OPTIMUM PITCH ANGLE IN ENGINE INSTALLATION OF A CIVIL JET AIRCRAFT

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Abstract

This paper mainly presents theoretical and computational approaches to the effects of engine angles in the pitch plane on the design optimization of a high-subsonic jet transport aircraft with twin wing-mounted turbofan engines. An iterative computer program ICP) was written for the preliminary design of a high-subsonic jet transport aircraft. A theory was developed for the preliminary design based on available theoretical methods and derivative-based techniques. The effects of changes to the jet engine angle in the pitch plane have been formulated and the results are compared with the ICP outputs which have been already validated using the data of few civil jet aircraft. In general, good agreement is found for small changes of the pitch engine angle to the particular aircraft specification. The results cover the emergency cases in which one engine is inoperative. The results indicate that a suitable jet engine angle in trimmed flight may reduce the total drag, the fuel used, the thrust required and/or maximize the range and endurance. The optimal engine angles for the minimum thrust required, the best range and the minimum fuel consumption in level flight and during take-off and climb, are presented.

1 Introduction

The conceptual design of a civil aircraft and the procedure for flying an aircraft from one airport to another, are complex. In this research project, the important parameter affecting the design of a civil jet aircraft is the engine angle.

One of the aims of this research paper was initially to satisfy the design requirements and specifications of a high-subsonic civil jet transport aircraft.

In order to achieve these goals, the convergent ICP was written in FORTRAN for the UNIX computer operating system. It is for the preliminary design of a high-subsonic jet transport aircraft and airworthiness regulations were adhered to throughout the design. By using a series of subroutines, the program is structured so that the effects of any change in the jet engine angle in pitch plane can be readily evaluated and further improvements and expansions can easily be made. The ICP uses a step-by-step method to integrate along the flight path and the output is presented after several iterations when there are no further changes.

The results of the theoretical method are compared with the ICP outputs which have been already validated using the data of B737, B747 and B757. Based on this method, an example of the design optimization of a passenger jet aircraft due to installing the engines at a right pitch angle is given. In general, good agreement is found for small changes of the pitch engine angle to the particular aircraft specification.

During the ICP development, it was realized that there is a significant difference between the optimum angles for the jet engine during take-off, climb and cruise. This suggests that the engine angle and thrust deflection should be adjusted to obtain better efficiency and meet both take-off and cruise demands. It was found that there are few published papers on the effects of engine angles on wing-mounted jet transport aircraft and that they present general trends instead of giving implicit equations and specific details. Therefore, this field of aircraft design optimization and theory needs to be considered in more depth.

By taking advantage of adjusting the engine angle, it would be possible in general to optimize the flight of a jet transport aircraft from the beginning of take-off up to the end of cruise phase. Estimates have been obtained for the resulting modification and optimization to the thrust required, flight range, endurance, maximum lift to drag ratio, required take-off runway length, total drag and induced drag.

The paper considers the design of a high-subsonic civil aircraft. A theoretical approach has been developed to examine how changes to the jet engine angle in a vertical plane would affect the aircraft's aerodynamics and performance. The conventional trim equations are extended to show how the cruising trim is affected by changes in the jet engine.

The reference aircraft has a constant engine angle of δ_e (or δ_{en}) to give better cruise performance at a particular design point. This angle is a small angle between the longitudinal engine axis and the fuselage longitudinal axis. It is defined as positive when the engine intake is upward. The reference line of the aircraft is defined as the fuselage longitudinal axis.

Throughout this paper, subscripts $_{fus}$, $_{n}$, $_{e}$ (or $_{en}$), $_{e\ell}$, $_{w}$ and $_{h}$ refer to the fuselage, nacelles, engines, elevator, wing and horizontal tail, respectively. Subscript $_{b}$ indicates the word 'body' which refers to the combination of wing, fuselage and nacelles of the aircraft. The words 'tail' and 'fin' refer to the horizontal and vertical parts of the tail, respectively. α is the aircraft angle of attack which is defined as the angle between the reference line and the freestream velocity. i is the average incidence of a lifting surface in the pitch plane with respect to the reference line. Consequently, the wing angle of attack is $\alpha + i_w$ and the tail angle of attack is $\alpha + i_h$. All angles are in radians.

It is assumed that C_T, C_L and C_D are the total thrust, lift and drag coefficients of the reference aircraft in steady cruising flight. The body and tail lift coefficients are positive in the upward direction.

Only the preliminary design stage of a jet transport aircraft is considered and no detailed study of the dynamic stability or the structure or the economics of the aircraft is given.

2 The Theory

How does jet engine angle directly affect the main aircraft design parameters? The full equations needed to analyse the jet aircraft are complex. An aircraft trim investigation is needed to evaluate how extra forces and moments due to jet engine angle can keep the aircraft in equilibrium more efficiently. The full equations are presented and then some approximations will be made to simplify the theory. The effects of jet engine angles on the lateral and directional trim equations are not considered here. It is also assumed that the aircraft is flying with no yaw in still air and the effects of any changes in wind component are neglected (see [1], [2], [3] for a discussion of wind effects).

The lift on the aircraft results in a rate of change of momentum of the free stream. The angle of the free stream to the horizontal is the small angle e. Ahead of the wing, the air flow is upwards with e negative and behind the trailing-edge of the wing, the air flow moves downwards and the downwash effects may decrease the AoA of the tail [4]. This upwash or downwash is considered constant across the span without any change in the spanwise direction. For more simplicity, it will be assumed that the engine thrust without vectoring is in line with the engine centre-line and with vectoring, is in line with the nozzle centre-line. If the airstream ahead of the engine is inclined to the nozzle centreline at angle $(\alpha + \delta_e + e)$ for the reference aircraft then a further force F_N normal to the engine centreline could be produced of order $\dot{m}U(\alpha + \delta_e + e)$ where $\dot{m} = \rho U A_e$ is the mass flow rate into the engine with the area A_e normal to the engine centre line where the intake velocity is approximately U in this research [5].

