HEAT TRANSFER CHARACTERISTICS OF LITHIUM COOLANT FLOWING THROUGH A CURVED BEND USING MODIFIED TOROIDAL COORDINATE SYSTEM

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Keywords: *Energy Equation, Modified Toroidal Coordinate System, Curved Bend, Liquid Metal Lithium*

Abstract

In recent years concentrated interest has been placed on Liquid-Metal heat transfer because of high heat transfer rates. The purpose of the paper was to develop a computer code for the solution of three dimensional energy equation in modified toroidal coordinate system to determine heat transfer characteristics of liquid metal Lithium across a curved bend. Energy and continuity equations were used for this purpose. In modified toroidal coordinate system there are two linear coordinate axes x, y, and one angular coordinate axis θ. Problems in curvilinear geometry like curvilinear channels in fusion reactors can be best handled by using this coordinate system.

In the first part of this paper, mathematical work was completed. Discretization equations were derived using control volume approach and power law numerical scheme. In the second part, computer code HTBEND was developed using these discretization equations. The code solves the energy equation in fully developed velocity profiles. The results of Nusselt number versus distance along the bend were obtained for liquid metal Lithium and water. The effect of bend radius on heat transfer was also determined. The computational heat transfer parameters visualization along with the flow was made through the square duct by heating different faces and for different radii of bend. A comparison was also done by solving the same problem with the help of commercial code Fluent, and quite a reasonable agreement was found. The paper contains a critical study of advantages and disadvantages of Lithium coolant compared with other coolants available.

Nomenclature

μ	Absolute viscosity
ν	Kinematic viscosity (μ / ρ)
ρ	Fluid density
g	Acceleration due to gravity
p	Pressure
Re	Reynold's Number
t	Time
u	Velocity in X- direction
v	Velocity in Y-direction
w	Velocity in Z-direction
D	Diffusion coefficient
F	Convective coefficient
C_p	Specific heat at constant pressure
Pr	Prandtl number
S	Source strength
ϕ	Dependent Variable

INTRODUCTION

The computational solution of the energy equation enables us to predict heat transport parameters during the flow of different fluids. Heat transfer behavior of liquid metal Lithium and Water has been

studied assuming fully developed parabolic velocity profile. Control volume method has been used to derive the discretization equation. Power law scheme has been used to make the algebraic equations because of its good accuracy and less computational cost. The fluid has been assumed to flow through a 90^0 square cross-sectional bend lying horizontally so that gravitational acceleration has no effect on the flowing fluid because modified toroidal coordinate system handles the curvilinear geometries in a good manner. The results have been interpreted in terms of Nusselt number verses distance along the bend.

All problems in fluid flow require that the continuity equation be satisfied. If steady state conditions prevail, all derivatives with respect to time are zero, and continuity equation becomes

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho \mathrm{v})}{\partial y}+\frac{\partial(\rho \mathrm{w})}{\partial z}=0 \qquad (1)$$

or

$$div(\rho V)=0 \qquad (2)$$

If a fluid is compressible, the density will vary in space, so Equation (2) applies for the steady state flow of a compressible fluid. For the steady state flow of an incompressible fluid the density is constant, and the continuity equation becomes

$$\frac{\partial \mathrm{u}}{\partial \mathrm{x}}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \qquad (3)$$

or

$$\text{div V} = 0 \qquad (4)$$

When flow is nonisothermal, the temperature of the fluid is a dependent variable which is a function of x, y, z, and t. Just as the continuity equation is a mathematical expression for the law of conservation of mass and gives the velocity distribution in space, the energy equation is a mathematical expression for the law of conservation of energy and gives the temperature distribution in space.

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) \qquad (5)$$

Equation (5) is the common form of the three dimensional energy equation and applies for the following conditions:

1. Perfect gases
2. Steady flow
3. $q' = 0$
4. ϕ negligible
5. $u\left(\frac{\partial P}{\partial x}\right)$, $v\left(\frac{\partial P}{\partial y}\right)$, $w\left(\frac{\partial P}{\partial z}\right)$ negligible

where

k = thermal conductivity of fluid.

q' = time rate of energy generation (from chemical reaction) in fluid per unit volume.

ϕ = dissipation function.

Modified Toroidal Coordinate System

In this coordinate system there are two linear coordinate axes x, y and one angular coordinate axis θ. Problems in curvilinear geometry like curvilinear channels in fusion reactors can be best handled by using this coordinate system (Fig.1).

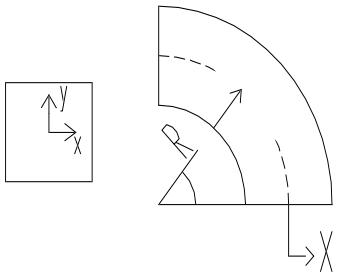


Fig. 1. Modified Toroidal Coordinate System

X : Distance in radial direction from pipe centre line.
Y : Distance normal to radial direction.
θ : Angle around loop in axial direction.

