NUMERICAL ANALYSIS OF FLAPPING WING

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Abstract

Micro Air Vehicles (MAVs) are very small and fly at low speeds. Therefore, their wings are flat plates or membranes. For MAVs, a flapping wing is proposed to generate lift and thrust simultaneously. In this study, the Immersed Boundary method is used to analyze the airflow around a flapping flat-plate wing numerically. The grid is not body-fitted but Cartesian and the body is expressed by the external force. The flow around a stationary plate is computed and the pressure contours, velocity vectors and streamlines show the plate exists in the grid.

1 Introduction

Thanks to recent technological developments, many mechanical parts and pieces of electrical equipment have become smaller and lighter. Thus, small aircraft for short-term observation, called Micro Air Vehicles (MAVs) are considered everywhere.

MAVs are smaller and fly at lower speeds than normal airplanes. So, the Reynolds number is very low and the knowledge gained from the existent airfoils cannot be applied to MAVs. And the wings of MAVs are flat plates or membranes.

For MAVs, a flapping wing that generates lift and thrust like birds and insects, is proposed, although this is still technologically quite challenging.

In this study, to numerically analyze the flow around a flat plate, the incompressible Navier-Stokes equations are solved with the Immersed Boundary Method [1].

2 Numerical Method

2.1 Immersed Boundary Method

The incompressible Navier-Stokes equations consist of the momentum equation

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \bullet \nabla \boldsymbol{u} = -\nabla p + \frac{\nabla^2 \boldsymbol{u}}{Re} + \boldsymbol{F} \qquad (1)$$

and the continuity equation

$$\nabla \bullet \boldsymbol{u} = 0 \qquad (2)$$

where t is the time, $\boldsymbol{u}$ is the velocity vector, p is the pressure, Re is the Reynolds number and $\boldsymbol{F}$ is the external force vector. All variables are nondimensional.

In this study, the computational grid is not body-fitted, but Cartesian. The body is expressed on the grid by the external force term $\boldsymbol{F}$ in Eq. (1). In expressing the body, the 'weight' is distributed over the grid points around it. Converting Cartesian grid x-y-z to uniform grid ξ-η-ζ, the weight d is imposed to the grid points by

$$d = \frac{1}{2}\left(1 + \cos\frac{\pi r}{2}\right) \quad (r < 2) \qquad (3)$$

where r is the distance between the body and the grid point.

2.2 Numerical Procedure

The Crank-Nicolson method is applied to the viscous term, and Eq. (1) becomes

$$\left(\frac{1}{\Delta t}-\frac{\nabla^2}{2Re}\right)\tilde{\boldsymbol{u}}=\boldsymbol{R}+\boldsymbol{F} \tag{4}$$

where $\boldsymbol{R}$ at the nth time step is

$$\boldsymbol{R}=\frac{\boldsymbol{u}^n}{\Delta t}-\boldsymbol{u}^n\bullet\nabla\boldsymbol{u}^n-\nabla p+\frac{\nabla^2\boldsymbol{u}^n}{2Re} \tag{5}$$

Then, to satisfy Eq. (2) at the next time step, a scalar variable φ is introduced:

$$\nabla\bullet\boldsymbol{u}^{n+1}=0 \tag{6}$$

$$\frac{\boldsymbol{u}^{n+1}-\tilde{\boldsymbol{u}}}{\Delta t}=-\nabla\varphi \tag{7}$$

The value of φ is computed by solving Poisson's equation induced from Eqs. (6) and (7)

$$\nabla^2\varphi=\frac{\nabla\bullet\tilde{\boldsymbol{u}}}{\Delta t} \tag{8}$$

The pressure of the next time step is

$$p^{n+1}=p^n+\left(1-\frac{\Delta t}{2Re}\nabla^2\right)\varphi \tag{9}$$

2.3 External Force Term

In the Immersed Boundary Method, the body is expressed by the external force. The velocity of the grid points around the body is forced to be that of the body. That is, the external force computed as the velocity of the grid points at r = 0 meets the condition

$$\boldsymbol{u}^{n+1}=\boldsymbol{u}_b^{n+1} \tag{10}$$

where $\boldsymbol{u}_b$ is the body velocity vector.

