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OPTIMUM FLIGHT TRAJECTORIES AND SENSITIVITY ANALYSIS FOR TERRAIN COLLISION AVOIDANCE SYSTEMS

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Abstract

Military aircraft are often flown in close proximity to terrain, which can result in controlled collision into terrain. In this paper, various methodologies are presented for computing optimal trajectories within a digital terrain 3D map to avoid collision. Two main issues are addressed in this paper. Firstly, various optimal flight trajectories are computed subject to different objectives, eg terrain following, minimum time, etc. while avoiding collision. These simulations show that it is possible to generate flyable trajectories within terrain to avoid collision. In practice, flight instruments are not fully accurate and the question is how sensitive is the optimum flight trajectory to input errors. This is essential to understand as terrain clearances can be small. To assess this, the optimal flight trajectories are flown again with a small change in the initial conditions. Optimal terrain following is considered as a minimax optimal control problem, which is solved using direct Within a very general framework for solving such problems, we are able to transform the non-smooth cost function into a constrained nonlinear programming problem. In the formulation, we are also able to solve optimal terrain avoidance manoeuvres. To ensure smooth derivatives of the terrain, we approximate it using B-Splines. The implementation and numerical case studies for different constraints, and initial aircraft position/velocity and the errors in trying to follow the optimal trajectory are discussed.

Nomenclature

$\dot{x}$	State vector in x direction
$\dot{y}$	State vector in y direction
$\dot{z}$	State vector in altitude
V	Velocity
γ	Climb angle
ϕ	Bank angle
$\dot{W}_x$	Wind factor in x direction
$\dot{W}_y$	Wind factor in y direction
$\dot{W}_z$	Wind factor = z direction
ρ	Air density
S	Span
C_L	Lift coefficient
C_D	Drag coefficient
T	Thrust
ε	Thrust angle with respect to fixed body
α	Angle of attack
D	Aerodynamic Drag
L	Aerodynamic Lift

$n_{z(turn)}$	Load factor for turn
$n_{z(Climb/descent)}$	Load factor for a climb
m	Weight
g	Gravity
ψ	Heading angle
τ	Non-dimensional time
η	Thrust settings
V_s	Speed of sound
v	Non dimensional speed
$\bar{x}$	Non dimensional x distance

Introduction

Military aircrafts are often required to fly close to terrain to avoid radar detection however; this is not easily achieved due to require quick reflexes and control required from the pilot and the need for the aircraft to function close to its operational constraints [1]. It has been noticed that, pilots adopt a similar response when confronted by mountainous terrain. That is, they pitch up to in order to clear the oncoming terrain. Unfortunately, there have been several accidents where pilots detected danger too late and were unable to pitch up in time and crashed into the oncoming terrain. The reasons are mainly due to the structural, propulsive and aerodynamic limitations of the aircraft. Not all pilots have the sheer confidence to fly as close to the terrain as possible. Introduction of navigational aids such as GPS Global Positioning Systems, GPWS Ground Proximity Warning Systems [2] and EGPWS Enhanced Ground Proximity Warning Systems [3] which act as advisory systems have reduced Controlled Flight into Terrain CFIT [4] accidents but an alternative collision avoidance system is required. The alternative solution will require the pilot to fly around the terrain rather than pulling up on the control stick to climb over the terrain. Of course, the pull up manoeuvre is best suited when the terrain is low or if the terrain is not lengthy but if it is otherwise, performing a lateral manoeuvre to flying around or close to the terrain may just give the pilot an increased chance of survival.