To express the governing equations in this coordinate system, the operators must be evaluated. This is done with the use of metric coefficients. The resulting differential equations are:

Continuity Equation

For steady state flow of an incompressible fluid

$$\frac{1}{R+x}\left[\frac{\partial}{\partial x}\left((R+x)u\right)+\frac{\partial}{\partial y}\left((R+x)v\right)+\frac{\partial}{\partial \theta}(w)\right]=0 \quad (6)$$

Energy Equation

For steady state flow of an incompressible fluid with negligible viscous dissipation and assuming physical properties constant during heat transfer and fluid flow

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\frac{w}{R+x}\frac{\partial T}{\partial \theta}\right)=$$

$$\frac{k}{R+x}\left[\frac{\partial}{\partial x}\left((R+x)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left((R+x)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial \theta}\left(\frac{1}{R+x}\frac{\partial T}{\partial \theta}\right)\right]+S \quad (7)$$

For hydrodynamically fully developed and thermally developing flow the above equation will be modified as

$$\rho C_p\left(\frac{w}{R+x}\frac{\partial T}{\partial \theta}\right)=$$

$$\frac{k}{R+x}\left[\frac{\partial}{\partial x}\left((R+x)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left((R+x)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial \theta}\left(\frac{1}{R+x}\frac{\partial T}{\partial \theta}\right)\right]+S \quad (8)$$

Equation (8) will be numerically solved by using this computer code. The comparison of above equations with the equations obtained in Cartesian coordinates will reveal that above equations have additional terms. These terms arise because of the coordinate transformation.

Discretization Method and Scheme

Finite Volume Method

This method is most popular as far as our problem is concerned. The computational domain is divided into a number of nonoverlapping control volumes such that there is one control volume surrounding each grid point. The differential equation is integrated over each control volume. Piecewise profiles expressing the variation of Φ between the grid points are used to evaluate the required integrals. The result is the discretization equation containing the values of Φ for a group of grid points.

The control volume in modified toroidal coordinate system is shown in "Fig. 2". Here E, W, N, S, T, B represent the directions east, west, north, south, top and bottom respectively. This characteristic exists for any number of grid points. . In recent years, Finite Volume Method receives considerable attention. It is similar to the finite difference method, but it has some features that are said to be akin to finite element method. In this study finite volume method has been used [3]. The power law scheme has been used to make algebraic equations from differential equations.

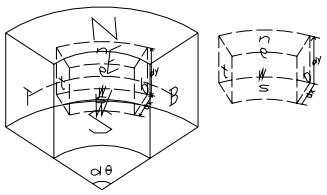


Fig. 2. Control Volume in Modified Toroidal Coordinate System

RESULTS AND DISCUSSION

In this computer code, bulk temperature of fluid and wall temperature were found. By using these temperature values Nusselt Number was computed by using the formula

$$Nu = \frac{q'' * D}{(T_w - T_b) * k} \qquad (9)$$

where,

Nu is Nusselt number, D is the diameter, T_w is the wall temperature, T_b is the bulk temperature of fluid, and k is thermal conductivity of fluid. Bulk temperature of the fluid is found by taking the average of temperatures at all the grid points for a particular plane. Maximum temperature along the wall has been taken as wall temperature for a particular plane.

When heat transfer occurs during laminar flow of a fluid, the transfer through the fluid is by conduction alone. No mixing of the fluid, like that occurring during turbulent flow, takes place. In practice it is difficult to obtain truly laminar flow during heat transfer except in very small passages. Natural convection currents are usually present, and under these conditions conduction alone is not the only mode of heat transfer to be considered. It is possible; since turbulence is absent, to set up continuity, momentum and energy differential equations for a given system. With the application of the appropriate boundary conditions, the solution of these equations will give the temperature and velocity at any point in the conduit. The results may be expressed in terms of the heat transfer coefficient or, if preferable, the Nusselt number.

