MULTI-DIMENSION LAPLACE WAVELET AND THE APPLICATIONS IN FLIGHT FLUTTER TEST

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Keywords：Laplace wavelet，flight flutter test，filter，mode parameter identification

Abstract

From the view of mode parameter identification, Laplace wavelet and correlation filter are introduced in this these. An improved method called Multi-dimension Laplace wavelet is presented. The construction and implement of the wavelet are studied. Numerical simulation and true flight flutter test data are used to analyse the features of the method. The result show that the new wavelet technique are reasonable and feasible with the great application potential in the data processing after flight flutter test.

1 Introduction

For the flutter flight test, there are a lot of characteristics, for example, the excitation force being not enough , the test point being limited, the modes being too dense, nonlinearity, the effective data sample being short, singnal noise ratio(SNR) and mode stability level being low, which bring too much difficulties to the mode parameter identification， meanwhile, flutter flight test is a high risk ,long term and high cost program, which require the data reduction should be quick(realtime or quasi-realtime) and accurate，only in this way, we can shorten the test cycle , improving the test fidelity and efficiency and providing an accurate and conservative test result [1~3].

Wavelet analysis is an important branch in modern signal processing field, whose progress has close relation with the fourier analysis and has better advantage in some degree [4~6]. What is more, wavelet analysis can choose time-frequency segment flexibly according to different requirements of the data analysis. In fact, the signal is composed of the slow-changing steady signal and the quick-changing signal, the wavelet transform happen to satisfy this requirement, and meanwhile, high resolution and signal reconstruction methodology of the wavelet transform can solve the dense mode problem in some degree and ameliorate the SNR of the measured signal.

However, the general wavelet analysis also has the shortcomings as follows:

- Much data decomposing is needed, therefore, the amount of the computation is too big and the analysis process is complicated.
- The mode parameters needed cannot be obtained directly from the wavelet transform, after the signal is processed by use of wavelet transform, other parameter identification methods are still needed to make the mode estimation.
- The transient time-changing characteristics of the identified system can not be followed up dynamically.

This paper mainly study the multi-dimension Laplace wavelet, which make the related filtering of the pulse response signal data to decompose the signal , gaining the mode parameters directly and following up the dynamic features of the system [7~9]. In this paper, numerical features and engineering application of the method studied are validated and discussed by means of plentiful simulation data and flight test data [10].

2 Laplace Wavelet

Laplace wavelet Ψ is a complex and analytical exponential function， which is defined as follows：

$$\psi(\omega,\zeta,\tau,t)=\psi_\gamma(t)$$

$$=\begin{cases} Ae^{\frac{-\zeta}{\sqrt{1-\zeta^2}}\omega(t-\tau)} e^{-j\omega(t-\tau)} & t\in[\tau,\tau+T] \\ 0 & else \end{cases} \tag{1}$$

T is the tense supporting of the wavelet, having a limited length being not zero. Coefficient vector $\gamma=\{\omega,\zeta,\tau\}$, it determine the characteristics of the wavelet. These coefficient is also related to the dynamic features of the mode。 Where: $\omega\in R^+$, damping ratio $\zeta\in[0,1]\subset R^+$, time parameter $\tau\in R$, coefficient A is a random normalization factor.

Fig.1 is a schematics of Laplace wavelet. The real plane and imaginary plane are respectively (a) and (b) of Fig.2.

