INTELLIGENT BASED TRAJECTORY PLANNING IN TERRAIN FOLLOWING FLIGHT

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Abstract

A novel approach in Terrain Following/Terrain Avoidance (TF/TA) flight is proposed that facilitates flying over unknown terrains. Intelligency is implemented using fuzzy decision making tools. This method can be used in off-line design in trajectory planning which has wide applications in TF/TA. A relationship between slope of terrain and aircraft height with speed of aircraft is constructed by fuzzy approach.

1 Introduction

Terrain Referenced Navigation (TRN) system currently may be provided on vehicles such as aircraft to provide accurate position registration relative to the terrain by referencing a vehicle position within a digital terrain map database. By virtue of the digital map and by knowing the vehicle position accurately, it can also provide the vehicle driver or pilot with warnings of impending obstacles and cues for avoidance action. In the case of aircraft, these warnings can be of Impending Controlled Flight In to Terrain (CFIT) and cues for low-level ***terrain following*** flight. Current Terrain Referenced Navigation systems for aircraft generally combine the aircraft Inertial Navigation System data with measurements of height above ground level, altitudes above mean sea level and terrain heights from the stored map data to provide high precision drift-free navigation. The high precision drift-free navigation generally includes accurate terrain referenced position registration, warning of impending controlled flight into terrain, situational awareness terrain display and ***terrain following*** cues. However, such a Terrain Referenced Navigation system only provides advisory information to the vehicle driver such as an aircraft pilot and in aircraft flight conditions, which are unduly hazardous and in which the pilot may be operating at less than normal or peak efficiency, there is a need for ensuring that the vehicle driver or aircraft pilot is assisted in driving the vehicle or flying the aircraft with increased safety. There is thus a need for a ***Terrain Following*** Apparatus for a vehicle, which can effectively assist, in a safe manner, the vehicle operator to drive or fly the vehicle [1].

The objective of terrain following/terrain avoidance (TF/TA) is to find the optimum flight path that maximizes survivability while satisfying appropriate flight path constraints. Chance of survival of an unmanned aerial vehicle in low-level flights strongly depends on height and Mach number. In some cases for long endurance and high reliability flight of space vehicles, Surveillance, Reconnaissance and Atmospheric Monitoring are required as mission. Due to existence unknown terrains and obstacles, mission may be crashed in low level flights.

There are many algorithms investigated for trajectory planning in TF, TA or TF/TA maneuver. TF/TA can be coupled or uncoupled. Uncoupled TF/TA is a maneuver in which the vehicle can perform TF or TA at a time independently. If the vehicle can pass over the terrain, TF maneuver is performed otherwise a TA maneuver for passing across the terrain should be performed. The TF/TA maneuver consists of a series of longitudinal and lateral arcs, separated by transition arcs. This goal can be achieved in two parts. First part, which is called open loop guidance, performs TF/TA off-line. The second part is called closed loop guidance, which performs TF/TA on-line. The focus of this paper is on the first part, but a fuzzy approach, which is state of the art in literature.

The terrain data can be forward looking radar or stored terrain maps. In conventional terrain following/terrain avoidance method, trajectory design is based on accurate terrain modeling. In this modeling all details of terrains are considered. To perform terrain following perfectly, the system must have sufficient data for the terrain ahead of the aircraft, but it is impractical to use all of the terrain information for the entire flight. Furthermore, small changes in terrains have little effects on the governing dynamics of aircraft and so they are not important.

Ref. [2] presents a new real-time optimization technique that efficiently generates a robust, optimum TF/TA trajectory. For this purpose dynamic programming is used. Ref. [3] has proposed a new numerical approach for the generation of optimal trajectories. Ref. [4] proposes trajectory generation based on minimizing a linear combination of flight time and terrain masking. Ref. [5] uses the method of steepest descent for path optimization. Ref. [6] uses an optimal control approach for terrain following maneuver of an aircraft. In this approach, path following and flight time are combined in cost function. Ref. [7] has used genetic algorithm in route optimization.