The value of heat transfer coefficient is governed by many factors:

- **Operating factors**

1. The geometrical shape of the channel
2. The flow rate of the fluid
3. The heat flux
4. The system temperature

- **Physical properties of the fluid**

Liquid metals have had limited application in the field of heat transfer for a number of years, but developments in handling and metering liquid metals and their suitability for high temperature, high heat transfer applications have led to a considerable amount of theoretical and empirical investigation in this field. The present interest in liquid metals in the field of heat transfer stems from their use in atomic reactors. In this application large quantities of heat must be removed rapidly at very high temperatures and at relatively low temperature differences. Under these conditions liquid metals have certain advantages over water at high pressure. With liquid metals, as with other fluids, the density, viscosity, thermal conductivity, and specific gravity are the properties to consider in heat transfer. Liquid metals are unique among fluids because of their high thermal conductivity and consequently low Prandtl number. Their viscosity is comparable to that of water. A summary of the properties of Lithium liquid metal and water is given in Table 1. One major advantage of liquid metals as heat transfer media is the fact that they are liquids over a wide temperature range. It is clear from Table 1 from their difference of melting points and normal boiling points. This wide temperature range is possible at pressures up to 1 atmosphere. If water is used as heat transfer media then a total pressure of 3000 psi will be required to ensure that the water will remain in liquid state.

Table:1 Physical Properties of the fluids used [5]

Property/Fluid	Lithium	Water
melting point $^{\circ\circ 0}$C	179	.....
Normal Boiling Point ^{0}C	1317	100
Density kg/m^3*10^{-3}	0.51	0.9958
Viscosity kg/ms*10^3	0.60	0.0086
Heat Capacity, kJ/kg K	4.19	4.179
Thermal Conductivity, W/m K	38.1	0.614
Prandtl number	0.065	5.85

It should be emphasized that the boundary conditions have a significant effect on the results. Here, the results have been taken using two fluids Lithium and water. Coolant velocity has been taken as 5 m/s. At first the results have been taken for the case of only top face heated with a heat flux of 5 MW/m^2 and three faces insulated. Then, the results were taken for the case of all faces heated with a heat flux of 5 MW/m^2 . Different radii of the bend have been used ranging from 0.05m to 1.0 m. Internal heat generation has been taken as 25 MW/m^3.

Analytical investigation of laminar heat transfer also involves a consideration of the hydrodynamic condition existing in the conduit. The heat transfer may be taking place at the entrance to the conduit, where the velocity profile is also developing, or it may take place far from the entrance, where the laminar velocity profile is fully developed. The problem handled in this project is where the flow is hydrodynamically fully developed assuming laminar parabolic velocity profile but thermally the flow is developing.

Lithium

Lithium top face heated at 5 Mw/m^2 with three faces insulated and internal heat generation of 25 Mw/m^3 for a bend radius of 0.05m was first studied. The results obtained are as shown graphically in "Figure 3 (a)". It is clear from the graph that Nusselt number has a maximum value of 8.28 and minimum value is 6.98. It is also clear from Table 2.

Now for the same fluid, all faces were heated at 5 Mw/m^2 and internal heat generation of 25 MW/m3 for a bend radius of 0.05m, . The results obtained are as shown graphically in "Figure 3 (b)". Maximum value of Nusselt number was 4.46 and minimum value was 3.77 .

As the radius of the bend is increased from 0.05m to 1.0m, the values of Nusselt number obtained are as shown in Table 2, the distance along the bend increases according to the relation

$$S = r\theta \qquad (10)$$

And maximum and minimum values of Nusselt number decrease. The graphical representation of this for Lithium for both cases of top and all faces heated is shown in "Figure 4".

For the same radius of the bend, the Nusselt number decreases with the distance along the bend for both cases of top and all faces heated. Now what is the reason that Nusselt number initially has a maximum value and then it decreases. "Figures 7" give the answer to this question. It is clear from these figures that wall temperature increases from 600 to 900 but the bulk temperature from 500 to 700. This means that wall temperature increases 300 degrees but bulk temperature increases about 200 degrees. It is clear from the equation (9) that Nusselt number is inversely proportional to difference of temperature. This temperature plot is also shown in "Figure 7".

As the problem is hydrodynamically fully developed but thermally the flow is not fully developed but it is developing one, so the trend of all the plots between Nusselt

Number and distance along the bend shows that at first the Nusselt Number decreases during thermally developing flow, and reaches a constant value when the flow is thermally developed. The plots have the same trend as shown in "Figure 6" from literature [2].

It is also clear from the Tables that for the same radius of the bend the value of the Nusselt Number with top face heated and three faces insulated is greater than that of with all faces heated. Because in the later case the opposite faces are being heated at the constant heat flux, so Nusselt number decreases. Similarly, when only top and right faces are heated and bottom and left faces are insulated, the value of the Nusselt number will be the maximum. This is true for both the fluids i.e., Lithium and water.