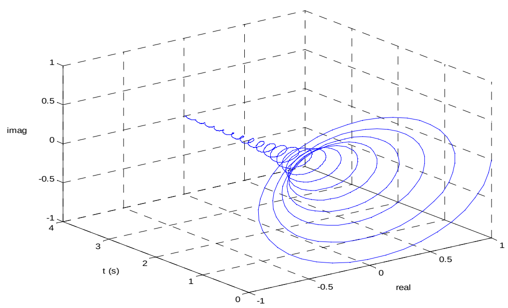

Fig.1 Laplace wavelet

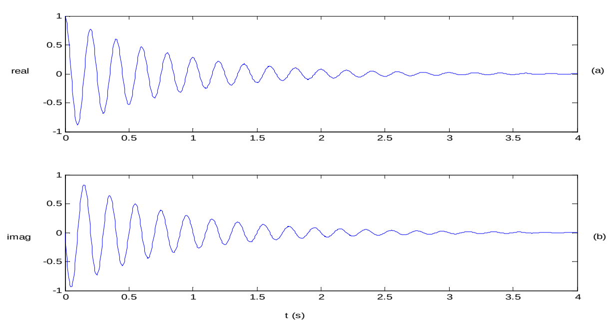

Fig.2 The real and imaginary projection of the Laplace wavelet

3 Multi-dimension Laplace Wavelet and Related Flitering

3.1 Multi-dimension Laplace Wavelet Definition

Multi-dimension Laplace wavelet is an extension of the Laplace wavelet, which is also a complex and analytical damped exponential function. Multi-dimension Laplace wavelet is defined as follows:

$$\psi(\omega_1,\zeta_1,\omega_2,\zeta_2,\ldots,\omega_n,\zeta_n\tau,t)=\psi_\gamma(t)$$

$$=\begin{cases} \sum_{i=1}^{n} A_i e^{\frac{-\zeta_i}{\sqrt{1-\zeta_i^2}}\omega_i(t-\tau)} e^{-j\omega_i(t-\tau)} & t\in[\tau,\tau+T] \\ 0 & else \end{cases} \tag{2}$$

where, n is the number of wavelet in the multi-dimension Laplace wavelet, T is the supporting length of the wavelet.

3.2 Related Filtering of Multi-dimension Laplace Wavelet

In order to describe an array of wavelet which can be used to make the signal decomposing, the concept of clan is introduced, clan is wave style database of the wavelet, here it refers to the wave style database of the multi-dimension wavelet. An array of certain wavelet parameters can determine a wavelet clan. Each parameter is a discrete vector.

$$\Omega_i=\{\omega_{i1},\omega_{i2},\cdots,\omega_{ip}\}\subset R^+ \tag{3}$$

$$Z_i=\{\zeta_{i1},\zeta_{i2},\cdots,\zeta_{iq}\}\in[0,1]\subset R^+ \tag{4}$$

$$TT=\{\tau_1,\tau_2,\cdots,\tau_m\}\subset R \tag{5}$$

Where, i=1,2,…,n， n is dimension of multi-dimension Laplace wavelet, Ω_i and Z_i is the frequency and damping parameter of the Laplace wavelet, TT is the displacement parameter of the Laplace wavelet in the time domain. Ψ is the wavelet clan，it's parameters are included in the following parameter collection:

$$\gamma=\{\omega_1,\zeta_1,\omega_2,\zeta_2,\cdots,\omega_n,\zeta_n,\tau\}\in\Gamma$$

$$=TT\times\prod_{i=1}^{n}(\Omega_i\times Z_i) \tag{6}$$

Define the correlation coefficient $k_r\in R$ between the multi-dimension Laplace wavelet and signal $f(t)$, it can be used to quantify the correlation degree between wavelet and signal.

$$k_\gamma=\sqrt{2}\frac{|\langle\psi_\gamma,f(t)\rangle|}{\|\psi_\gamma\|_2\|f\|_2} \tag{7}$$