2 Aircraft Dynamics

Due to complexity of optimization process in TF/TA problem, most papers have used kinematics or a simple dynamics for aircraft. Using a simple dynamics guaranties finding an optimal solution. Most investigators in TF prefer using a simple dynamics rather than a complex one, and then will compensate the problems are made due to this simplification in controller design. By considering aircraft as a point mass, the longitudinal dynamics is as follow:

$$\dot{x} = V\cos\gamma \tag{1}$$

$$\dot{y} = V\sin\gamma \tag{2}$$

$$m\dot{V} = T\cos\alpha - D - mg\sin\gamma \tag{3}$$

$$mV\dot{\gamma} = L + T\sin\alpha - mg\cos\gamma \tag{4}$$

Where:

$$\begin{aligned} L &= 0.5\rho V^2 S C_L, \quad C_L = C_{L\alpha}\alpha \\ D &= 0.5\rho V^2 S C_D, \quad C_D = C_{D0} + \kappa C_L^{\,2} \end{aligned} \tag{5}$$

The air density is a function of altitude and the maximum of the thrust is a function of altitude and Mach number, that is, $\rho(h), T_{\max}(h, M)$.

The governing equations can be more complicated and depending on the analysis of a desired trajectory anyone or combination of the control variables can be selected, but usually these controls are subjected to physical limitations or constraints like as $\alpha_{\min} \le \alpha \le \alpha_{\max}$, $0 \le T \le T_{\max}$. The governing equations will be introduced in spatial form too.

3 Cost Function

Various cost functions can be used in off-line guidance. These cost functions can be used for a minimum time, minimum fuel consumption or other strategies. Ref. [4] suggests the following cost function:

$$J = \int_{t_0}^{t_f} \left[k + (1-k)(h - F(x))^2 \right] dt \quad ; 0 \le k \le 1 \tag{6}$$

Where *F(x)* is the terrain profile as Fig. 1 and *h* is a specified terrain clearance. *K* is a constant that determines the scenario. K equal to 1 represents the minimum time scenario and k equal to zero represents terrain clearance minimization. For in between values a combined scenario will be considered. Ref. [5] presents the following cost function:

$$J = \int_{\circ}^{tf} (W_1 C_t^2 + W_2 h^2 + W_3 f_{TA}) dt \tag{7}$$

The first term minimizes flight time, the second term minimizes vehicle altitude respect to terrain and the third one minimizes threat avoidance. W$_1$, W$_2$, W$_3$ are constant weights.

4 Optimal Trajectory Planning

By eliminating time between (1) to (4) aircraft dynamics equations reduces and will be written in spatial form by the following equations:

$$y' = \tan\gamma \tag{8}$$

$$\gamma' = \frac{T\sin\alpha + L}{mV^2\cos\gamma} - \frac{g}{V^2} \tag{9}$$

$$V' = \frac{T\cos\alpha - D}{mV\cos\gamma} - \frac{g}{V}\tan\gamma \tag{10}$$

Where x is independent variable. The prime indicates differentiation with respect to independent variable x.

Based on these equations, performance index (6) for minimum time scenario (*k=1*) becomes:

$$J = \int_0^{x_f} \frac{1}{V\cos\gamma} dx \tag{11}$$

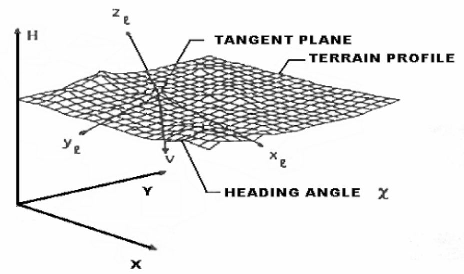


Fig. 1. Terrain Profile [5]

By solving (8) for γ:

$$\gamma = \tan^{-1}(y') \tag{12}$$

The path curvature is derived by differentiating of (12) and using (9) as follow:

$$y'' = \left(\frac{T\sin\alpha + L}{mV^2\cos\gamma} - \frac{g}{V^2}\right)(1 + \tan^2\gamma) \tag{13}$$

It means the path y(x) over the terrain should be differentiable twice.