Performance Model

The equations of motion below are based on the assumption of a point mass aircraft [5]. The full point mass model equations of motion are described in detail in [6]. Sideslip and unsteady aerodynamic effects were neglected at this stage. The Earth is assumed to be non-rotating.

$$\dot{x} = V\cos\gamma\cos\phi + Wx \qquad (1)$$

$$\dot{y} = V\cos\gamma\sin\phi + Wy \qquad (2)$$

$$\dot{z} = V\sin\gamma + Wz \qquad (3)$$

$$\dot{V} = \frac{T\cos(\alpha+\varepsilon)-D}{m} - g\sin\gamma - \dot{W}_x\cos\gamma\cos x \qquad (4)$$

$$-\dot{W}_y\cos\gamma\sin x - \dot{W}_z\sin\gamma (4)$$

$$\dot{\gamma} = \frac{\left[L+T\sin(\alpha+\varepsilon)\right]\cos\phi}{mV} + \frac{g}{V}\cos\gamma + \qquad (5)$$

$$\frac{\dot{W}_x\sin\gamma\cos\chi}{V} + \frac{\dot{W}_y\sin\gamma\cos\chi}{V}$$

$$-\frac{\dot{W}_z\cos\gamma}{V}(5)$$

$$\dot{\psi} = \frac{\left[L+T\sin(\alpha+\varepsilon)\right]\sin\phi}{mV\cos\gamma} + \frac{\dot{W}_x\sin\chi}{V\cos\gamma} \qquad (6)$$

$$-\frac{\dot{W}_y\cos\chi}{V\cos\gamma}(6)$$

To reduce the time taken for simulations, non-dimensional equations were introduced [6]. This was important because all the states possessed non uniform values. Therefore via introducing the non-dimensional equations, all the states are transformed to uniform values. The lift and drag coefficients for $\overline{V} < 1.15$ are given by [6]. For the value of thrust, thrust data has been used from a supersonic aircraft F4, more information is shown in [7]. Constraints are put on the angle of attack, bank angle as well as rate of change of angle of attack and roll rate to account for physical limitation of the control devices and aircraft performance:

$$-8 \le \dot{\alpha} \le 8 \qquad (7)$$

$$-15 \le \dot{\phi} \le 15 \qquad (8)$$

$$-20 \le \alpha \le 20 \qquad (9)$$

$$-70 \le \phi \le 70 \qquad (10)$$

The final constraint which was implemented was the load factor. This was implemented to ensure that the aircraft would operate within its structural limitations. It was important to consider and implement this factor so that the simulations would represent a realistic scenario.

$$-4 \le n_z\ \text{climb} \le 9 \qquad (11)$$

$$0 \le n_{z(turn)} \le 7.5 \qquad (12)$$

Software used

The softwares used for the optimisation process are ***Snopt*** [8] and ***Direct*** [9]. ***Terrain Generator*** [10] is used to create the terrain model and ***Matlab*** [11] was used as the working environment for simulations. For more information on the use of these softwares please refer to the referencing section.

As required by ***Direct***, for an optimised collision avoidance problem, a path constraint is a requisite. Therefore the addition of obstacles is done in the path constraint file. It is assumed that terrain that is being avoided is available from a known terrain database. The principal idea is to keep the flight trajectory free of obstacles whilst keeping the aircraft as close to the terrain as possible. To take into account obstacles such as buildings and other low lying obstacles, a clearance height h_c is utilised so that the aircraft maintains a safe tolerance above the terrain. More information is given in [10].

In the next part of this paper, the methodology and the equations required in relation to the sensitivity studies are discussed. It is assumed that the aircraft is flying an optimised trajectory. The scenario is set as; the aircraft is flying at the optimised trajectory. Generally the indicated speed, altitude or lateral position will guide the pilot, however if he chooses to do otherwise then this could lead to a problematic situation. The level of confidence and trust the pilots have in flight instruments plays a critical role. Generally pilots do not tend to adhere to the guidance provided by flight instruments which can lead to fatal accidents. In attempt to combat this situation, certain tolerances in the states errors have been allowed. Eqns 13 to 14 shows the tolerance parameters utilised for this purpose.

$$V_{\min} \le V_{\max} \qquad (13)$$

$$Altitude_{\min} \le Altitude_{\max} \qquad (14)$$

$$\text{Y Distance}_{\min} \le \text{Y Distance}_{\max} \qquad (15)$$

Cost function

In this study, minimum time, where the aircraft has to clear the terrain in the shortest possible time possible and minimum clearance to the terrain (terrain following) was used for the first part of the simulations [6]. For the second part of this paper, a sensitivity analysis was done on the lateral position, velocity and altitude. The cost function for both scenarios are shown below.

$$J_1 = t_f \tag{16}$$

$$J_2 = \int_{t_0}^{t_f} h(t)dt = \|h(t)\|_{l_1} \tag{17}$$

The Mayer and Bolza components of the cost function, shown in [6], were used in the performance index.