To be able to use the computational and experimental results in the theoretical approach, the trim equations are simplified by making some assumptions such as ignoring upwash effects at the engine inlets and neglecting small changes of some terms due to jet deflections.

The equation for vertical forces on the aircraft consists of the body lift, the tailplane lift, the lift due to the elevator, the vertical force from the engine thrust and the aircraft weight. It is possible to include the effects of elevator deflection in the tail lift for a specific trim condition at cruise [5]. The dimensionless equation for vertical forces in steady horizontal flight is

$$C_{Lb,v} + \eta_h \frac{S_h}{S} C_{Lh,v} + C_{T,v} \sin(\alpha_v + \delta_e + \delta_v) + C_{TN,v} \cos(\alpha_v + \delta_e + \delta_v) - \overline{W}_v = 0 \quad (1)$$

where C_{Lh} includes any elevator deflection, C_{TN} is the normal force coefficient of the jet engines due to turning the airstream particularly at the inlet front faces [5]. η_h is called the tail efficiency factor or the dynamic pressure ratio. $q = \frac{1}{2}\rho U^2$ is the dynamic pressure of the freestream at cruise conditions.

The jet engines have several effects on the lift including the direct lift of the engine thrust, the lift component of the engine normal force, the lift due to the nacelle and the influence of the jet-induced flows upon the tail, wing and aft fuselage. Of course, depending on the altitude of the aircraft some of these can be negative. The engine normal force is determined from momentum considerations of the turning of the airstream and approximately equals the mass flow into the inlet times the change in vertical velocity. By ignoring upwash effects at the engine inlets, the engine normal force coefficients of the reference aircraft may be estimated from

$$C_{TN} \approx \frac{2A_e}{S}(\alpha + \delta_e) \quad (2)$$

where C_{TN} is likely to be about 3% of the total thrust. C_{TN} in this approximate form simplifies the theory.

The parameter η_h represents the flow interference about the tail due to the wake and trailing vortices produced by the wing ahead of the tail. It is equal to the ratio of the dynamic pressure at the tail to that in the freestream or

$$\eta_h = q_h/q = \left(U_h/U\right)^2 \quad (3)$$

A T-tail and a cruciform tail which are normally clear of most wake effects, are expected to have higher η_h values than a conventional tail. Several methods are available to determine η_h, but they do not include interference from the engine exhausts. It can be shown that from Stinton's method [6]

$$\eta_h = 1 - 3.6\sqrt{C_{Dowing}/\hat{x}_h} \quad (4)$$

For comparison, Roskam's method [7] (Part-6) produces $\eta_h = 0.96$ for a Boeing 757-200 and Stinton's method produces the value of 0.89 which is closer to the typical value of 0.9 presented by [5].

Trim drag is caused by the tail lift as a result of the requirement to trim the aircraft. By taking advantages of the extra forces and moments that thrust-vectoring provides, it is possible to optimise the longitudinal characteristics of the aircraft so that the effective trim drag is less than the trim drag without thrust deflection. The dimensionless equation for horizontal forces on the aircraft in steady horizontal flight is

$$C_{Db,v} + \eta_h \frac{S_h}{S} C_{Dh,v} + C_{Dm} - C_{T,v} \cos(\alpha_v + \delta_e + \delta_v) + C_{TN,v} \sin(\alpha_v + \delta_e + \delta_v) = 0 \quad (5)$$

where the miscellaneous drag coefficient C_{Dm} is due to the remaining drag contributors including fin, fuselage base, fuselage upsweep, non-vectored interference, leakage and protuberances. It can be estimated from the methods of [5], [7] (Part-6) and [8].

3 Minimum Thrust Required

In this section, the angle of the engine thrust line which gives the most efficient flight will be considered. Since the main part of the flight is at cruise level, the emphasis is on the minimum thrust required at cruise conditions and depends on which parameter of H_C, M_C or C_{LC} is kept constant. The effects of U, M, H and C_L on the minimum thrust required have been summarised by [2], [5], [9] and [10]. Only the effect of engine angles on the minimum thrust required during level flight is considered here.

The vertical and horizontal force equations for the reference aircraft in steady cruising flight, neglecting the small C_{TN} terms, simplify to

$$C_L = \overline{W} - C_T sin(\alpha + \delta_e) \qquad C_D = C_T cos(\alpha + \delta_e) \tag{6}$$

Using $C_D = C_{Do} + KC_L^2 + C_{Dcomp}$ in eq. (6), C_L is eliminated to give

$$C_T = \frac{1}{sin(\alpha + \delta_e)} \left[\overline{W} + \frac{cotan(\alpha+\delta_e)}{2K} - \sqrt{\left(\overline{W} + \frac{cotan(\alpha+\delta_e)}{2K}\right)^2 - \overline{W}^2 - \frac{C_{Do}+C_{Dcomp}}{K}} \right] \tag{7}$$

For small $(\alpha + \delta_e)$, eq. (7) reduces to

$$C_T \approx \left(C_{Do} + C_{Dcomp} + K\overline{W}^2\right) \left[1 - 2K\overline{W}(\alpha + \delta_e) + \zeta(\alpha + \delta_e)^2\right] \tag{8}$$

where $\zeta = \frac{1}{2} + K(C_{Do} + C_{Dcomp}) - 3(K\overline{W})^2$.