Water

As the radius of the bend is increased from 0.05m to 1.0m, the values of Nusselt number obtained for water are as shown in Table 3. It has been observed that Nusselt number of water is greater than that of Lithium. For example, for the case of top face heated for a bend radius of 0.40 m, the maximum value of Nusselt number is 7.636 and for the case of All faces heated Nusselt number is 4.09 as is clear from "Figure 8". The Nusselt number decrease with the bend radius for water is also different from those of the liquid metal Lithium as is clear from "Figure 5". The reasons are:

1. Water is liquid but lithium and Lithium are in liquid metal form.
2. Thermal conductivity of water is small as compared to Lithium. So as clear from the relation (9) Nusselt number is inversely proportional to the thermal conductivity. This is the reason that water has a larger value of Nusselt number. Another point in the case of water is its smaller thermal entry length. In the case of Lithium thermal entry length is greater than that of water as is clear from all the plots of Nusselt number versus distance along the bend. In general, the higher the Prandtl number, the shorter the entry length [5]. Thermal entry lengths are much shorter for turbulent flow than for the laminar counterpart. It was seen that Nusselt number also increases due to increase in the velocity. It is due to increase in heat transfer when the velocity is increased.

The results obtained were compared with those obtained under the similar conditions for the linear duct for the same fluid. It was found that with increasing bend radius the results obtained were approaching to those obtained for linear duct. However, for the bend radius of 1.0 m there was a difference of 40% between the two values. This is because in a bend fully developed parabolic velocity profile is hardly achieved. This is only possible for a very long radius of the bend. Further, when high heat fluxes are involved the parabolic velocity distribution is disturbed even for a linear duct. This is because the viscosity of the liquid decreases with temperature increase. If the bend radius is increased beyond 1.0 m, this difference will be accordingly reduced.

The results of CFD Analysis using commercial code Fluent for both the fluids, i.e., Liquid Lithium and Water are shown in Figures 9-13. This analysis shows a reasonable agreement with the predictions of HTBEND.

Table: 2 Maximum and Minimum Values of Nusselt Number at various Bend Radii for Lithium.

Radius of the bend	Faces heated	0.05 m	0.10 m	0.15 m	0.20 m	0.40 m	0.60 m	0.80 m	1.0 m
Maximum	Top	8.31	7.75	7.63	7.58	7.46	7.37	7.29	7.22
	All	4.50	4.17	4.10	4.07	4.00	3.96	3.62	3.88
Minimum	Top	7.43	7.17	6.96	6.79	6.27	5.93	5.68	5.48
	All	4.02	3.87	3.76	3.67	3.40	3.22	3.09	2.99

Table: 3 Maximum and Minimum Values of Nusselt Number at various Bend Radii for Water.

Radius of the bend	Faces heated	0.05 m	0.10 m	0.15 m	0.20 m	0.40 m	0.60 m	0.80 m	1.0 m
Maximum	Top	8.33	7.80	7.70	7.67	7.636	7.629	7.625	7.623
	All	4.51	4.20	4.14	4.12	4.09	4.092	4.090	4.08
Minimum	Top	7.66	7.63	7.63	7.62	7.610	7.600	7.590	7.580
	All	4.14	4.11	4.10	4.09	4.08	4.077	4.072	4.06

Acknowledgement

This work was supported by the office of NESCOM, University of Engineering and Technology (UET)Taxila, Pakistan by providing the computational support.

References

[1] Suhas V. Patankar, " Numerical heat transfer and fluid flow '' , Hemisphere publishing corporation, Mc Graw Hill Book Company , 1980.
[2] James G. Knudsen, Donald L. Katz, " Fluid dynamics and heat transfer " , Mc Graw Hill Book Company New York, 1958.
[3] Vijay Gupta, Santosh K. Gupta, " Fluid mechanics and its applications" , Wiley Eastern Limited New Delhi, 1991.
[4] A. K. Jains, " Fluid Mechanics" ,Khanna Publishers, Delhi, 1993.
[5] J. P. Holman, "Heat Transfer " , Mc Graw Hill Book Company London, 1992.