Terrain Modeling

A three dimensional complex terrain model for investigation was developed. The terrain utilised for the optimization problem was modelled utilising a matrix of elevation data provided by the terrain generation program. Terrain models were constructed which were representative of complex terrains. The terrain profiles were created via utilising third party software ***Terrain Generator*** [10] as shown in figure 1. After the terrain profiles have been created, the files are exported into Matlab in the text file format. Utilising B-splines [12], the terrain data is provided as a set of x and y coordinates, and a matrix of z coordinates representing the elevation as shown in figure 2. To obtain the solution, interpolated values of the elevation data are required. In addition, gradients of the constraints are calculated via utilisation of finite differences. This implies that the smooth derivatives of the terrain data are required for the solution algorithm to be effective. It is possible to provide C continuity by approximating the data with a tensor product cubic B spline of the form shown in Equation 16 and the example is shown in figure 3. It is critical to provide a relatively accurate guess of the controls when utilising B splines. Hence, a good guess would produce results which are relatively accurate.

$$h_T(x,y) = \sum_{i=1}^{n_1} \sum_{j=1}^{n} c_{i,j} B_i(x) B_j(y) \tag{18}$$

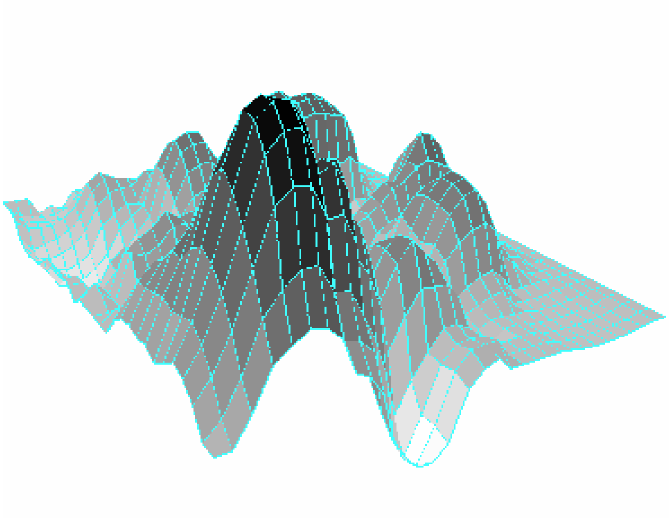

Figure 1: Three dimensional plots from *Terrain generator*

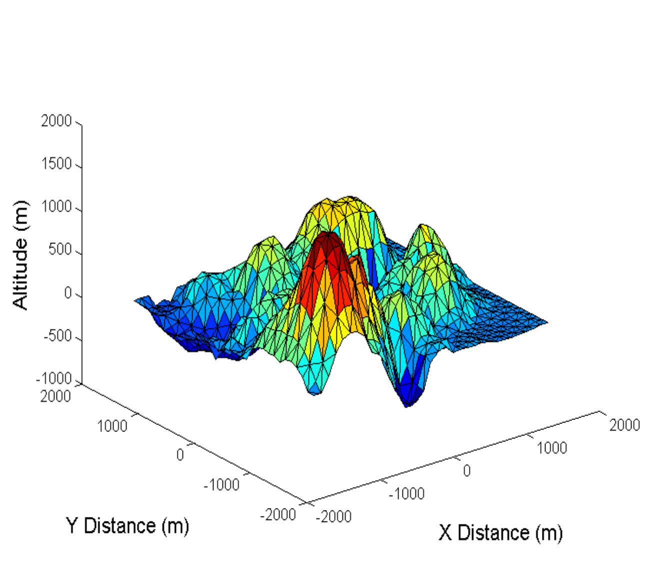

Figure 2: Three dimensional terrain profile without using B splines

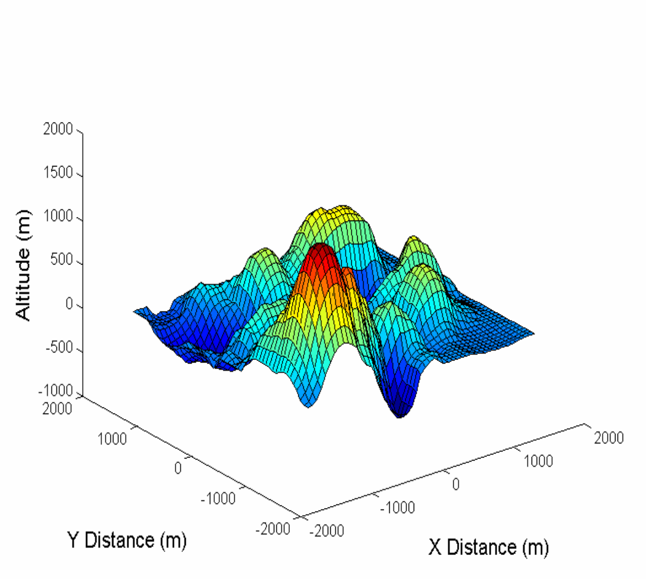

Figure 3: Three dimensional plot using B splines

Results

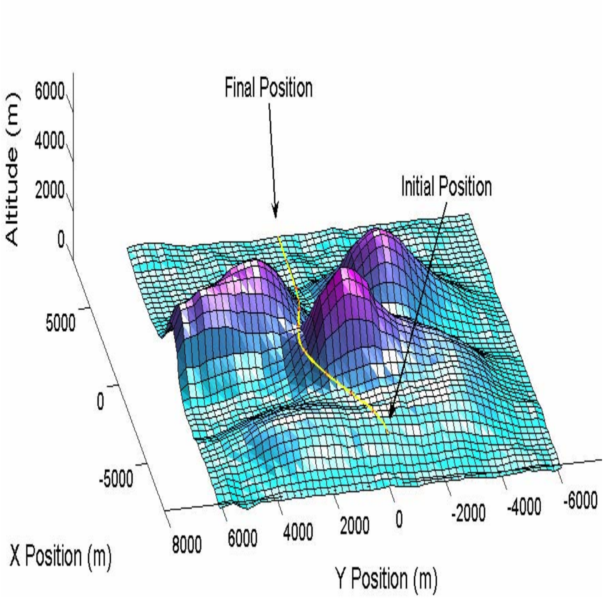

Figure 4: Minimum Time Scenario

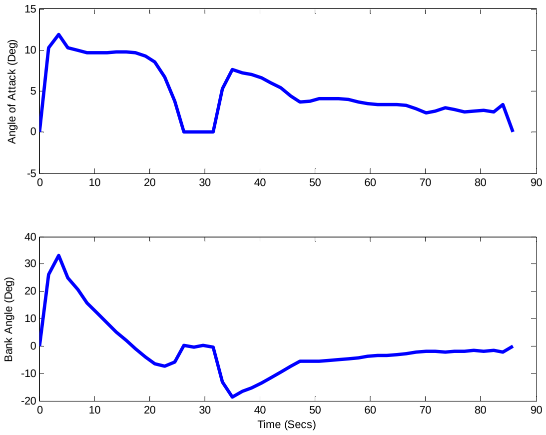

Figure 5: Controls for minimum time scenario