To find the engine angle at which thrust is a minimum, the derivative with respect to engine angle from eq. (8) is set to zero. Theoretically, the minimum thrust required occurs when the condition $\frac{\partial C_T}{\partial \delta_e} = 0$ which for small $(\alpha + \delta_e)$, gives

$$(\delta_e)_{min\ thrust} \approx \frac{K\overline{W}}{\zeta} - \alpha \tag{9}$$

Substituting this angle into eq. (8) yields

$$(C_T)_{min} \approx \left(C_{Do} + C_{Dcomp} + K\overline{W}^2\right) \left(1 - \frac{(K\overline{W})^2}{\zeta}\right) \tag{10}$$

There can be a small extra drag due to changes of engine angle. It mainly depends on δ_e and the engine geometry and position. It might alter the C_D slightly but this effect is ignored here [7] (Part-6 & Part-2) and [8].

Obviously, instead of turning the whole engine to an optimum angle, it is easier to deflect the exit nozzles. By using non-axisymmetric exhausts, the jet direction can be altered effectively while the nacelles are fixed. This analysis will be extended to take-off and climb calculations later and the data are given in the output of the ICP (see the appendix). Similarly, Mair & Birdsall [2] also considered the effects of varying thrust direction on the thrust required during level flight and they got more or less similar results.

$$C_T = \frac{1}{sin(\alpha + \delta_e)} \left[\overline{W} + \frac{cotan(\alpha+\delta_e)}{2K} - \sqrt{\left(\overline{W} + \frac{cotan(\alpha+\delta_e)}{2K}\right)^2 - \overline{W}^2 - \frac{C_{Do}+C_{Dcomp}}{K}} \right] \tag{11}$$

4 Range Optimization

The longest part of the flight is at cruise level and therefore, again the emphasis is on cruising range. The distance or the length of time that an aircraft can fly on a given quantity of fuel is most important. An adjustment in the engine angle which leads to an increase in range is desirable. As fuel is used the aircraft weight reduces and a climb-cruise or a speed reduction is needed in order to keep $UL/(SFC\,D)$ at a maximum.

The conditions required during the cruise depend on H_C, M_C & C_{LC}. Several options are available and for each of them, the optimum δ_e may be different. The U, M, C_L & C_D for optimum range, endurance, thrust, drag, fuel used, L/D, ML/D or $UL/(SFC\,D)$ have been frequently evaluated in the past by, for example, [2], [5], [6], [8], [9], [10], [11] and [12].

The integral $\int_{W_1}^{W_2} \frac{U\,d\overline{W}}{SFC\,C_T}$ in the Breguét equation should be maximised for each step of the range. The value of this integral is a measure of the cruise performance and is named here as the 'range factor'. Some references such as [5] named the term $UL/(SFC.D)$ as the 'range parameter' which does not involve the weight change. However, this is different from the range factor used in this section. The ratio U/SFC may be taken out of the integral without a significant error as discussed in [13]. Using eq. (6) in the integral for a very small segment of range

$$\int \frac{d\overline{W}}{C_T} = \int \frac{dC_L + \sin(\alpha+\delta_e)dC_T + C_T\cos(\alpha+\delta_e)d\delta_e}{C_T}$$

$$= \int \frac{\cos(\alpha+\delta_e)}{C_D} dC_L + \int \frac{C_T\cos^2(\alpha+\delta_e)}{C_D} d\delta_e + \int \frac{\sin 2(\alpha+\delta_e)}{2C_D} dC_T \qquad (12)$$

Then, using eq. (10)

$$\int \frac{d\overline{W}}{C_T} \approx \frac{-C_T}{2C_D}(\alpha+\delta_e)^3 - \frac{\zeta}{2}(\alpha+\delta_e)^2 + \frac{3C_T}{2C_D}(\alpha+\delta_e) + \zeta + costant \qquad (13)$$

where from eq. (6),

$$C_T = \sqrt{C_D^{\,2} + (\overline{W} - C_L)^2} \qquad (14)$$

and $\zeta = \left[K\left(C_{Do} + C_{Dcomp}\right)\right]^{-1/2} Arctan\sqrt{\frac{KC_L^{\,2}}{C_{Do} + C_{Dcomp}}}$

with where K is the total induced-drag factor of the aircraft. There is a small extra drag due to nozzle deflection which mainly depends on δ_e and the engine geometry and position. It might alter the C_D slightly but this effect is ignored here. It can be shown that the engine angle for best range can be determined from

$$(\delta_e)_{best\ \ range} \approx \sqrt{\left(\frac{\zeta C_D}{3C_T}\right)^2 + 1} - \frac{\zeta C_D}{3C_T} - \alpha \qquad (15)$$

From eq. (15), the optimum δ_e for the best range is 3.59^o. For the minimum thrust required, this optimum angle is found from eq. (9) as 3.42^o which is about 6% lower. This comparison will be extended to different flight conditions at cruise.

In the case when the aircraft flies at constant M_C and H_C, the change of thrust coefficient in eq. (12) is cancelled ($dC_T = 0$) and it can be shown that

$$(\delta_e)_{best\ range} \approx \mathrm{Arc}\tan(C_D/C_L) - \alpha \qquad (16)$$

This produces answers within 1% of eq. (15).

A conventional civil jet aircraft will have its engines fixed at a value which is optimum for a design point in the cruise. This point is expected to be at a position somewhere up to the first 30% of the range. The design point of the aircraft which satisfies the best range requirement (or maximum ML/D), is located at a distance of 160km near to the beginning of cruise. The gain from the jet engine angle must be considered either side of this position and this is briefly discussed in [13].

It is important to firstly find the optimum engine angle at the design point of the aircraft. By changing the design altitude and Mach number at cruise, the magnitude of this angle will be varied. For example, the design engine angle should be reduced as much as 33% if the Mach number increases from 0.78 to 0.84. Also, it must have a 16% reduction when the flight altitude rises from 8km to 14km. The optimum engine angles which satisfy the minimum thrust requirement during cruise will be introduced later.