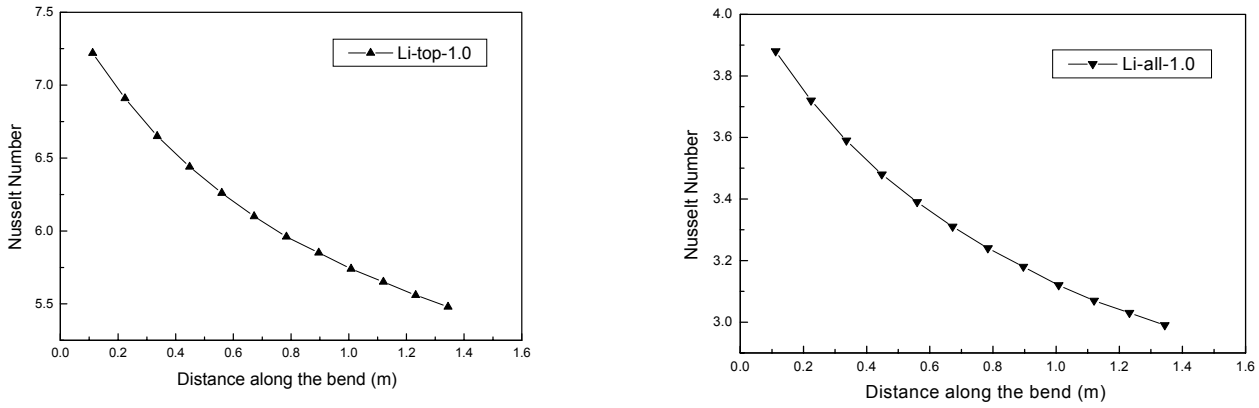


Fig. 3. Nusselt No. for Lithium,(a) top face heated , three faces insulated, (b) all faces heated at 5 MW/ m^2, internal heat generation of 25 MW / m^3, 5 m/s coolant velocity, bend radius of 1.0 m .

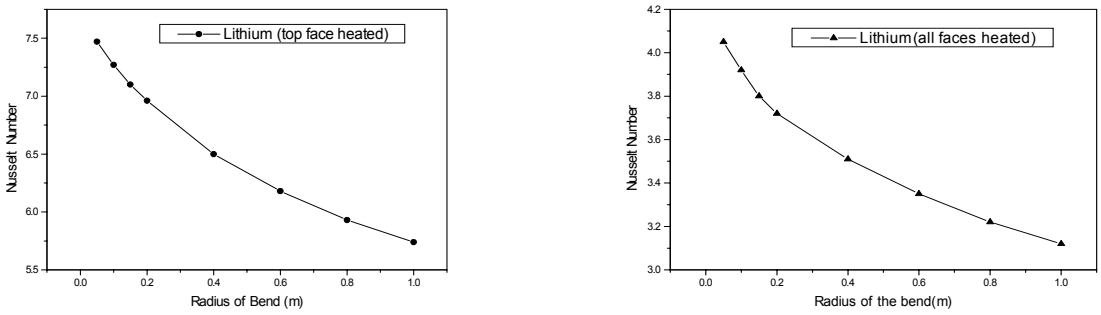


Fig. 4 Nusselt Number vs. Radius of the bend for the case of Lithium at a particular plane

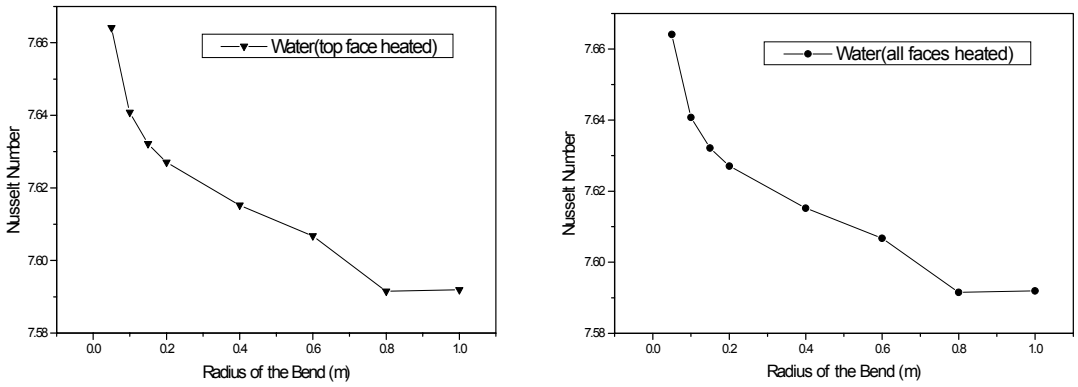


Fig. 5. Nusselt Number vs. Radius of the bend for the case of Water at a particular plane.