By means of equation (7), it is possible to calculate the correlation value at every time point between signal *f(t)* and multi-dimension Laplace wavelet which is determined by the frequency and damping combination parameter vector $\{\omega_1,\zeta_1,\omega_2,\zeta_2,\cdots,\omega_n,\zeta_n\}$, meanwhile, the mode analysis can also be made. All of the elements k_r form a matrix K, whose dimension is determined by parameter vector $\{\omega_1,\zeta_1,\omega_2,\zeta_2,\cdots,\omega_n,\zeta_n,\tau\}$, that is 2n+1. The peak value of K is denoted as k_{max} , meanwhile, vecto $\{\overline{\omega_1},\overline{\zeta_1},\overline{\omega_2},\overline{\zeta_2},\cdots,\overline{\omega_n},\overline{\zeta_n},\overline{\tau}\}$ is defined as the parameter value of the Laplace wavelet when the k_γ equals k_{max}. For a given $\tau_0\in TT$, it is also possible to determine the peak value $k(\tau_0)$ of k_γ in the order of falling dimension.

$$\begin{aligned}k_{\max} &= \max(K)\\ &= k_{\{\overline{\omega_1},\overline{\zeta_1},\overline{\omega_2},\overline{\zeta_2},\cdots,\overline{\omega_n},\overline{\zeta_n},\overline{\tau}\}}\\ &= \max\left(k\left(\tau\right)\right)\end{aligned} \tag{8}$$

In the definition of k_γ, the normalization factor is set to be $\sqrt{2}$ in order that $k_{\max}$ can be 1.

4 Numerical Simulation

4.1 Simulation Mechanism

According to the mechanism of structure flutter, i.e. the coupling of multi-order structural mode, the following mathematical model generally can be used to simulate the dynamic response signal under pulse excitation:

$$f(t)=\begin{cases}\sum_{j=1}^{M} e^{-\omega_j\xi_j t}\sin(\omega_j(t-t_0))+Bn(t) & t\ge t_0\\ Bn(t) & t<t_0\end{cases} \tag{9}$$

Where: ω_j and ζ_j are the frequency and damping of structure mode, B is the noise magnitude coefficient, *n(t)* is the GAUSS white noise, M is the number of mode order.

4.2 The Application Characteristics Analysis of Multi-dimension Laplace Wavelet

4.2.1 Three Mode Simulation

Dynamic response signal of structure flutter test under pulse excitation is produced by numerical simulation, meanwhile, the application characteristics of multi-dimension Laplace wavelet is also studied.

At first, make the three mode simulation signal according to equation (9) which M is 3. *f(t)* is a real exponential damped sinusoidal signal which corresponds to three mode impact response signal. The mode parameters of the system are as follows: ω_1=8Hz、 ζ_1=0.04 、 ω_2=14Hz、 ζ_2=0.06、 ω_3=10Hz、 ζ_3=0.04. The noise magnitude coefficient B=0.01, signal sampling rate f_s=200Hz. According to the definition of multi-dimension Laplace wavelet, it could obtain a three dimension Laplace wavelet group. When n equals to 3 which can be used to analyse the above signal. Defining a group of parameter according equation (6) where n is 3, the interval of each parameter vector needn't be unanimous, which can be defined according to resolution requirement. Here, the vectors are defined as follows according equations (3)、(4) and (5):

Ω_1={5:0.5:18}　　Z_1={0.005:0.005:0.1}
Ω_2={5:0.5:18}　　Z_2={0.005:0.005:0.1}
Ω_3={5:0.5:18}　　Z_3={0.005:0.005:0.1}
TT={-2:0.1:2}

Through correlation filtering computation, a correlation coefficient matrix K can be obtained whose dimension is determined by the dimension of Ω_1、Z_1、Ω_2、Z_2、Ω_3、Z_3、and TT. Here, dim(Ω_1)=dim(Ω_2)=dim(Ω_3)=28, dim(Z_1)=dim(Z_2)=dim(Z_3)=20,dim(TT)=21.Mean while, the wavelet supporting length T used to make the correlation filtering computation equals to 3 seconds. K is a seven-dimension matrix. Computing the time point of the maximum value k_{max} of coefficient matrix K, On this time point, the three corresponding wavelet parameter are $\{\overline{\omega_1},\overline{\zeta_1}\}$, $\{\overline{\omega_2},\overline{\zeta_2}\}$ and $\{\overline{\omega_3},\overline{\zeta_3}\}$. Meanwhile, the initial time of mode response in signal *f(t)* can be determined, i.e. $t=\tau$.