Using eq. (15), the engine angles at the design point of the aircraft for the range optimization, are given in Table 1 at different H_C and M_C . The results are from the iterative computer program and the corresponding range factors are given in the brackets. The optimum engine angles which satisfy the best range or minimum thrust requirements at different positions in the flight path, are also presented separately. Table 1 shows that when the aircraft flies at lower altitude or higher speed, the design engine angle should be increased to provide the best range condition. Note that the range factor reduces while the optimum engine angle increases and vice versa.

5 Take-off Analysis

Conventional aircraft are usually designed to have certain take-off and landing runway lengths. The primary objective of this section is to discuss the engine angle for better take-off performance. It is assumed that all the installed thrust is used to provide maximum acceleration. This section only investigates the ideal case when the take-off proceeds well. Emergency cases have been considered for the reference aircraft after an engine failure and the results are presented in [13],[16],[17],[18],[19].

The take-off path is defined here as the combination of three segments; ground run, rotation and transition. On the ground, the aircraft accelerates up to the lift-off speed U_{TO} without C_L and C_D changing significantly. During the rotation, the aircraft is pitched nose up and starts to rotate so that at the lift-off point, the lift will be just more than the weight and the aircraft leaves the ground. The aircraft centre of gravity follows almost a circular path during the rotation phase. Then, the path angle remains constant during transition to climb and the aircraft increases speed and clears the obstacle height ($h_{TO} \geq 10.7$m). If all engines are operating, it could clear the obstacle with plenty to spare.

The ground run, the rotation roll on the ground, the rotation after lift-off and the transition to climb are analysed separately in the iterative computer program. The results of the take-off analysis are given in the appendix for the cases when all engines are operating and when one engine is inoperative. However, the ground run and the rotation roll are considered as one phase here and also the rotation after lift-off and the transition to climb are investigated together. The proximity of the ground affects the C_L, C_D and α at take-off [2], [8], [10], [14], [15]. These effects have been taken into account in the iterative computer program.

Although the engine angle effects on landing performance have not been analysed in the paper, they are as important as for take-off and should be considered in the future. The results for the landing analysis of the reference aircraft are presented in [13] and [17]. In the following two sub-sections, the equations of motion for the ground run and the transition phases will be analysed in order to obtain the optimum engine angles. In practice, a step-by-step analysis should be applied to determine the optimum engine angle as a function of other take-off parameters.

5.1 Ground Run

On the ground, the aircraft AoA depends on the undercarriage geometry but it is sufficient to assume an average constant AoA. The runway may not be exactly horizontal and in general, the runway can have a small angle θ to the horizontal. For an uphill runway θ is positive. The equation of motion for an aircraft accelerating on a runway is

$$a = \frac{UdU}{dx} = \frac{g}{\overline{W}} \begin{Bmatrix} C_T\,cos(\alpha+\delta_e) - C_D \\ -C_{D\,extra} - \overline{W}\,sin\theta - \\ \mu\left[\overline{W}\,cos\theta - C_L - C_{LGE} - C_T\,sin(\alpha+\delta_e)\right] \end{Bmatrix} \quad (17)$$

where μ is the rolling friction coefficient and to ease the integration is taken as constant during the ground run. Values of μ given by [5] and [11] can be selected in the computer program for various runway surfaces. $C_{D\,extra}$ is an extra drag coefficient for the aircraft which is produced by changing the engine angle or deflecting the nozzles and elevator.

The nozzle contribution on $C_{D\,extra}$ depends on the jet characteristics, exhaust nozzle deflection and nozzle geometry, and sometimes is called the 'momentum drag coefficient' [15]. The elevator contribution on $C_{D\,extra}$ depends on the geometry and aerodynamic characteristics of the horizontal tail and the change of elevator angle.

This contribution may be estimated from Roskam's method [7] (Part-2 & Part-6). The term C_{LGE} in eq. (17) is the change in lift coefficient due to ground effects and well presented in [6] and [10].

The ground-run distance may be calculated from $s_G = \int_0^{U_{TO}} \frac{d(U^2)}{2a}$. The take-off velocity U_{TO} must not be less than $1.1U_S$ where U_S is the stalling speed. Using eq. (17), the integral may be solved to give [5] and [9]

$$s_G = \frac{1}{2g\Psi} ln\left(1 - \frac{\Psi U_{TO}^2}{sin\,\theta + \mu\, cos\,\theta}\right) \qquad (18)$$

where

$$\Psi = \frac{\rho S}{2W}\left\{ \begin{array}{l} C_T cos(\alpha+\delta_e) - C_D - C_{Dextra} \\ \qquad + \mu[C_L + C_{LGE} + C_T sin(\alpha+\delta_e)] \end{array} \right\} \qquad (19)$$

To determine δ_e at which the aircraft acceleration during the ground run is a maximum or the runway length on the ground is a minimum, $\frac{da}{d\delta_e} = \frac{ds_G}{d\delta_e} = \frac{d\Psi}{d\delta_e} = 0$ is used. If α, C_D, C_L and C_T are independent of the change of δ_e, it can be shown that

$$(\delta_e)_{best\ ground\ run} = Arc\tan(\mu) - \alpha \qquad (20)$$

5.2 Transition

The transition is assumed to start from the lift-off point and the aircraft accelerates from U_{TO} to an initial climb speed, U_{climb}, which must not be less than $1.2U_S$. By analysing the aircraft rotation after lift-off and the transition to climb, the equation of motion during the transition phase may be written as

$$a = \frac{U dU}{d\ell} = \frac{g}{\overline{W}} \left[C_T cos(\alpha+\delta_e) - C_D - C_{Dextra} - \overline{W} sin\beta\right] \qquad (21)$$

with $C_L + C_T sin(\alpha+\delta_e) = \overline{W} cos\beta$ and $\tan\beta \approx h_{TO}/s_R$ where β is an average initial climb angle.