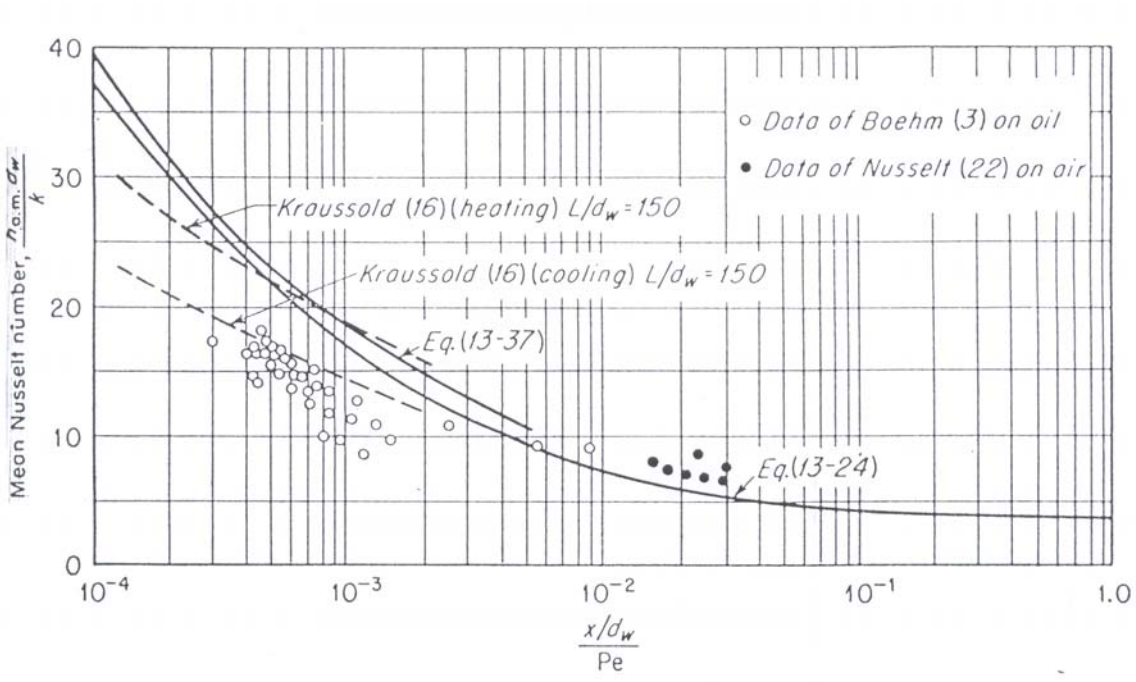


Fig.6. Nusselt Number variation for fully developed laminar flow.[2]

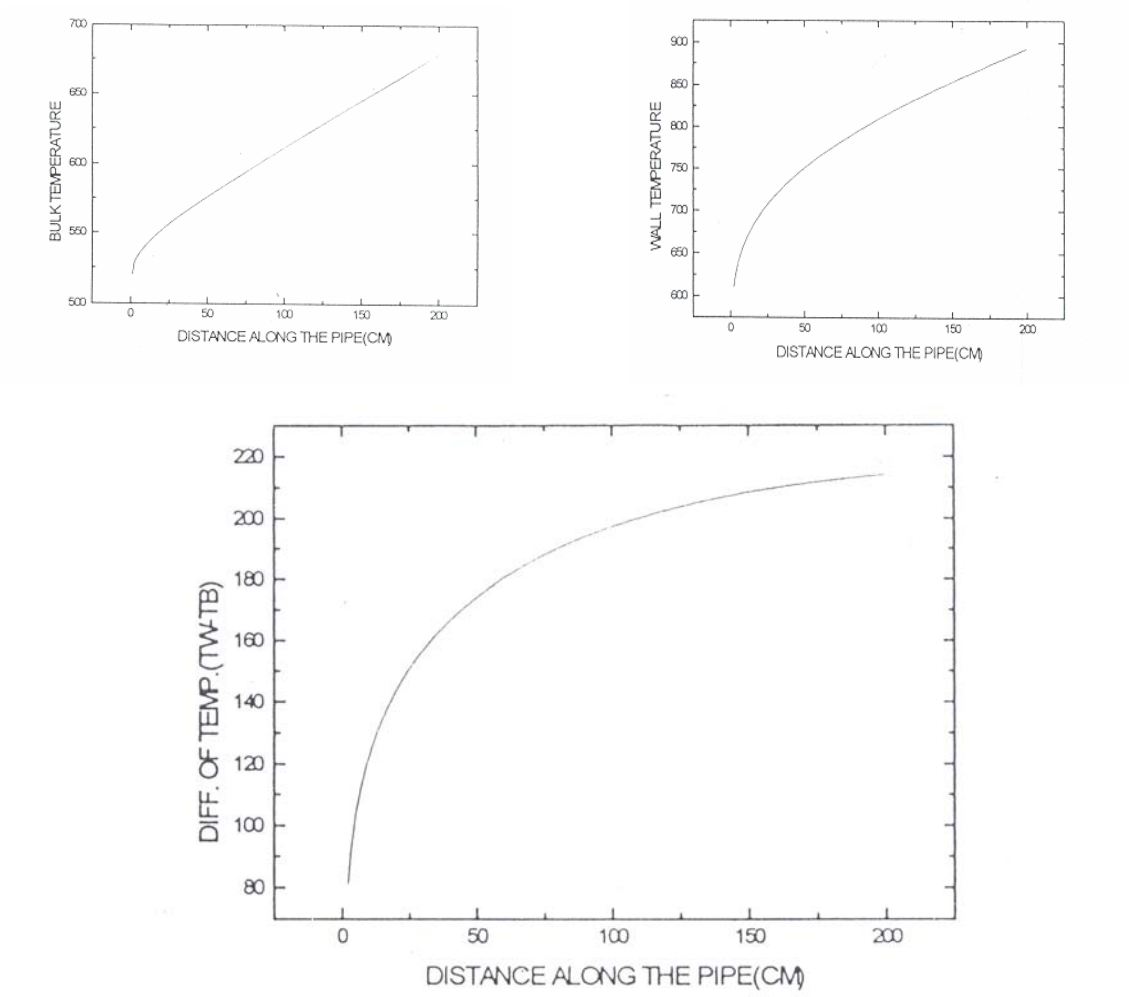
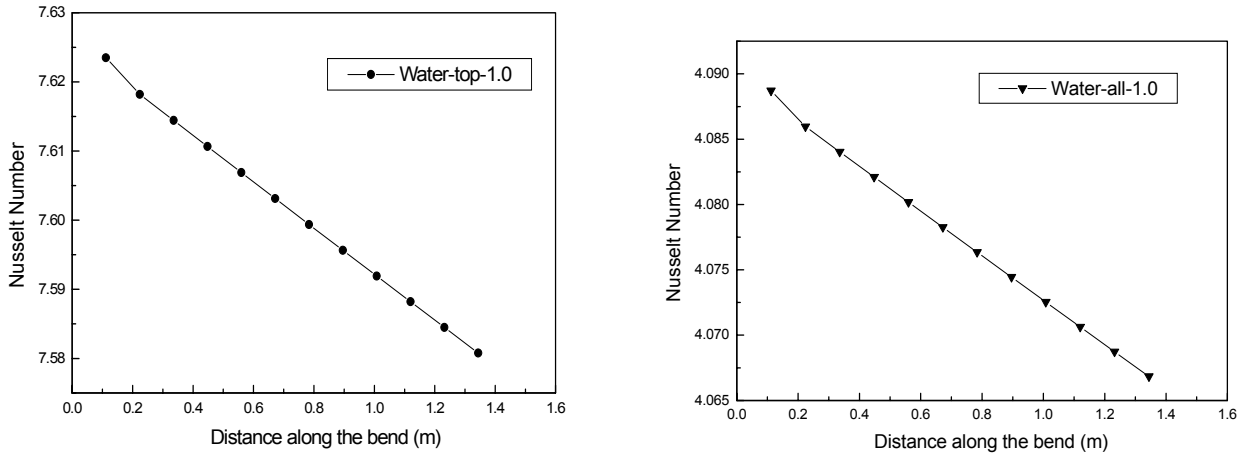