By solving (13) for $\frac{1}{V\cos\gamma}$ and replacing in (11), the performance index based on control variables is:

$$J = \int_0^{x_f} \left(\frac{y''}{1+\tan^2\gamma} + \frac{g}{V^2}\right)\left(\frac{mV}{T\sin\alpha + L}\right) dx \tag{14}$$

By breaking interval $[0\, x_f]$ to subintervals, (14) can be written as the following summation [7]:

$$J = \sum_{i=0}^{N-1} \int_{i\Delta x}^{(i+1)\Delta x} \left(\frac{y''}{1+\tan^2\gamma} + \frac{g}{V^2}\right)\left(\frac{mV.d}{T\sin\alpha -}\right. \tag{15}$$

Now considering the desired path y(x) as follow:

$$y(x) = a_\circ + a_1 x + a_2 x^2 + \ldots + a_n x^n \tag{16}$$

By using (8):

$$1+\tan^2\gamma = 1+y'^2 \qquad (17)$$

The second derivative of y(x) in (15) is:

$$y'' = 2\times 1a_2 + 3\times 2a_3x + n(n-1)a_nx^{n-2} \qquad (18)$$

By substituting (17) and (18) in performance index in (15) a parameter optimization problem is resulted and solving for each sub intervals results to a set of equations which can be solved by least square method. So the flight path y(x) will be determined. In this work the values of V, α and η are computed by a ***fuzzy decision-making*** in each sub interval.

5 Intelligent Approach to TF Problem

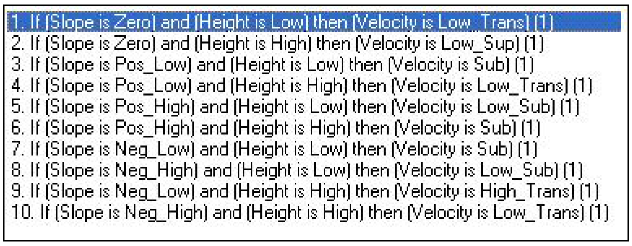
In the present research, a terrain is recognized based on fuzzy logic so the small changes in terrain are omitted automatically. By default, the aircraft should perform a terrain following maneuver in vertical plane, but based on some conditions the vehicle can perform a terrain avoidance maneuver in horizontal plane automatically. Making an aircraft intelligence means a decision making through passing over or across a terrain. Our work is confined to terrain following maneuver. For implementing intelligence, first, slope angle of terrain, height of vehicle, speed and throttle setting are considered as linguistic variables and trajectory design is performed off-line based on various speeds of the vehicle.

For passing over the terrain, some rules are required. So a rule base should be created. Membership functions of slope angle and those of height, speed and throttle setting are implemented in Fig. 3 to Fig. 5. Passing aircraft over or across a terrain is performed based on interaction between the terrain and the vehicle, which are implemented in cost function.

Variable speeds and the importance of resolution for angles upper than 45 degrees can generate so much rules and membership functions for slope and height and also a combination of them. Each rule can be effective in an optimum solution. The dependency of membership functions to speed is of importance in optimum solution and this causes expansion and contraction in membership functions. The fuzzy rule base is used to map the input variables to the output variable. Ten rules are considered:

- If Slope is Zero and Height is Low Then Velocity is Low_Trans
- If Slope is Zero and Height is High Then Velocity is Low_Sup
- If Slope is Pos_Low and Height is Low Then Velocity is Sub
- If Slope is Pos_Low and Height is High Then Velocity is Low_Trans
- If Slope is Pos_High and Height is Low Then Velocity is Low_Sub
- If Slope is Pos_High and Height is High Then Velocity is Sub
- If Slope is Neg_Low and Height is Low Then Velocity is Sub
- If Slope is Neg_Low and Height is High Then Velocity is Low_Sub
- If Slope is Neg_High and Height is Low Then Velocity is High_Trans
- If Slope is Neg_High and Height is High Then Velocity is Low_Trans