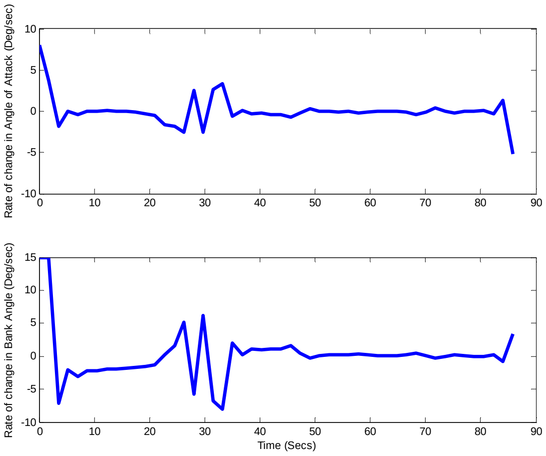

Figure 6: Rate of changes for minimum time scenario

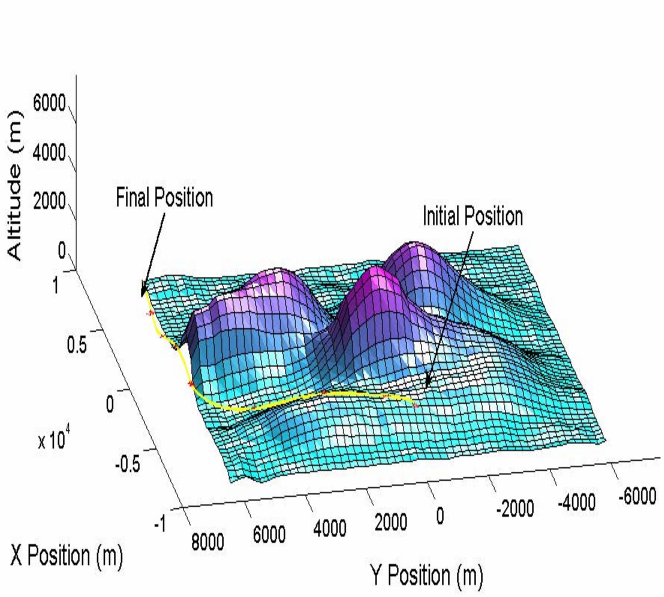

Figure 7: Minimum clearance scenario

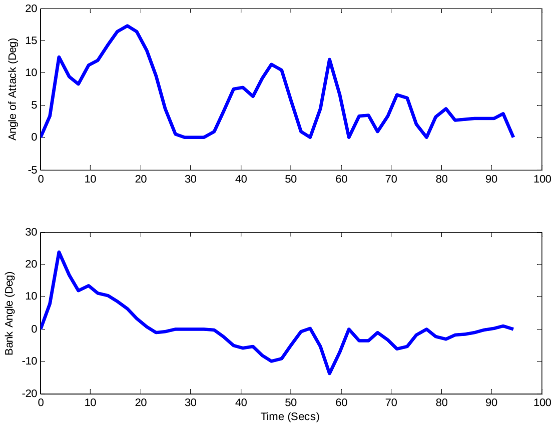

Figure 8: Controls for minimum time scenario

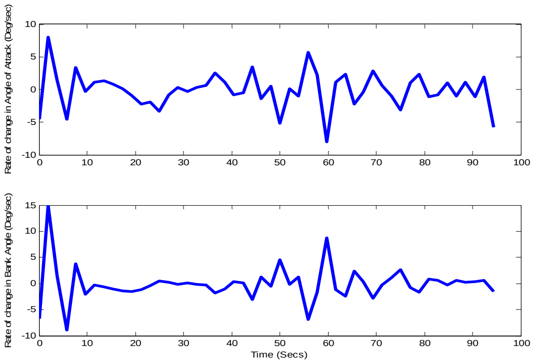

Figure 9: Controls for minimum clearance scenario

For the terrain, the aircraft demonstrated a unique feature whilst generating an escape trajectory. The aircraft flew between the peaks of the terrain model via performing a well coordinated lateral manoeuvre as shown in Figure 4. The time taken for this trajectory was 80 seconds. The control plots as detailed in the appendix exhibits little variation for angle of attack, bank angle and rates of change in comparison to the minimum clearance scenario. Figure 7 shows that the aircraft flew around the terrain profile whilst achieving a clearance eof 1.65 metres above the safety clearance.