Analyse and processing above simulation signal by use of three-dimension Laplace wavelet group, the sample length is $T\times f_s$=600.

Fig.3 shows the correlation filtering result of the Laplace wavelet and simulation signal. Fig.3 (a) is the simulation signal *f(t)*. Fig.3 (b) is the maximum value $k(\tau)$ of correlation coefficient. Fig.3 (c) and (d) are frequency and damping ratio's relation with τ of first order wavelet which corresponds to the max value of correlation coefficient. Fig.3 (e) and (f) are frequency and damping ratio's relation with τ of second order wavelet which corresponds to the maximum value of correlation coefficient. And Fig.3 (g) and (h) are frequency and damping ratio's relation with τ of third order wavelet which corresponds to the max value of correlation coefficient.

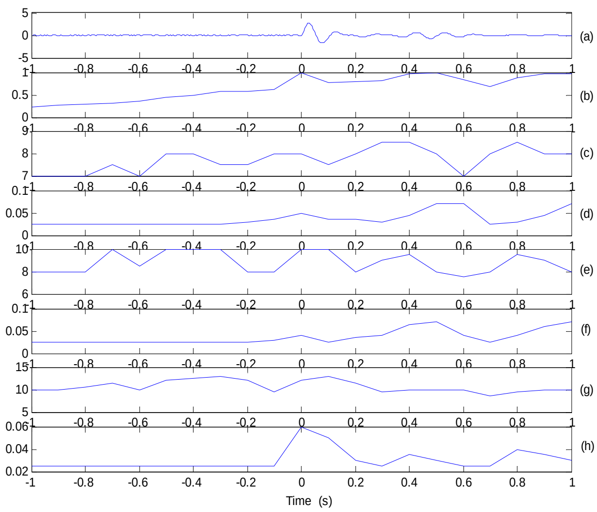

Fig.3 Three dimension Laplace wavelet group and the result curve of the correlation filtering to the three mode simulation signal

It could be known from Fig. 3 (b) that, with the simulation signal *f(t)* being increasingly covered by the multi-dimension Laplace wavelet, the maximum value k_{max} of correlation coefficient is gradually reaching 1. That is to say, when the supporting length of the wavelet more and more cover the response portion of the signal, the function of inner product in correlation filtering appears more and more clear .When the starting point of wavelet response and pulse response coincide with each other, $k(\tau)$ reaches the max value k_{max} for the first time. It shows from that time on, the simulation signal which length is 600 matches the dynamic characteristic of some wavelets in the wavelet group very much. That is to say, three-dimension Laplace wavelet parameter $\{\overline{\omega}_1,\overline{\zeta}_1,\overline{\omega}_2,\overline{\zeta}_2,\overline{\omega}_3,\overline{\zeta}_3\}$ which corresponds to k_{max} matches the three mode parameters $\{\omega_1,\zeta_1\}$, $\{\omega_2,\zeta_2\}$ and $\{\omega_3,\zeta_3\}$ of simulation signal *f(t)* very much. The wavelet parameter $\{\overline{\omega}_1,\overline{\zeta}_1,\overline{\omega}_2,\overline{\zeta}_2,\overline{\omega}_3,\overline{\zeta}_3\}$ is the three mode parameters of simulation signal. At the time τ corresponding to k_{max}, the information of mode parameters brought out by correlation filtering can be obtained from Fig.3. Table 1 shows the comparison between theory mode parameters and the identified mode parameters.