To satisfy Eq. (10) and the NS Eqs, the velocity becomes

$$\boldsymbol{u}^{n+1}=d\boldsymbol{u}_b^{n+1}+(1-d)\boldsymbol{u}_m^{n+1} \tag{11}$$

where $\boldsymbol{u}_m$ is the solution of the NS Eqs. and is

$$\boldsymbol{u}_m^{n+1}=\frac{\boldsymbol{R}+\frac{\nabla_o^2\tilde{\boldsymbol{u}}}{2Re}}{\frac{1}{\Delta t}-\frac{\nabla_c^2}{2Re}}-\Delta t\nabla\varphi \tag{12}$$

The Laplacian operator is divided into two parts: the term of the subscript "i,j" and the others. In the case of the second-order central difference, the operators are

$$\nabla_c^2 u=-\frac{4}{h^2}u_{i,j} \tag{13}$$

$$\nabla_o^2 u=\frac{u_{i+1,j}+u_{i-1,j}+u_{i,j+1}+u_{i,j-1}}{h^2} \tag{14}$$

where h is the distance of the grid points.

From Eqs. (5), (11) and (12), the external force vector $\boldsymbol{F}$ becomes

$$\boldsymbol{F}=-d\left(\boldsymbol{R}+\frac{\nabla_o^2\tilde{\boldsymbol{u}}}{2Re}\right)+d\left(\frac{1}{\Delta t}-\frac{\nabla_c^2}{2Re}\right)(\boldsymbol{u}_b^{n+1}+\Delta t\nabla\varphi) \tag{15}$$

Because the value of φ in Eq. (15) is unknown, Eqs. (4) and (8) are solved iteratively during a time step.

2.4 Computational Grid

As described above, the computational grid is Cartesian. To decrease the amount of computation, the flow field is symmetrical to the y = 0 plane and the grid is made on the range y>0 (shown in Fig. 1).

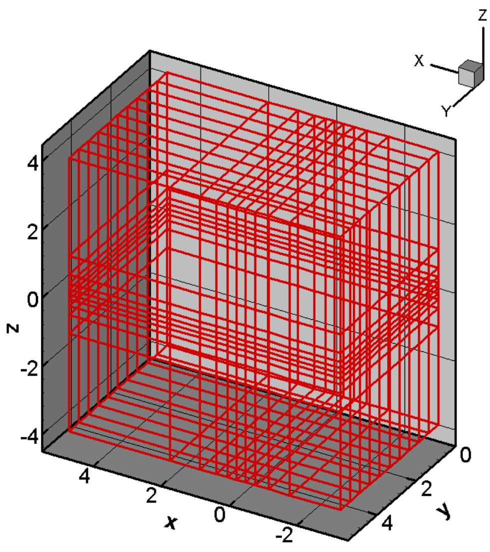

Fig. 1. Computational Grid

3 Results and Discussion

3.1 Computational Condition

In this study, the flapping of a small bird with a wing span of about 30cm is considered. And it is assumed that the chord length is 6.5cm, the frequency is 12.0Hz and the flight speed is 7.0m/s. As a result, the reduced frequency k ($=\pi fc/2U$) is 0.35.

The initial condition is the impulsive start. The shape of the plate is rectangular and the aspect ratio is 5.0. The Reynolds number is 100.

3.2 Stationary Plate (α = 0deg.)

The flow around the flat plate with an angle of attack α as 0deg. is computed.

The history of the aerodynamic force is shown in Fig.1. The lift coefficient is always zero because the angle of attack is zero and the plate has no camber. The drag coefficient converged at 0.55. In the case of a two-dimensional plate, the drag coefficient is calculated by the analytical solution [2]

$$C_d = 2\left(\frac{1.328}{Re^{1/2}} + \frac{2.67}{Re^{7/8}}\right) \qquad (16)$$

and becomes 0.36. The result is larger than the analytical solution because of the effect of the three-dimensional case.