The horizontal length during transition is $s_R = \int_{U_{TO}}^{U_{climb}} \frac{d(U^2)}{2a} cos\,\beta$. This integral is solved to $s_R = \frac{cos\,\beta}{2g\Phi} ln\left(1 + \frac{0.23\Phi U_S^2}{1.21\Phi U_S^2 - sin\,\beta}\right)$ where $\Phi = \frac{\rho S}{2W}\left[C_T cos(\alpha+\delta_e) - C_D - C_{Dextra}\right]$.

It can be shown that eq. (21) reduces to

$$s_R \approx \frac{0.115 U_S^2\, cos\beta}{g\left[1.21\Phi U_S^2 - sin\beta\right]} \approx \frac{\left(g h_{TO} + 0.115 U_S^2\right)\left[C_L + C_T\, sin(\alpha+\delta_e)\right]}{0.915 g\left[C_T\, cos(\alpha+\delta_e) - C_D - C_{Dextra}\right]} \qquad (22)$$

Similar to the method used in the previous section, δ_e can be estimated for the maximum acceleration or the minimum length during the transition phase. When the aircraft leaves the ground, it hopefully flies with the maximum lift over drag ratio to provide better performance. It is more practical here to determine δ_e at which the aircraft lift to drag ratio is a maximum. Using eq. (20),

$$\frac{C_L}{C_D + C_{Dextra}} = \frac{cos\ \beta - \frac{C_T}{\overline{W}} sin(\ \alpha + \delta_e\ )}{\frac{C_T}{\overline{W}} cos(\ \alpha + \delta_e\ ) - sin\ \beta - \frac{a}{g}} \qquad (23)$$

It can be shown that the condition $d\left(\frac{C_L}{C_D + C_{Dextra}}\right) \Big/ d\delta_e = 0$ for maximum lift over drag ratio is satisfied when

$$(\delta_e)_{besttransition} = 2Arctan\left[\frac{\cos\beta - \sqrt{\cos^2\beta + \left(\frac{a}{g} + \sin\beta\right)^2 - \left(\frac{C_T}{\overline{W}}\right)^2}}{\frac{C_T}{\overline{W}} + \frac{a}{g} + \sin\beta}\right] - \alpha \qquad (24)$$

6 Climb

For the iterative computer program, the climb calculation is repeated for each step in altitude until the cruise altitude is reached. The M_C may be reached before the cruise altitude. If the velocity is still less than U_C at the beginning of cruise, the aircraft must continue to accelerate to reach U_C . The results from the iterative computer program indicate that the initial angle of climb, β, increases at the beginning of the climb and after a short time, it continuously decreases. Using a similar procedure to that for the transition phase, it can be shown that δ_e at which the lift over drag ratio is a maximum, may be determined from eq. (24) with the data from the climb phase.

Using the equations given in the previous sections, the optimum engine angles can be compared here for the ground run, transition and climb. During the ground run, the engine angle of the aircraft should be set at 2.3^o and then, it must be increased to 8^o during the transition after lift-off. At the beginning of the climb, the engine angle is increased to 10.7^o and then the engine angle should be reduced continuously. These results have been presented in the appendix. Possible constraints and technical difficulties which may occur during the take-off phase, are not considered in this research project.

7 Engine Angles & Nozzle Deflections

Step-by-step calculations were used in the iterative computer program to estimate the optimum engine angles of the reference aircraft at different flight phases and cruise conditions.

Using the data produced by this program [13],[16],[17], the variation of optimum angles for the engines and the flight path of the reference aircraft from the beginning of the take-off ground run until the end of cruise can be obtained. $C_{D\ extra}$ has not been taken into account in the calculations. The minimum thrust condition was evaluated keeping $\overline{W}$ constant whereas it is varying in the maximum range integration.

The calculation of the optimum engine angle for the aircraft is not a straight forward procedure and the assumptions which have been made must be mentioned [13],[18],[19]. First of all, δ_v is the angle of the thrust vector and may be slightly different from the nozzle angle. By ignoring this possible difference, δ_v is assumed to be the same as the nozzle angle. Secondly, the variation of $C_{L,v}$, $C_{D,v}$ and $C_{T,v}$ in terms of δ_v should be taken into account which make the current theoretical procedure more complicated. When the extra-circulation and jet-induced effects are included, it is not easy to solve the equations for the aircraft to determine δ_v which satisfies the optimum requirements. To a first approximation, the changes in δ_e , which are evaluated for various conditions and flight phases in this paper, should be the same as changes in δ_v when δ_e is constant. Therefore, the engine nozzle deflection of the aircraft is estimated from

$$(\delta_v)_{optimum} \approx (\delta_e)_{optimum} - (\delta_e)_{fixed} .$$

$(\delta_e)_{optimum}$ is the optimum engine angle at each position in the flight path and is determined in the step-by-step calculation of the iterative computer program. It is varied from one phase to another but may be taken as the engine angle for the best ground run, the best transition, the best climb, etc. $(\delta_e)_{fixed}$ is the fixed angle of the jet engines under the wing and is a constant for different flight phases and cruise conditions.

Due to the effect of upwash on the intake of the jet engine at cruise, it might be more efficient to move down the inlet a few degrees. In addition, the fuselage will deflect the streamlines and it might be necessary to also move the inlet slightly toward the fuselage in the yaw plane. The intake of a high-bypass ratio turbofan engine may need to have an angle of order of 4^o downward and 2^o toward the fuselage [7] (Part-2) and [8].

8 Conclusions

A theoretical approach has been developed to examine how changes to the jet engine angle in a vertical plane would affect the aircraft's aerodynamics and performance. The emphasis is on the longitudinal trim, thrust, lift and the drag at cruise. The conventional trim equations are extended to show how the cruising trim is affected by changes in the jet engine.