Table1 Three mode simulation result

	Theory value		Estimated value		Error	
sign	ω (Hz)	ζ	$\overline{\omega}$ (Hz)	$\overline{\zeta}$	$\Delta\omega$ (Hz)	$\Delta\zeta$
mode1	8	0.05	8	0.05	0	0
mode2	10	0.04	10	0.04	0	0
mode3	12	0.06	12	0.06	0	0

After correlation filtering has been completed to the pulse response of the system by useing of three-dimension Laplace wavelet, the whole process of system dynamic response can be obtained by following up the changes of $k(\tau)$.The wavelet parameters which corresponds to k_{max} give the mode parameters of the system. Therefore, the correlation filtering using Multi-dimension Laplace wavelet can be considered as mode filtering.

4.2.2 The effect of wavelet supporting length

During correlation filtering computation by use of Laplace wavelet, the wavelet supporting length is an important parameter, whose change will directly result in the change of Laplace wavelet length and affect the matrix *K* further.

The wavelet supporting length is defined as *T*=3s、2s、1s、0.5s respectively, use 2-dimension Laplace wavelet to make correlation filtering for the dual-mode simulation signal in equation (9) where M is 2. Table 2 shows the results.

We can see from Table 2 that the change of the wavelet supporting length T basically do not affect the mode identification result. Theoretically, as long as T contains at least one period of the minimum mode frequency, the mode frequency can be identified. But the mode damping is reflected in many periods of the signal, so the wavelet supporting length can not be too short, otherwise the mode damping ζ identified will have errors to some extent.

Table 2 The simulation results of the changeable supporting length

Comparing item	Mode	Theory value		Estimated value		Error	
		$\omega(Hz)$	ζ	$\overline{\omega}(Hz)$	$\overline{\zeta}$	$\Delta\omega(Hz)$	$\Delta\zeta$
T=3s	mode1	10	0.04	10	0.04	0	0
	mode 2	12	0.06	12	0.06	0	0
T=2s	mode 1	10	0.04	10	0.04	0	0
	mode 2	12	0.06	12	0.06	0	0
T=1s	mode 1	10	0.04	10	0.04	0	0
	mode 2	12	0.06	12	0.06	0	0
T=0.5s	mode 1	10	0.04	10	0.035	0	0.005
	mode 2	12	0.06	12	0.055	0	0.005

4.2.3 Dense mode analysis

In the actual program, the case in which two or more mode frequency are very close can be often met, this is the dense mode problem. Here, we set the second-order mode as example, and analyse two-dimension Laplace wavelet's resolution capacity to the dense mode.

The wavelet supporting length T=3s is chosen for calculating. A set of parameters according equation (6) where n is 2 need to be defined. For the Ⅰ、Ⅱ、Ⅲ simulation signal computation, the given parameter vectors are defined as follows according equations (3)、(4) and (5):

$\Omega_1=\{6.25:0.25:14\}$, $Z_1=\{0.005:0.005:0.1\}$

$\Omega_2=\{6.25:0.25:14\}$, $Z_2=\{0.005:0.005:0.1\}$

$TT=\{-2:0.1:2\}$

For the Ⅳ simulation signal computation, the parameter vectors are defined as follows:

$\Omega_1=\{6:0.1:10\}$, $Z_1=\{0.002:0.002:0.1\}$

$\Omega_2=\{6:0.1:10\}$, $Z_2=\{0.002:0.002:0.1\}$

$TT=\{-2:0.1:2\}$

Use the above Laplace wavelet parameter to make the correlation filtering computaion, the result obtained is shown in Table 3.

Table 3 Simulation results for the effect of dense modes

Comparing item	Sign	Theory value		Estimated value		Error	
		$\omega(Hz)$	ζ	$\overline{\omega}(Hz)$	$\overline{\zeta}$	$\Delta\omega(Hz)$	$\Delta\zeta$
set Ⅰ	mode 1	10	0.04	10	0.04	0	0
	mode 2	14	0.06	14	0.06	0	0
set Ⅱ	mode 1	10	0.04	10	0.04	0	0
	mode 2	12	0.06	12	0.06	0	0
setⅢ	mode 1	8	0.04	8	0.04	0	0
	mode 2	9	0.06	9	0.06	0	0
setⅣ	mode 1	8	0.04	7.9	0.046	0.1	0.006
	mode 2	8.5	0.06	8.3	0.054	0.2	0.006

It can be seen from Table 3 that, for the dense mode, the two-dimension Laplace wavelet correlation filtering algorithm can obtain satisfactory result.