The above rules can be implemented in *FUZZY* toolbox in Matlab as follow:



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1. If (Slope is Zero) and (Height is Low) then (Velocity is Low_Trans) (1)
2. If (Slope is Zero) and (Height is High) then (Velocity is Low_Sup) (1)
3. If (Slope is Pos_Low) and (Height is Low) then (Velocity is Sub) (1)
4. If (Slope is Pos_Low) and (Height is High) then (Velocity is Low_Trans) (1)
5. If (Slope is Pos_High) and (Height is Low) then (Velocity is Low_Sub) (1)
6. If (Slope is Pos_High) and (Height is High) then (Velocity is Sub) (1)
7. If (Slope is Neg_Low) and (Height is Low) then (Velocity is Sub) (1)
8. If (Slope is Neg_High) and (Height is Low) then (Velocity is Low_Sub) (1)
9. If (Slope is Neg_Low) and (Height is High) then (Velocity is High_Trans) (1)
10. If (Slope is Neg_High) and (Height is High) then (Velocity is Low_Trans) (1)
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Fig. 2. Fuzzy Rules

There are three membership functions. Slope of terrain and Height of vehicle are input membership functions and Mach number is out put membership function. A more membership functions in terrain slope, vehicle height and vehicle speed a more realizable solution will be achieved. These membership functions are implemented in Matlab as follow:

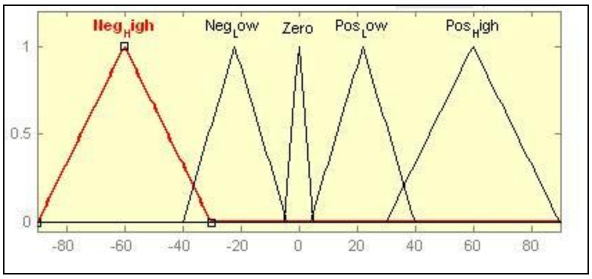

Fig. 3. Slope Membership Function

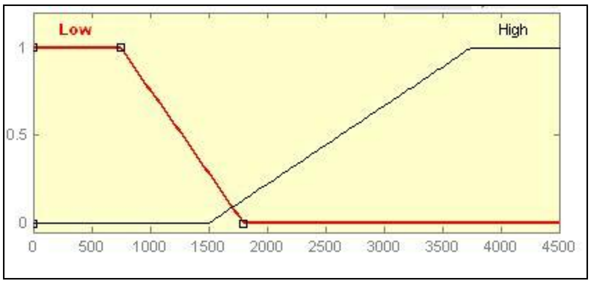

Fig. 4. Height Membership Function

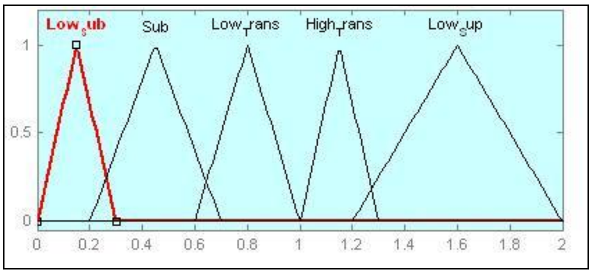

Fig. 5. Mach Membership Function

Figure 6 illustrates rules and the defuzification method. By the defuzzification, a crisp value will be computed for Mach number. In figure 6 the Mach number is computed for typical values of slope equal to 34.7 degree and height equal to 2800 meters.