The pressure contours are shown in Fig. 3. The pressure contours and the streamlines on the z = 0 plane are shown in Fig. 4. In the figures, the contours show that the pressure is high between (x, y, z) = (0, 0, 0) and (0, 2.5, 0) which is where the leading edge of the plate is. In Fig. 4, the streamline near the wing root goes straight, but that near the wing tip goes outside at the leading edge and inside behind the trailing edge.

The pressure contours and the velocity vector on the y = 0 plane are shown in Fig.5. The velocity vector shows the boundary layer is formed on the range 0<x<1, and in the wake, the velocity around z = 0 recovers to the freestream velocity.

These results show the plate exists there.

3.3 Stationary Plate (α = 6deg.)

The flow around the flat plate in witch the angle of attack α is 6deg. is computed.

The history of the aerodynamic force is shown in Fig.6. The lift and drag coefficients converged at 0.65 and 0.59, respectively. The lift curve slope is 6.1/rad and this value is larger against the aspect ratio. The drag coefficient is larger than that of α=0deg. because of the induced drag. From these values, the airplane efficiency factor becomes 0.65.

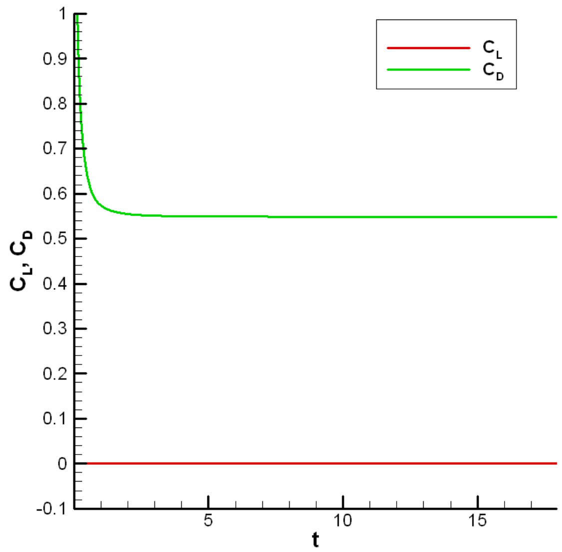

Fig. 2. The Aerodynamic Force (α = 0deg.)

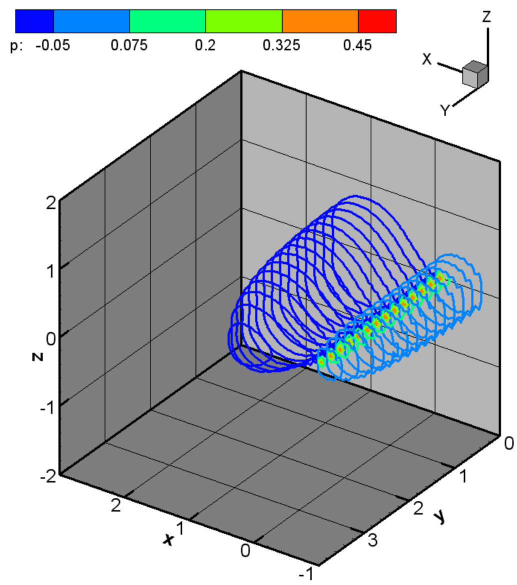

Fig. 3. The Pressure Contours (α = 0deg.)

The pressure contours and streamlines on the $y = 0$ plane are shown in Fig. 7. The contours become concentrated around the place where the plate exists and slope downward. The angle of the slope is equal to the angle of attack. The streamlines below the plate are parallel to it and those above it go upward to avoid it.

4 Conclusion

The flow around the three-dimensional flat plate is reproduced in the Cartesian grid by the Immersed Boundary method.