4.2.4 The effect of noise for mode identification

The signal noise ratio (SNR) is defined as follows：

$$SNR=10\times\log(Ey/Ex) \quad (10)$$

Where, Ey is the energy of the signal and Ex is the energy of the noise.

Fig.4 shows the calculating result curves when the simulation signal is added 50% noise. It is a double mode problem using two-dimension Laplace wavelet group to make the signal analysis and processing. Fig.4 (a) is the simulation signal $f(t)$, Fig.4 (b) is the maximum of the correlation coefficient $k(\tau)$, Fig4 (c) and (d) is frequency and damping ratio's relation with τ of first order wavelet which corresponds to the maximum value of the correlation coefficient, Fig. 4 (e) and (f) is frequency and damping ratio's relation with τ of second order wavelet which corresponds to the maximum value of the correlation coefficient. The identified and theoretical mode parameters are compared in Table 4.

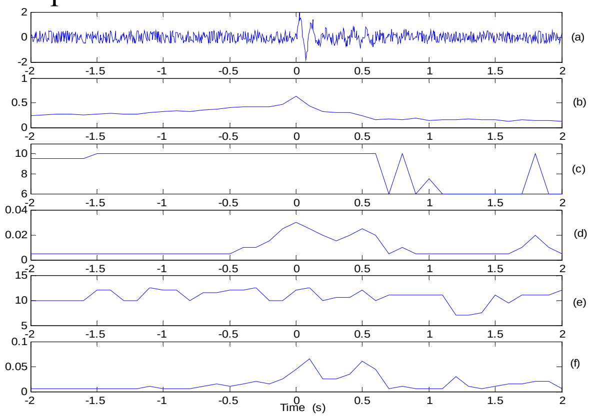

Fig.4 The calculating results of the simulation signal added 50% noise.

Table 4 The simulation results of the noise effect

Noise coefficients	SNR	Mode	Theory value		Estimated value		Error	
B	SNR		$\omega(Hz)$	ζ	$\overline{\omega}(Hz)$	$\overline{\zeta}$	$\Delta\omega(Hz)$	$\Delta\zeta$
0.05	18.71	mode1	10	0.04	10	0.04	0	0
		mode 2	12	0.06	12	0.06	0	0
0.1	12.83	mode 1	10	0.04	10	0.04	0	0
		mode 2	12	0.06	12	0.055	0	0005
0.2	6.41	mode 1	10	0.04	10	0.04	0	0
		mode 2	12	0.06	12	0.055	0	0.005
0.3	3.34	mode 1	10	0.04	10	0.035	0	0.005
		mode 2	12	0.06	12	0.055	0	0.005
0.4	0.63	mode 1	10	0.04	10	0.04	0	0
		mode 2	12	0.06	12	0.06	0	0
0.5	-1.36	mode 1	10	0.04	10	0.03	0	0.01
		mode 2	12	0.06	12	0.045	0	0.015

It can be seen from above result, the change of SNR will not greatly affect mode frequency estimation during using two-dimension Laplace wavelet. With the SNR decreasing, k_{max} and the confidence are also decreasing, However, the mode frequency identified is accurate and the damping estimated is basically to be steady.

5 Data Analysis for Flight Flutter Test

In this section, the two-dimension Laplace wavelet is used to make the flutter flight test data analysis and processing for some aircraft. The configuration of the tested aircraft is as follows: two heavy bombs on the middle and outboard carrying point respectively. The test use bonkers for excitation, which are arranged on the middle and outboard bombs, the vibration sensors on the bombs are used to measure the structural response.