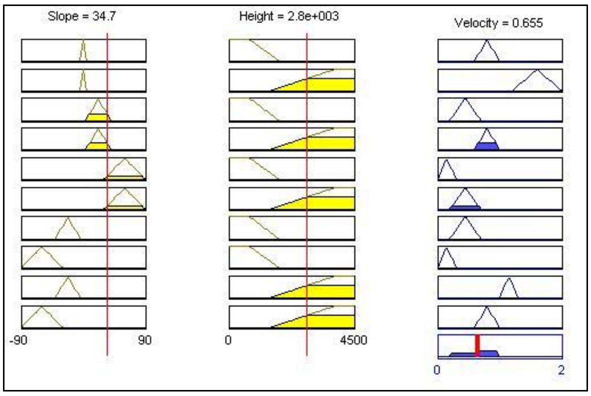

Fig. 6. Rules and Defuzzification

The correlation minimum inference method geometrically sums the consequences of the active associations, and centroidal defuzzification is used to find the crisp output for Mach number.

The rule surface which describes the decision making area, is shown in Figure 7.

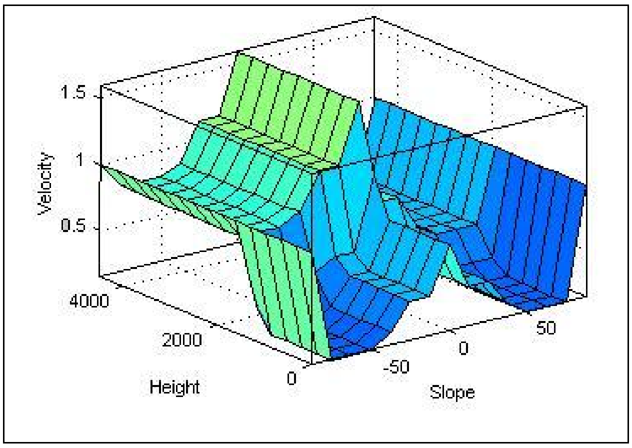

Fig. 7. Rule Surface

6 Simulation and Case Study

By using the proposed method a simulation is performed. Lift and drag are computed as (5). Ref. [8] has used the following relations for $C_{L\alpha}$ and C_{D0} versus Mach number M:

$$CL_{\alpha} = 4.198 - 0.3425M + 1.0125M^2 \tag{19}$$

$$Log(C_{D0}) = -2.02037 - 0.078043\,Log(M) + 0.05707\,Log(y) \tag{20}$$

Also thrust is modeled as follow:

$$T = (\eta^6 + N(M,y))T_{\max}(M,y) \tag{21}$$

About fifty case studies are simulated in Ref [9]. In this paper, a pure terrain following flight is simulated. A typical terrain is used as an input for simulation. Figure 8 illustrates the terrain and trajectory of vehicle. Figure 9 to 13 illustrate Mach, Velocity, Flight Path Angle, Angle of Attack and Throttle Setting respectively in spatial domain and figure 14 to 18 illustrate these variables in time domain.