The procedure which has been used to calculate the optimum engine angles for the minimum thrust required and for the best range, may be used to estimate the optimum δ_e for the best endurance, for the best rate of climb, and for the minimum time-to-climb, time-to-descent and fuel-to-climb trajectories.

The effects of changes to the jet engine angle in the pitch plane have been formulated to see how the main aircraft design parameters interact. The results of the theoretical method are compared with the computational results which have been already validated. In general, good agreement is found for relatively small changes of the pitch engine angle to the particular aircraft specification. By using a series of subroutines, the convergent ICP is structured so that the effects of any change in the jet engine angle in pitch plane can be readily evaluated and further improvements and expansions can easily be made. The results cover the emergency cases in which one engine is inoperative. The results of the theoretical approach indicate that a suitable engine angle in trimmed flight may reduce the total drag, the fuel used, the thrust required and/or maximize the range and endurance. The optimal engine angles for the minimum thrust required, for the best range and for the minimum fuel consumption in level flight and during take-off and climb, are presented.

The complexity of the equations and the number of parameters involved in the program makes it difficult to see how any particular engine angle affects the design parameters. The author concedes that in general, many approximations have been made in the present theory and design procedure, some of which may be an oversimplification.

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Table 1. Design engine angles and range factors (in the brackets) for range optimization.

	M_C = 0.78	M_C = 0.80	M_C = 0.82	M_C = 0.84
H_C = 8km	3.89° (28.75×10^6m)	4.18° (27.08×10^6m)	4.56° (25.08×10^6m)	5.10° (22.65×10^6m)
H_C = 10km	3.49° (32.08×10^6m)	3.77° (30.05×10^6m)	4.15° (27.61×10^6m)	4.69° (24.65×10^6m)
H_C = 12km	3.15° (35.48×10^6m)	3.43° (33.03×10^6m)	3.81° (30.08×10^6m)	4.36° (26.55×10^6m)
H_C = 14km	2.86° (39.11×10^6m)	3.13° (36.15×10^6m)	3.51° (32.62×10^6m)	4.07° (28.44×10^6m)

APPENDIX - Computer Program Results

The convergent iterative computer program uses a step-by-step method to integrate along the flight path and the output is presented in this appendix after 35 iterations when there are no further changes. The iterated results of the step-by-step calculations for each phase will be presented in Tables A.1 to 7. Flight at constant C_L and M has been selected as cruise conditions for the aircraft and the results are given in Table A.4a. In addition, two cruise alternatives including flight at constant H and M and flight at constant H and C_L are also considered and the results are given in Tables A.4b & c.

Number of iterations, NIT = 35
Number of engines, NEN = 2
Mach number at cruise, MCR= 0.81
FAR take-off runway length, STOFAR = 1848.6m or 6065ft
FAR landing runway length, SLFAR = 1552.3m or 5093ft
Required take-off runway length, STO = 1607.4m
Required landing runway length, SEND = 807.3m
Required landing runway length in emergency, SL = 931.4m
Number of passengers, NPAS = 200
Number of crew including two pilots, NCREW = 11
External length of fuselage, LF = 50.0m
External height of fuselage, HF = 4.0m
External breadth of fuselage, BF = 4.0m
Wing aspect ratio, AR = 8.0
Horizontal tail aspect ratio, ARHT = 2.0
Flight range, RANGE = 5000.0km or 2698.3 Nautical miles
Runway altitude at take-off, ALTAP = 0
Runway altitude at landing, ALTL = 0
Flight altitude at the beginning of cruise, ALTCR = 12.0km or 39370ft
Air density at the beginning of cruise, ROCR = 0.3108kg/m^3
Wing area (platform), SW = 175.20m^2
Horizontal tail area (platform), SHT = 44.36m^2
Wing span, BW = 37.44m
Horizontal tail span, BHT = 9.48m
Wing mean chord, CHORDM = 5.37m
Horizontal tail mean chord, CHORDHT = 1.36m
Velocity at the beginning of cruise, VCR = 238.98m/s or 464.6knots
Equivalent airspeed at the beginning of cruise, EAS = 120.37m/s or 234.0knots
Stalling speed at landing, VSTL = 52.65m/s or 102.4knots
Touch-down velocity at landing, VTD = 60.54m/s or 117.7knots
Take-off velocity at lift-off point, VTO = 71.32m/s or 138.6knots
Stalling speed at take-off, VSTTO = 64.84m/s or 126.0knots
Climb velocity at 15.3m, VCLIMB = 77.81m/s or 151.3knots
Maximum lift coefficient at landing, CLML = 3.0
Maximum lift coefficient at take-off, CLMTO = 2.40
Maximum lift coefficient at cruise, CLMCR = 1.130
Wing sweepback angle at quarter chord, BLANDA = 0.5236 rad. or 30.0°
Wing thickness to chord ratio, TOC = 0.110
All-up weight at take-off, WTO = 996.2kN or 101.5ktons
Empty weight (zero fuel), WEM = 718.2kN or 73.2ktons
Structural weight, WST = 482.7kN or 49.2ktons
Take-off thrust on a hot day, FTOHD = 330.9kN
Take-off thrust on a cold day, FTOCD = 384.8kN
Landing weight, WEND = 775.7kN or 79.1ktons
Landing weight in emergency, WL = 820.9kN or 83.7ktons
Aircraft weight at the beginning of cruise, WCR = 967.7kN or 98.6ktons
Fuel used, WFUSED = 220.5kN or 22.5ktons
Weight of one engine, WEN = 41.5kN or 4.2ktons

