left(1+\frac{\kappa-1}{2}M^2\right)^{\frac{\kappa}{\kappa-1}}}$$

Again, after some algebraic, the total pressure ratio over the "Atomixing" process becomes:

$$\frac{p_{03}}{p_{02}} = \frac{Z(M_2)}{Z(M_3)}$$

Inspection of the graphical appearance of the Z-function in **Fig.7** shows that the stagnation pressure ratio is always less than one, for all cases of "Atomixing" processes.

The following computational example shows the exit Mach number and the stagnation pressure for a supersonic "Atomixing" process starting with an assumed inlet Mach number M_2 = 3.5. The fuel air ratio is varied between 0.01 and 0.07 (this last value is the maximum allowable for hydrocarbon fuels as mentioned earlier in the text):

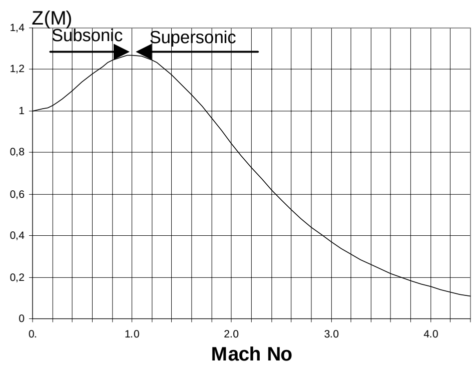

Fig.7. The shorthand Z(M) as a function of the Mach No.

f	M_3	p_{03}/p_{02}
0.01	3.32	0.85
0.03	3.00	0.64
0.05	2.76	0.52
0.07	2.54	0.43

Table 5. Outlet Mach number M_3 and stagnation pressure ratio p_{03}/p_{02} vs. fuel-air ratio f in ideal supersonic "Atomixing" at const. area with inlet Mach number M_2 = 3.5

Obviously, the stagnation pressure losses are rather high, much higher than in a corresponding subsonic case. For instance, at the stoichiometric fuel-air ratio 0.07, the relative pressure loss is around 57% (without any flame holders or turbulence generators!) Another peculiarity here is the rapid outlet Mach number decrease with increasing fuel-air ratios. For instance, using a fuel-air ratio f = 0.3 in the calculation, no solution exists for M_3. This implies that the flow has encountered a "dissipative choking". Physically, the inlet conditions of the combustor are changed in a transient so that the outlet state M_3=1.0 is established.

3.2.2.2. Pressure losses in "ideal" combustion

As mentioned above, the practical supersonic combustor should be free from any constrictions or geometries that can cause unwanted shocks. This includes also the flame-stabilizing baffles that are used in conventional, subsonic combustors. Supposing a supersonic combustion can be realized without such blocking geometries, what are the pressure losses in an "ideal" supersonic combustion at constant area? The potential size of pressure losses and accompanying other effects are analyzed below.

The basic equations are in principle the same as in the case of fuel-air mixing. The only difference is that in this ideal case, the fuel-air ratio f = 0; however there exists a stagnation temperature ratio as prescribed by the heat addition process.

The momentum equation for the combustion zone is, with reference to **Fig.5.** (Ideal combustion is between pos 3 and 4)

$$p_3 + \rho_3 u_3^2 = p_4 + \rho_4 u_4^2$$

The continuity equation:

$$\rho_3 u_3 = \rho_4 u_4$$

The energy equation:

$$q_{3\to4} = c_p\left(T_{04} - T_{03}\right)$$

where $q_{3\to4}$ (kJ/kg air) is the heat added to the air at constant chemical composition; symbolically representing the combustion process.

Using the same pressure- temperature relations for compressible flow as above in the mixing analysis, together with the shorthand Mach number functions $Y(M)$ and $Z(M)$, the outcome is:

$$\sqrt{\frac{T_{04}}{T_{03}}} = \frac{Y(M_4)}{Y(M_3)}$$

and

$$\frac{p_{04}}{p_{03}} = \frac{Z(M_3)}{Z(M_4)}$$

Again, analyzing the shape of the Y- and Z-functions, the qualitative conclusions for combustion processes are that the Mach number is always changing towards a limiting value 1.0 both for subsonic and supersonic combustion and that the stagnation pressure ratio is always less than one. By plotting the equations in a h-s (enthalpy-entropy) diagram, it can be shown that limit (M=1) cannot be passed from either (subsonic or supersonic) side in a continuous change of state. However, starting in the supersonic regime, it is possible for the flow to "jump" to subsonic Mach numbers by a shock wave and thereafter follow the subsonic combustion branch. The Mach number M= 1 is called the "critical Mach number" and a flow condition at which this occurs is called "thermal choking". A more detailed study of the graphics of the Y-function makes possible the characterization of four regions of combustion processes:

* A case starting with subsonic Mach numbers and ending with subsonic, higher values. In combustion science it is termed "weak deflagration".