Fig.5(a) shows the structural response measured by vertical sensor when the bonker on the outboard bombs is excited, the flight altitude is 5km and the Mach number is 0.749. Figure 5(c) 、(d)、 (e) and(f) shows the identified structure mode result after using two-dimension

Laplace wavelet correlation filtering. According to the correlation coefficient k_{max} in Fig 5(b), we can confirm that the mode frequency is 10.25 Hz and the damping is 0.07.

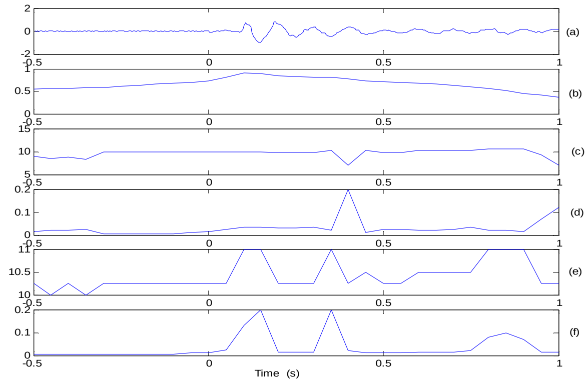

Fig.5 The signal results on the outboard bomb when the flight altitude is 5km and the mach number is 0.749

Fig.6 (a) shows the structural response measured by vertical sensor when the bonker on the outboard bombs is excited, the flight altitude is 5km and the mach number is 0.914. Fig.6 (c) 、(d)、 (e) and (f) shows the identified structure mode result using two-dimension Laplace wavelet correlation filtering. According to the correlation coefficient k_{max} in Fig. 6(b), we can confirm that the mode frequency is 10.25 Hz and the mode damping is 0.08.

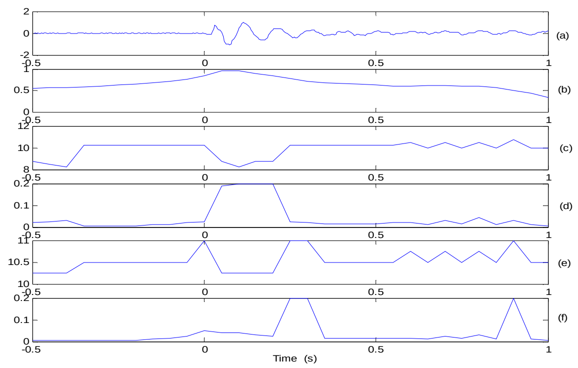

Fig.6 The signal results on the outboard bomb when the flight altitude is 5km and the mach number is 0.914

Fig.7 shows the classical result curves of the relation between the symmetrical pitching structure mode frequency and damping and mach number, the flight altitude is 5km.The damping is higher than 0.03, the damping changing tendency indicates that the aircraft have enough flutter margin.

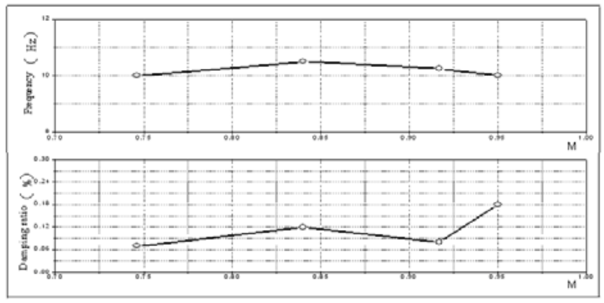

Fig.7 The relation of the identified symmetrical pitching structure mode frequency and damping with mach number for a certain aircraft at 5km

6 Conclusion

In order to improve and ameliorate modern flutter flight test data reduction technology , This paper study the method by which wavelet can be introduced to the flutter flight test data reduction according to the basic characteristics and reduction requirement of the flutter flight test data. Research shows : the multi-dimension Laplace wavelet correlation filtering algorithm brought out in this paper is applied to the analysis of the pulse response signal and retraction of mode parameters, which has great prospect for the future flutter flight test.

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