Engine weight, WE = 83.1kN or 8.5ktons
Lift at the beginning of cruise, LCR = 964.2kN
Drag at the beginning of cruise, DCR = 61.1kN
Thrust at the beginning of cruise, FCR = 61.2kN
Engine angle w.r.t. velocity vector at cruise, ALPHAECR = 0.0590rad. or 3.382deg.
Wing incidence w.r.t. velocity vector at cruise, ALPHAWCR = 0.1114rad. or 6.382deg.
Induced-drag factor at cruise, K = 0.04780
Compressibility drag at cruise, CDCOMPCR = 0.00342
Zero-lift drag coefficient at the beginning of cruise, CDOCR = 0.01749
Zero-lift drag coefficient at take-off, CDOTO = 0.0569
Zero-lift drag coefficient at landing, CDOL = 0.1287
Throttle setting of engine required at cruise, TSCR = 80.0%
Specific fuel consumption at the beginning of cruise, SFCCR = 1.803×10^{-4} N/(N.sec)
Total time of flight, FTIME = 6.05 hours
Wing Oswald efficiency factor at cruise, EPSIL = 0.8324
Lift coefficient at lift-off, CLLO = 1.9835
Average lift coefficient during take-off ground run, CLTO = 0.4184
Average drag coefficient during take-off ground run, CDTO = 0.0653
Average lift coefficient during landing ground run, CLL = 1.0184
Average drag coefficient during landing ground run, CDL = 0.1787
Lift coefficient at the beginning of cruise, CLCR = 0.620
Drag coefficient at the beginning of cruise, CDCR = 0.03929
Average lift over drag at cruise, LODL = 15.811
Lift-curve slope at cruise, CLALPHA = 5.346rad.$^{-1}$

Table A.1. Take-off ground run.

Distance /m	**Velocity /(m/s)**	**Acceleration /(m/s^2)**	**Weight /kN**	**Lift /kN**	**Drag /kN**	**Thrust /kN**
0.0	0.00	0.00	996.17	0.00	0.00	330.93
8.6	6.95	2.84	996.07	2.00	0.31	324.08
34.9	13.91	2.75	959.68	8.00	1.25	317.57
79.9	20.86	2.67	995.86	17.98	2.80	311.38
144.6	27.82	2.59	995.76	31.96	4.98	305.49
230.2	34.77	2.51	995.65	49.94	7.79	299.90
338.1	41.72	2.43	995.54	71.92	11.21	294.58
469.7	48.68	2.35	995.42	97.89	15.27	289.54
626.9	55.63	2.27	995.30	127.85	19.94	284.74
811.6	62.59	2.19	995.18	161.81	25.23	280.19
1026.2	69.54	2.10	995.05	199.77	31.15	275.86
1080.2	71.15	2.08	995.02	209.09	32.61	274.89

Table A.2a. Transition after lift-off.

Altitude /m	**V /(m/s)**	**Path Angle /deg.**	**Engine Angle /deg.**	**Weight /kN**	**Lift /kN**	**Drag /kN**	**Thrust /kN**	**Radius /km**
0.0	71.15	0.00	2.29	995.02	209.09	32.61	274.89	0.00
0.0	71.32	0.50	4.36	994.91	996.16	96.53	274.79	6.90
5.3	74.57	1.60	5.96	994.74	1170.45	116.85	272.87	6.90
10.7	77.81	3.19	7.97	994.69	1361.98	183.93	270.77	0.00

Table A.2b. Transition after lift-off in emergency.

Altitude /m	**V /(m/s)**	**Path Angle /deg.**	**Engine Angle /deg.**	**Weight /kN**	**Lift /kN**	**Drag /kN**	**Thrust /kN**	**Radius /km**
0.0	71.15	0.00	2.29	995.02	209.09	32.61	274.89	0.00
0.0	71.32	0.20	4.36	994.94	996.16	96.53	137.39	6.90
5.3	74.57	1.09	5.96	994.91	1170.45	116.85	136.43	6.90
10.7	77.81	1.09	5.96	994.84	980.61	112.41	135.38	0.00

Table A.3 Climb.

Altitude /km	M	V /(m/s)	Path Angle /deg.	Engine Angle /deg.	Weight /kN	Lift /kN	Drag /kN	Thrust /kN	Gradient (%)
0.01	0.23	77.8	3.19	7.97	994.7	1362.	183.9	270.8	5.57
0.76	0.26	87.9	9.42	10.69	992.2	931.2	70.2	256.4	16.60
1.51	0.29	98.0	8.97	8.79	990.4	941.2	65.0	242.6	15.78
2.26	0.33	108.1	8.43	7.36	988.7	948.6	61.5	229.4	14.82
3.01	0.36	118.4	7.74	6.51	987.1	978.1	61.3	216.9	13.60
3.76	0.39	128.5	7.19	5.59	985.7	977.9	59.2	204.8	12.61
4.51	0.43	138.6	6.62	4.86	984.2	977.7	57.9	193.2	11.60
5.26	0.46	148.7	5.99	4.32	982.9	977.5	57.1	180.7	10.50
6.01	0.50	158.8	5.27	3.96	981.5	977.4	56.7	166.4	9.23
6.76	0.54	168.8	4.60	3.67	980.2	977.0	56.5	153.1	8.04
7.51	0.58	178.9	3.96	3.46	978.8	976.4	56.4	140.4	6.93
8.26	0.61	189.0	3.37	3.30	977.4	975.7	56.3	128.5	5.89
9.01	0.65	199.1	2.81	3.20	975.9	974.7	56.3	117.0	4.91
9.76	0.70	209.2	2.28	3.14	974.3	973.6	56.3	106.0	3.99
10.51	0.74	219.3	1.76	3.12	972.6	972.1	56.7	95.4	3.08
11.26	0.78	229.3	1.25	3.19	970.6	970.3	58.4	85.9	2.17
12.01	0.81	239.1	0.68	3.38	967.7	967.8	61.2	76.7	1.19

Table A.4a. Cruise at constant lift coefficient & Mach number.

