Application of Gaussian Complex Wavelet in PIO Detection

TIAN Fu-li, GAO Zheng-hong
Northwestern Polytechnic University, China

YU Zhi-gang
Chinese Flight Test Establishment, China

Key Words: *Wavelet Time-Frequency Analysis, Hyperbola-Gaussian Complex Wavelet, Pilot Induced Oscillation Detection*

Abstract

The validity and feasibility of Gaussian complex wavelet time-frequency analysis for identifying the Pilot Induced Oscillations (PIO) are studied. The time-frequency relationship of Gaussian complex wavelet is researched, and both Gaussian and Hyperbola-Gaussian complex wavelet filters are structured. A set of the character and judging rules for identifying PIO is determined. An event of flight recorded lateral PIO time history are processed and analyzed for demonstrating the feasibility of real time PIO detection. The results indicate that both Gaussian and Hyperbola-Gaussian complex wavelets can identify PIO accurately, but the later is better in real-time PIO detecting. The roll rate is more effective than roll angle in timely PIO detection.

Introduction

PIO is a kind of oscillations caused by unfavorable pilot-vehicle interactions, which will greatly threaten the flight safety. During the last decades, although the great progress was made in the development of PIO design criteria, the predictive evaluation of pilot in the loop flight simulation and flight test, PIO suppression & prevents technique and others, it was still quite difficult to eliminate PIO. Therefore, since the late of 20 century, the great efforts have been made to explore PIO identification techniques and further to develop the on-board detection and compensation devices [1] [2]. The safety of flight test can be ensured by implementing the PIO real-time on-board warning device in the phase of the development flight test of new type aircraft. PIO detection adopts the pattern-recognition method, which generally consists of data acquisition, data preprocess, features extraction and classifying decision.

This paper primarily refers to the methods of PIO features extraction and PIO detection, including both Gaussian and hyperbola-Gaussian complex wavelet transforms definition of PIO features, PIO identification.

The early study of PIO features extraction was based on the Fourier transform method. It was found that even the windowed Fourier transform (WFT) still exist some shortcomings with its fixed window width. So the wavelet transform has been considered as a compromised way to solve the localization contradiction between time and frequency domain.

It is presented in this paper that two ways of detecting PIO onset with complex wavelet transforms for lateral stick displacement and the roll angle or roll rate. It also studied on the validity and feasibility of extracting PIO features.

1 Wavelet Transform

1.1 Definition of wavelet transform

Definition 1: When $\psi \in L^2 \cap L^1$, and $\hat{\psi}(0)=0$ ($\hat{\psi}$ is the ψ of Fourier transform), the family of functions generated in the following formula can be called analysis or continuous wavelet, and ψ can be called the mother wavelet. A continuous wavelet is defined as

$$\psi_{a,b}(t)=|a|^{-\frac{1}{2}}\psi\left(\frac{t-b}{a}\right) \quad b\in \mathbf{R}, a\in \mathbf{R}-\{0\}$$

Definition 2: If ψ is the mother wavelet, and $\{\psi_{a,b}\}$ is the continuous wavelet, when $f\in L^2$, the continuous wavelet transform will be defined as

$$W_f(a,b)=\langle f,\psi_{a,b}\rangle=|a|^{-\frac{1}{2}}\int_R f(t)\overline{\psi\left(\frac{t-b}{a}\right)}dt$$

Definition 3: If $\psi \in L^2 \cap L^1$, and

$$C_\psi = \int_R \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty$$

, ψ will be an admissible wavelet, and the above formula will be admissibility condition. According to the basic definition [3] of the wavelet transform, Both Gaussian complex wavelet and hyperbola-Gaussian complex wavelet have been presented for the on-line detecting and off-line analyzing of PIO.

1.2 Gaussian complex wavelet

Establish a mother wavelet function as

$$\psi(t) = \frac{1}{2\sqrt{\pi}} e^{-\left(\frac{t}{2}\right)^2 + j\mu t}$$

So that its corresponding continuous wavelet function will be

$$\psi_{a,b}(t) = |4\pi a|^{-\frac{1}{2}} e^{-\left(\frac{t-b}{2a}\right)^2 + j\mu\left(\frac{t-b}{a}\right)}$$

Through Fourier transforming, its frequency domain expression can be written as

$$\hat{\psi}_{a,b}(\omega) 2a\sqrt{\pi}\, e^{-\left(\frac{\omega-\frac{1}{a}\mu}{\frac{1}{a}}\right)^2 - jb\omega}$$

Figure 1 illustrates the shape of a Gaussian complex wavelet and its spectrum as the changing of scale *a* (*b*=40).