Fig.7. Bulk Temperature, Wall Temperature and the Difference of Temperature Vs. Distance along the pipe



(a) (b)

Fig. 8. Nusselt No. for Water,(a) top face heated , three faces insulated, (b) all faces heated at 5 MW/ m^2, internal heat generation of 25 MW / m^3, 5 m/s coolant velocity, bend radius of 1.0 m.

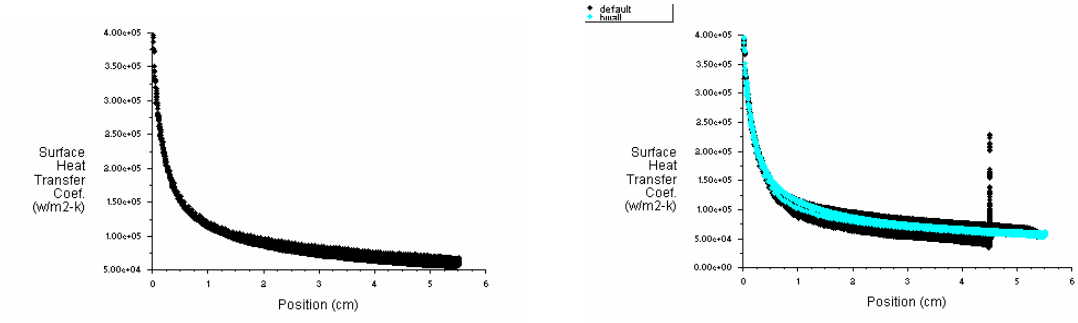


Fig.9 Surface Heat Transfer Coefficient along X- axis for Lithium top and all faces heated for a Bend Radius of 5 cm.