* A case starting with subsonic Mach numbers and ending with supersonic Mach numbers. This is termed "strong deflagration" and is probably non-existent because entropy laws would be violated.

* A case starting with supersonic Mach numbers and ending with supersonic, lower values. This is termed "weak detonation".

* A case starting with supersonic Mach numbers and ending with subsonic values (involving shocks). This is termed "strong detonation".

(* In a general sense, there exist also a whole branch of quasi-constant volume combustion processes, but as this text deals with stationary combustion, the non stationary phenomena must be left aside)

The following computational example shows the exit Mach number and the stagnation pressure ratio for a supersonic combustion process starting with an inlet Mach number equal to the outlet value for the mixing case above. The fuel air ratio is also here varied between 0.01 and 0.07. The combustor inlet stagnation temperature T_{02} corresponds to a flight Mach number 10 and flight altitude 30 km. The ideal, stagnation temperature increase $\Delta T_0 = T_{04} - T_{03}$ is calculated taking the energy density of the fuel H_f = 45 MJ/kg, c_p= 1.0 kJ/kg, deg and using $f \ll 1$

f	M_3	ΔT_0 2-3 deg. K	$\frac{T_{04}}{T_{03}}$	M_4	$\frac{p_{04}}{p_{03}}$	$\frac{p_{04}}{p_{02}}$
0.01	3.3	450	1.09	2.7	0.58	0.49
0.03	3.0	1350	1.28	1.8	0.38	0.25
0.05	2.8	2250	1.46	1.1	0.36	0.19
0.07	2.5	3150	1.64	no sol.	---	---

Table 6. Combustor outlet Mach number and stagnation pressure ratio vs fuel air ratio in an ideal supersonic combustion case. See also Fig 5.

The **Table 6** above shows that thermal choking is encountered around $f = 0.07$. In practice, there will then be an inlet Mach number transient forced upon the flow that can seriously disturb the intake function and a thermal choking situation should be avoided.

The column p_{04}/p_{02} is the stagnation pressure ratio including the mixing process. At f =0.05 (max permissible due to thermal choking) the pressure ratio represents a stagnation pressure loss of 81%. To what extent such levels of pressure losses constitute limitations to hypersonic ramjets should be assessed more closely. However, the rule of the thumb is that the impact of pressure losses decreases with increasing flight Mach number.

3.2.3 The exhaust nozzle

After the combustion zone the high energy combustion gases are expanded through an exhaust nozzle to a supersonic jet, (ideal expansion to ambient condition **see Fig 4**). The gases are expanded from the pressure level at the combustor exit to ambient pressure through some kind of divergent nozzle. For the exhaust nozzle in **Fig.3** the nozzle perimeters are between the atmospheric streamlines and an angled ramp. The angled ramp can also be designed as a parabolic curve. The thrust is the mass flow multiplied with the difference between exhaust velocity and flight speed.

The exhaust velocity can be calculated from:

$$\frac{u_e^2}{2} = c_p \cdot T_{04} \cdot \left(1 - \left(\frac{p_a}{p_{04}}\right)^{\frac{\kappa-1}{\kappa}}\right)$$

The net thrust; $Th = \dot{m}_a \cdot (u_e - u_a)$

The specific impulse I_{spf}, (as the thrust per fuel mass flow or as the air specific impulse I_{spa} ,(the thrust per air mass flow), divided with fuel-air ratio).

$$I_{spf} = \frac{Th}{\dot{m}_f} = \frac{I_{spa}}{f}$$

4. Conclusion

An analysis based on "first principles" is an alternative tool to make early (preliminary?) performance predictions and cycle optimizations at different operating points. By applying first principles of mechanics and thermodynamics, the deviating nature of the **SCRJ** principle as compared to concurrent aero propulsion technology becomes evident. To realize the **SCRJ**, the reward is the extremely high (theoretical) thermal efficiency of the hypersonic ramjet. But in practice this benefit is offset by entropy-producing stagnation pressure losses in the intake and in fuel-air preparation and combustion; together with entropy-related stability issues of the internal supersonic flow (as illustrated by the possible occurrence of unintentional shock patterns and existence of critical Mach numbers).

The current opinion of this author is that the supersonic fuel-air preparation process is the matter that needs most attention and a realization of engineering solutions. The supersonic fuel-air preparation process is of an extremely complex structure, constituting interacting subsonic, transonic and supersonic, two-phase flow including disintegrating liquid jets. To what extent this can be purposefully modelled by current CFD techniques remains to be proven. But fortunately, as it happens, the conventional supersonic wind tunnel could provide an almost-full-dimensional scale experimental tool with realistic internal Mach numbers as encountered in the **SCRJ**; from around Mach 2 to Mach 4. The thermodynamic performance of different fuel injection system can be investigated, the physical functioning of different fuels together with the necessary length coordinate to produce a combustible mixture.