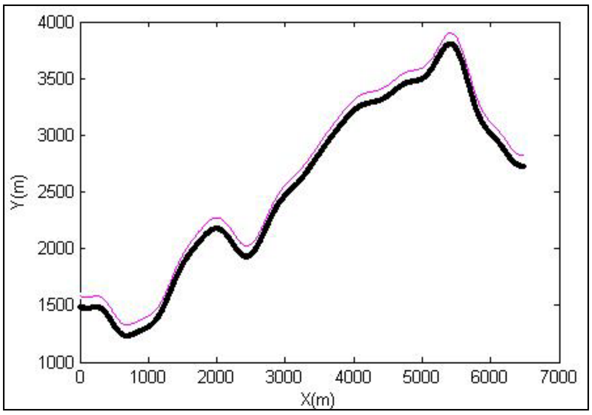

Fig. 8. Terrain and Trajectory

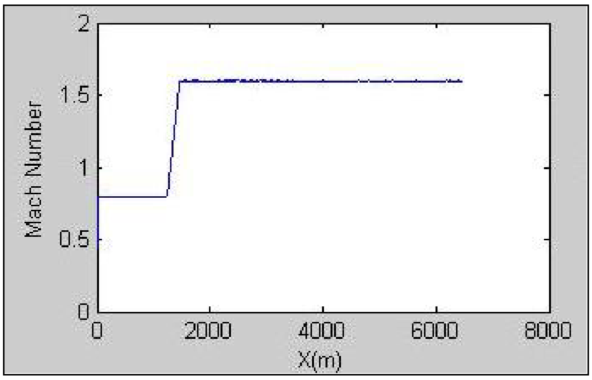

Fig. 9. Mach vs. Range

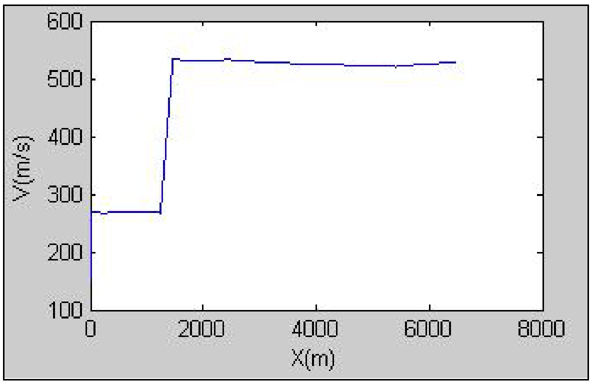

Fig. 10. Velocity vs. Range

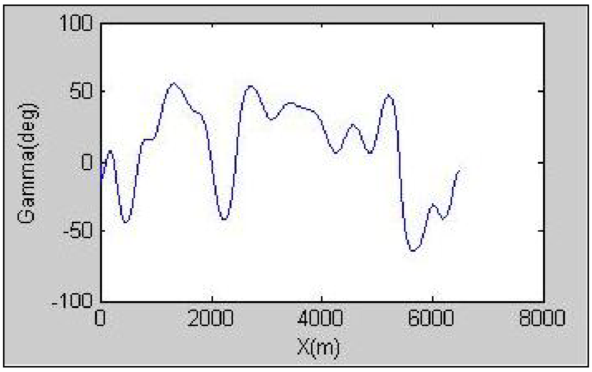

Fig. 11. Flight Path Angle vs. Range

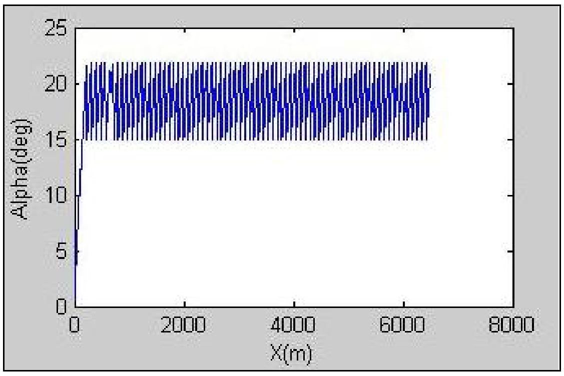

Fig. 12. Angle of Attack vs. Range

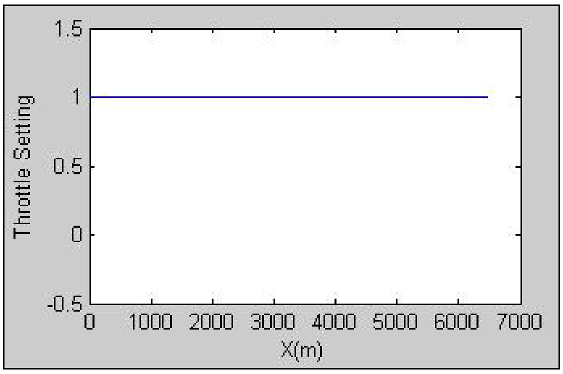

Fig. 13. Throttle Setting vs . Range

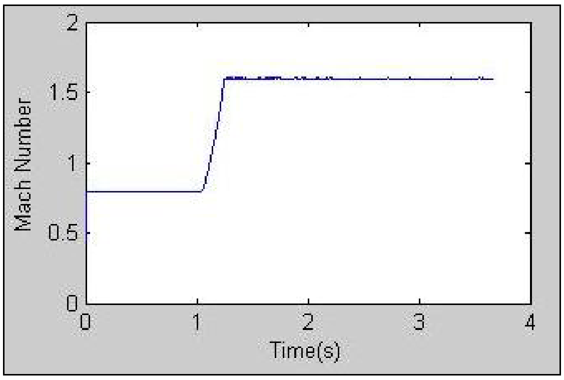

Fig. 14. Mach vs. Time

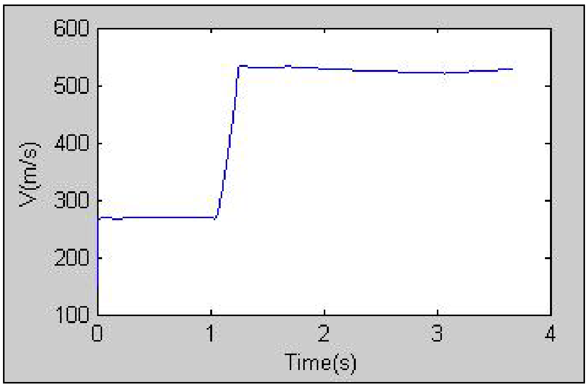