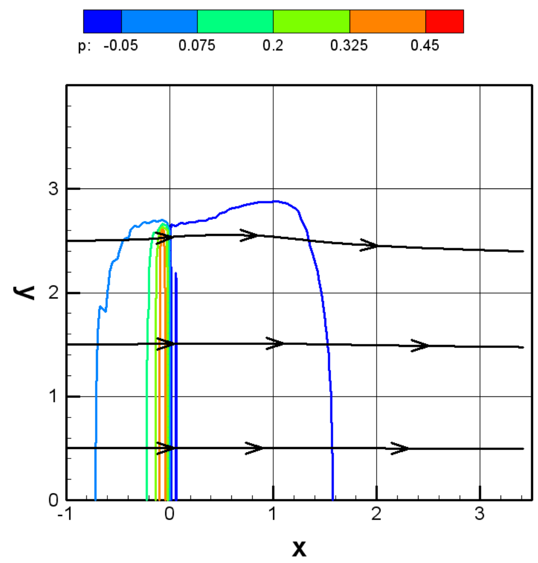

Fig. 4. The Pressure Contours and Streamlines (*z*=0)

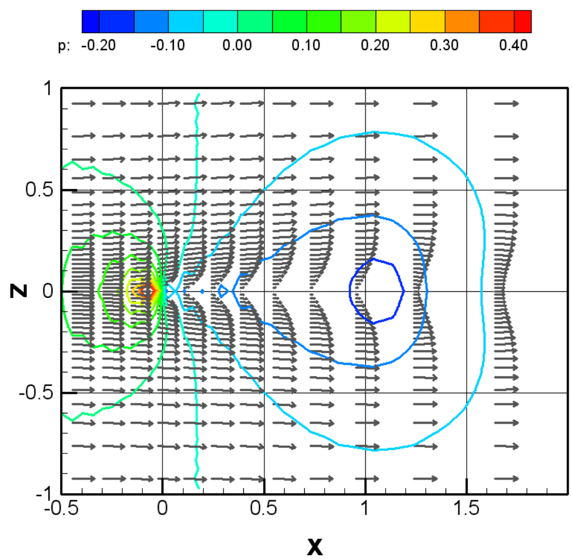

Fig. 5. The Pressure Contours and Velocity Vector (*y*=0)

The grid is not body-fitted but Cartesian. The velocity around the flat plate is forced to be that of the plate by the external force. The pressure contours, streamlines and velocity vectors show where the flat plate exists.

References

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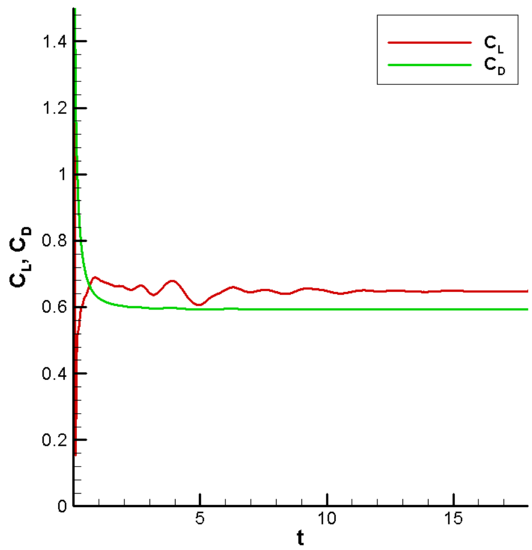

Fig. 6. The Aerodynamic Force (α = 6deg.)

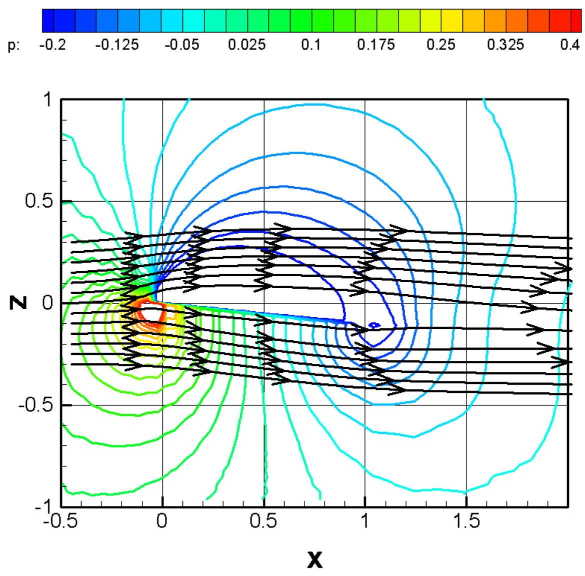

Fig. 7. The Pressure Contours and Streamlines (*y*=0)