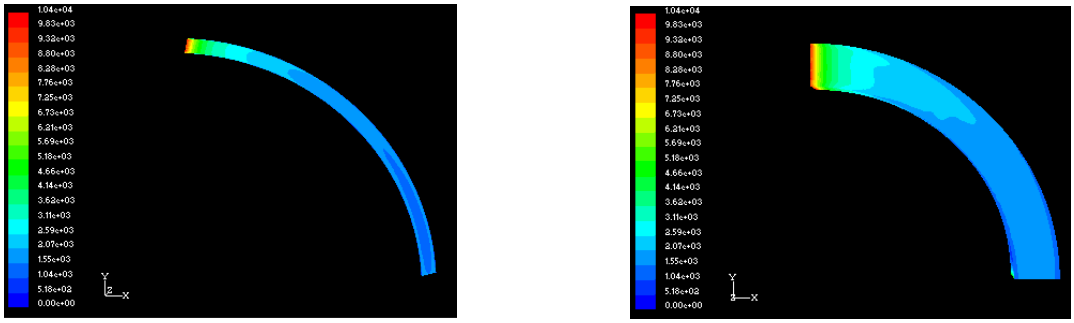


Fig.10 Surface Nusselt Number Distribution for Lithium top and all faces heated for a Bend Radius of 5 cm.

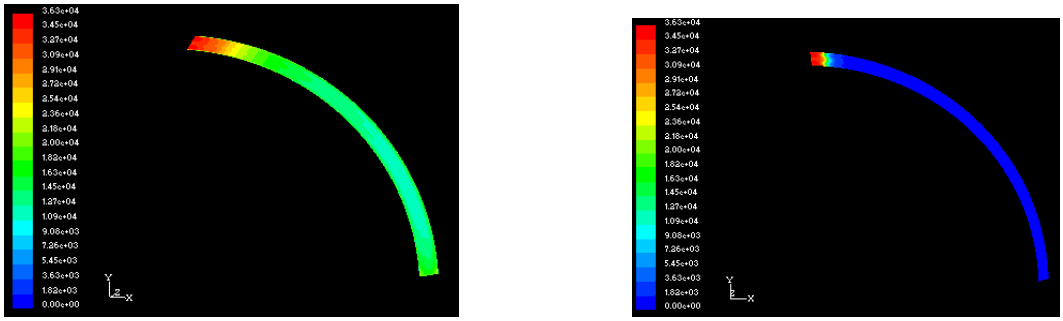


Fig.11 Surface Nusselt Number Distribution for Water top and all faces heated for a Bend Radius of 5 cm.

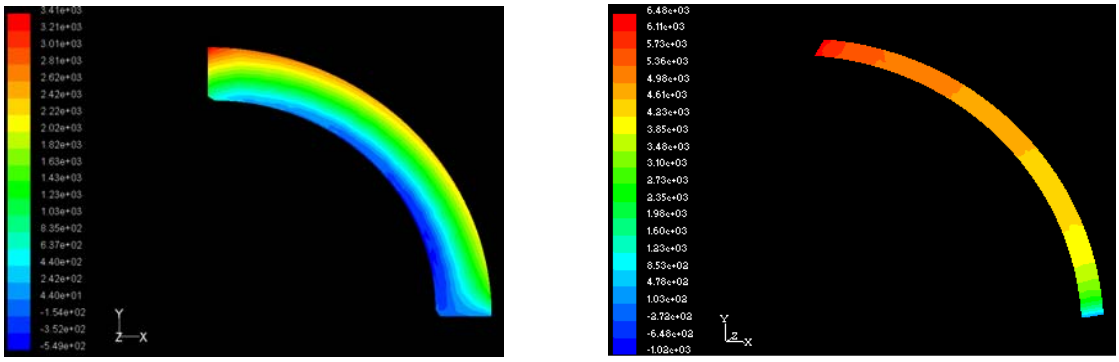


Fig.12 Pressure Distribution for Lithium and water top face heated for a Bend radius of 5 cm.

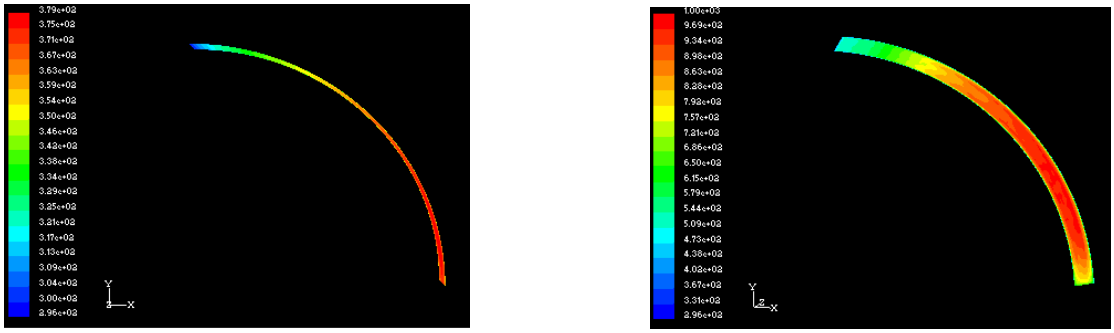


Fig.13 Temperature Distribution for Lithium and water top face heated for a Bend radius of 5 cm.