Altitude /km	Distance /km	M	V /(m/s)	Engine Angle /deg	Weight /kN	Lift /kN	Drag /kN	Thrust /kN
12.01	0.00	0.81	239.1	3.38	967.7	967.8	61.2	76.7
12.13	459.13	0.81	238.7	3.38	946.7	943.2	59.7	59.8
12.27	918.25	0.81	238.7	3.38	926.2	922.8	58.4	58.5
12.41	1377.38	0.81	238.7	3.38	906.2	902.8	57.1	57.2
12.54	1836.51	0.81	238.7	3.38	886.6	883.3	55.9	56.0
12.68	2295.64	0.81	238.7	3.38	867.4	864.2	54.7	54.8
12.82	2754.77	0.81	238.7	3.38	848.6	845.4	53.5	53.6
12.96	3213.90	0.81	238.7	3.38	830.2	827.1	52.3	52.4
13.10	3673.03	0.81	238.7	3.38	812.3	809.2	51.2	51.3
13.24	4132.16	0.81	238.7	3.38	794.7	791.7	50.1	50.2
13.38	4591.29	0.81	238.7	3.38	777.5	774.6	49.0	49.1
13.39	4633.45	0.81	238.7	3.38	775.9	773.0	48.9	49.0

Table A.4b Alternative cruise at constant altitude & Mach number.

Altitude /km	Distance /km	M	V /(m/s)	Engine Angle /deg.	Weight /kN	Lift /kN	Drag /kN	Thrust /kN
12.01	0.00	0.81	239.0	3.43	970.4	970.4	61.3	76.9
12.01	458.95	0.81	239.0	3.30	949.4	945.9	59.8	59.9
12.01	917.90	0.81	239.0	3.16	928.8	925.6	58.6	58.7
12.01	1376.84	0.81	239.0	3.02	908.7	905.7	57.4	57.5
12.01	1835.79	0.81	239.0	2.88	889.0	886.2	56.3	56.4
12.01	2294.74	0.81	239.0	2.75	869.7	867.0	55.2	55.3
12.01	2753.70	0.81	239.0	2.62	850.7	848.3	54.2	54.2
12.01	3212.66	0.81	239.0	2.50	832.1	829.8	53.2	53.2
12.01	3671.61	0.81	239.0	2.37	813.9	811.7	52.2	52.3
12.01	4130.57	0.81	239.0	2.25	796.0	793.9	51.3	51.3
12.01	4589.51	0.81	239.0	2.13	778.3	776.5	50.4	50.4
12.01	4631.66	0.81	239.0	2.12	776.7	774.9	50.3	50.4

Table A.4c. Alternative cruise at constant altitude & lift coefficient.

Altitude /km	Distance /km	M	V /(m/s)	Engine Angle /deg	Weight /kN	Lift /kN	Drag /kN	Thrust /kN
12.01	0.00	0.809	239.0	3.48	959.2	959.2	60.8	76.2
12.01	459.05	0.802	236.6	3.48	938.4	934.8	58.5	58.6
12.01	918.10	0.793	234.0	3.48	918.3	914.8	56.5	56.6
12.01	1377.14	0.785	231.5	3.48	898.8	895.4	54.6	54.7
12.01	1836.19	0.776	229.1	3.48	879.8	876.6	52.9	53.0
12.01	2295.24	0.768	226.7	3.48	861.3	858.1	51.3	51.4
12.01	2754.29	0.760	224.3	3.48	843.2	840.2	49.9	50.0
12.01	3213.35	0.752	221.9	3.48	825.6	822.6	48.5	48.6
12.01	3672.41	0.744	219.6	3.48	808.3	805.4	47.3	47.4
12.01	4131.46	0.736	217.3	3.48	791.3	788.5	46.1	46.2
12.01	4590.50	0.729	215.0	3.48	774.7	772.0	45.0	45.1
12.01	4632.66	0.728	214.8	3.48	773.2	770.5	44.9	45.0

Table A.5a. Landing ground run with thrust reversing.

Distance /m	Velocity /(m/s)	Deceleration /(m/s^2)	Weight /kN	Lift /kN	Drag /kN	Reversing Thrust /kN
0.0	57.6	-4.28	775.92	826.16	101.51	0.00
59.1	52.0	-5.37	775.88	271.57	43.11	129.26
109.2	46.4	-5.63	775.84	216.19	34.32	131.03
151.6	40.8	-5.87	775.80	167.13	26.53	132.88
187.3	35.2	-6.08	775.76	124.37	19.74	134.81
216.7	29.6	-6.26	775.73	87.91	13.96	136.82
243.3	24.0	-4.66	775.71	57.77	9.17	0.00
268.5	18.4	-4.76	775.71	33.93	5.39	0.00
286.7	12.8	-4.84	775.71	16.39	2.60	0.00
298.2	7.2	-4.88	775.71	5.17	0.82	0.00
303.2	1.6	-4.91	775.71	0.25	0.04	0.00
303.4	0.2	-4.80	775.71	0.00	0.00	0.00

Table A.5b. Landing ground run in emergency without thrust reversing.

Distance /m	Velocity /(m/s)	Deceleration /(m/s^2)	Weight /kN	Lift /kN	Drag /kN
0.0	57.31	-3.63	820.90	871.32	113.05
81.6	51.70	-3.86	820.90	268.79	47.05
150.6	46.10	-4.08	820.90	213.72	37.41
208.8	40.50	-4.27	820.90	164.95	28.87
257.4	34.90	-4.43	820.90	122.49	21.44
297.4	29.30	-4.57	820.90	86.34	15.11
329.4	23.70	-4.69	820.90	56.49	9.89
354.2	18.10	-4.78	820.90	32.95	5.77
372.0	12.50	-4.84	820.90	15.72	2.75
383.2	6.90	-4.89	820.90	4.79	0.84
387.8	1.30	-4.90	820.90	0.17	0.03
388.0	0.00	-0.00	820.90	0.00	0.00