Fig. 15. Velocity vs. Time

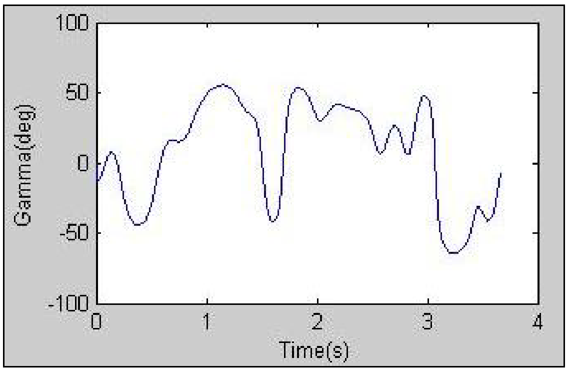

Fig. 16. Flight Path Angle vs. Time

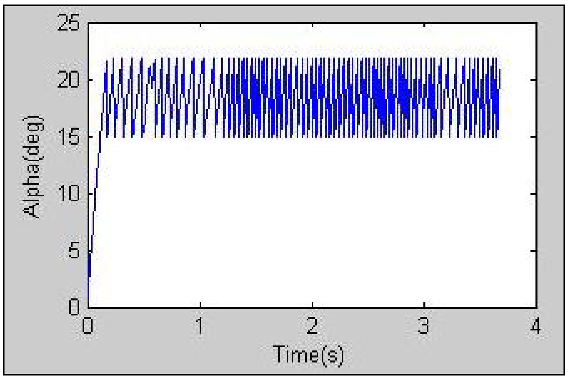

Fig. 17. Angle of Attack vs. Time

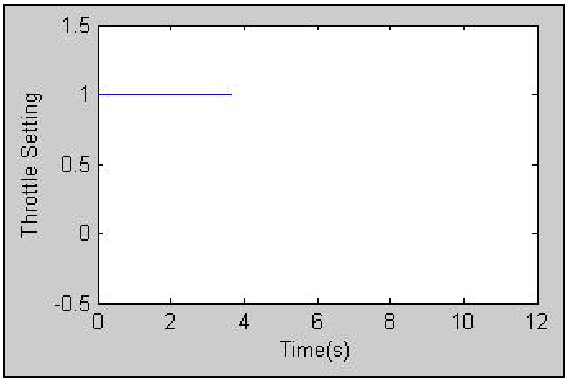

Fig. 18. Throttle Setting vs . Time

Figure 9 shows that the vehicle can fly over the terrain with a supersonic speed and this speed is almost constant up to the end of the range. If the terrain were not limited, this speed would not be constant.

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