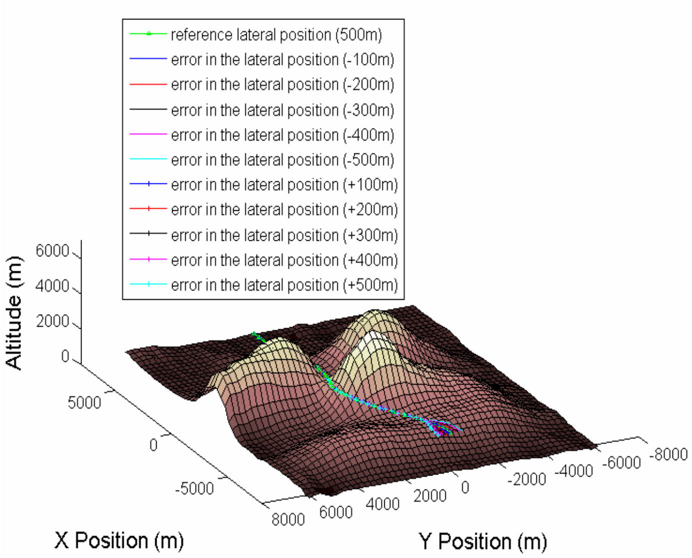

Figure 10: Error in lateral position for minimum time scenario

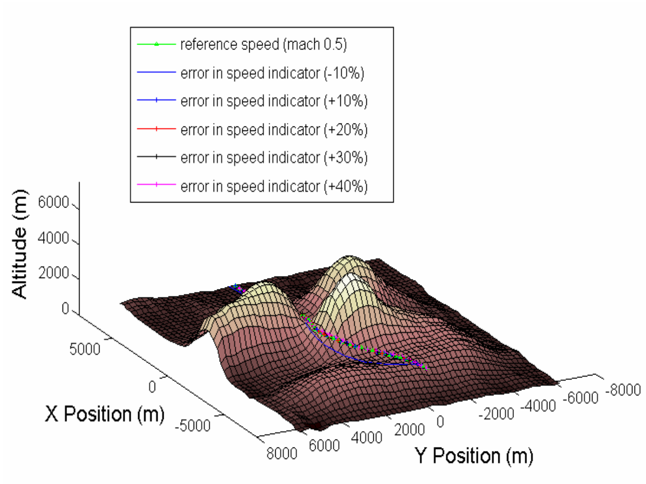

Figure 11: Error in speed indicator for minimum time scenario

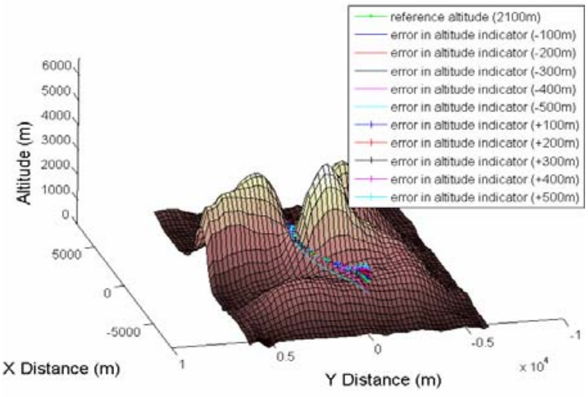

Figure 12: Error in altitude indicator for minimum time scenario

Figure 10 exhibits the plot for minimum time scenario for terrain. It is evident that when lateral position errors of ± 500 metres were introduced, the aircraft did not deviate from the optimised trajectory. Figure 11 depicts the plot for errors introduced in the speed indicator. The results show that when the aircraft was flown at a speed which was less than 10% of the reference speed, it deviated significantly from the optimised trajectory. Figure 12 shows that for errors in the altitude of -400 metres to – 500 metres with respect to the reference, the aircraft deviated significantly from the optimised trajectory.

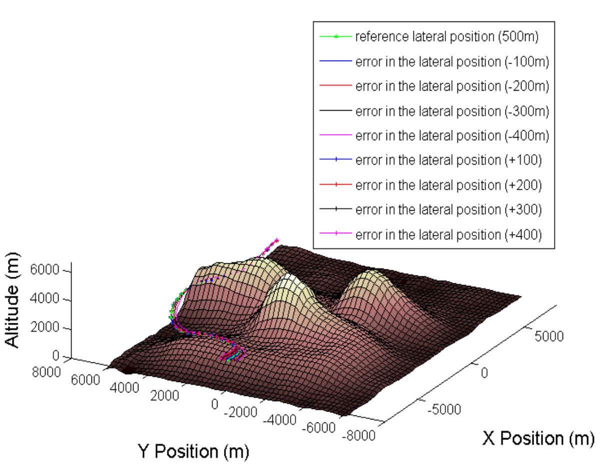

Figure 13: Error in lateral position for minimum clearance scenario

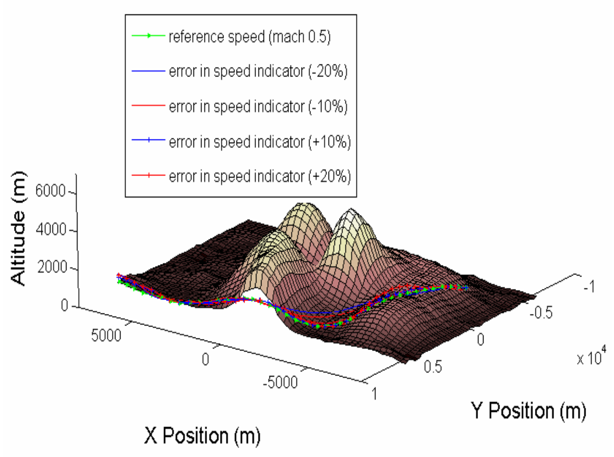

Figure 14: Error in speed indicator for minimum clearance scenario

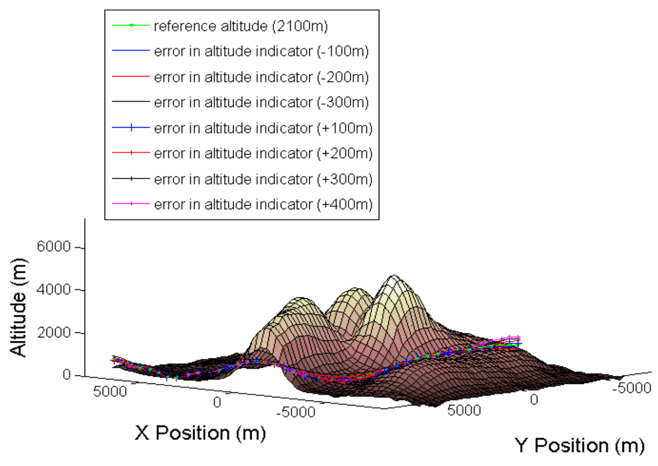

Figure 15: Error in altitude indicator for minimum clearance scenario

Figure 13 shows plot with lateral position errors for terrain. The plot shows that when the aircraft was flown at -400 metres from the reference lateral position, it deviated from the optimal trajectory. Figure 14 illustrates the plot with errors in the speed indicator. It was found that the aircraft deviated from the optimal trajectory when flown at speeds -10% and -20% of the reference speed. Figure 15 shows that when the aircraft was flown at an altitude of -200 metres and -300 metres with respect to the reference, it deviated significantly from the optimised trajectory.

Conclusion

The objective of this paper was to show that generation of trajectories around a three dimensional obstacle is possible using numerical method. In this paper more focus was shown on that the method is possible and the controls. With this method, any terrain representation is possible as long as the x, y coordinates are available. It was critical to investigate the effect of errors in the lateral position, speed and altitude indicators of the aircraft. As discussed, the variations in speed resulted in the different paths produced. The results obtained for this thesis demonstrate that collision avoidance is possible via performing a lateral manoeuvre. However the results are preliminary indicating that further work needs to be conducted. The limitations set on the rate of change in angle of attack, bank angle and thrust settings have not been adhered to in accordance with the civil regulations. Therefore, the results obtained may not apply for a real life scenario. Additionally, it is essential that the equations of motion for a rigid body which includes the rolling moments and side slips are implemented. The addition of flight controls such as elevators, ailerons and stick force would ensure that the simulations are representative to a real life case. It is essential to consider the real time scenario as it would ensure that the aircraft is able to detect the terrain beforehand rather than just possessing a known database

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