.

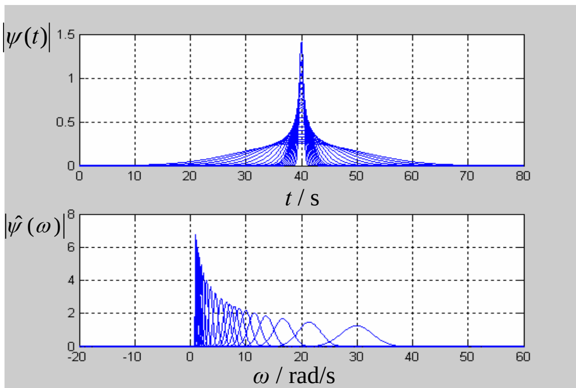

Fig. 1. Time Wave Pattern and Spectral Shape of Gaussian Complex Wavelet

The continuous wavelet transform of the function $f \in L^2$ is

$$W_f(a,b) = |a|^{\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \overline{\frac{1}{2\sqrt{\pi}} e^{-\left(\frac{t-b}{2a}\right)^2 + j\frac{\mu}{a}(t-b)}} dt$$

1.3 Hyperbola-Gaussian Complex Wavelet

Figure 1 show that while representing the signal feature with Gaussian complex wavelet, it is necessary to take the data information of the left and right sides of time point b. The lower the frequency, the bigger the time span will be. The time delay caused by transforming the data of right side of time b is not acceptable for PIO warning and compensation. Therefore, in this paper, a kind of hyperbola-Gaussian complex wavelet is generated through the time domain transform from (-∞, 0) to (-∞,∞).

$$\psi_{a,b}(t) = \begin{cases} |4\pi a\sigma|^{-\frac{1}{2}} e^{-\left(\frac{b-t-\frac{c}{b-t}}{2a\sigma}\right)^2 + j\mu\left(\frac{b-t-\frac{c}{b-t}}{a}\right)} & t < b \\ 0 & t \ge b \end{cases}$$

Figure 2 illustrates the shape of hyperbola-Gaussian complex wavelet and its spectrum as the changing of scale *a* (*b*=50).

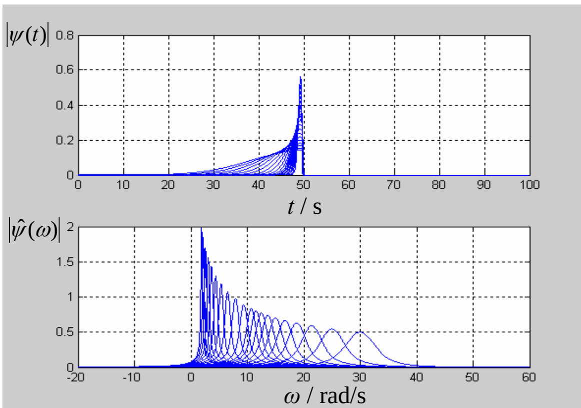

Fig. 2. Time Wave Pattern and Spectral Shape of Hyperbola-Gaussian Complex Wavelet

It is easy to see that the wavelet shape is almost similar to that of Gaussian complex wavelet of the left side of time point b. The later example application verified that it is available for hyperbola- Gaussian complex wavelet transform to realize on-line extraction of PIO characters. The extraction calculations with complex wavelet transform is so time-wasting that the FIR (Finite Impulse Response) filter arithmetic be adopted for ensuring real time PIO detecting. The expression of FIR filter is as follows:

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

Where x is the input, y is the complex wavelet transform of x, and h is the conjugated wavelet function of N samples.

2 Character and Judging Rules of PIO Oscillation

The characters and judging rules, established for PIO detecting from the stick displacement and roll angle or roll rate, are as follows:

1) The main frequency range of identifying PIO is generally between 0.3 to 1.8 Hz. Of course, the narrower the frequency range of interest, the more specific the identification criteria can be.
2) Magnitudes of the complex wavelet transform of the lateral stick displacement and the roll angle or the roll rate must be over certain values.
3) Absolute value of the phase difference of the complex wavelet transform of the roll angle and the lateral stick displacement are over 100°. As it approaching to 180°, the PIO trend will become more obviously. However, the phase difference of the complex wavelet transform of the roll rate and the lateral stick displacement are at about -90°.
4) The magnitude change rates with time of the complex wavelet transform of lateral stick displacement and roll angle or the roll rate are equal to or greater than 0.
5) The correlation degree of the magnitudes of complex wavelet transform of the control input and the aircraft response are at least greater than 50%. In practical, the threshold of the correlation degree is determined by the simulation tests.

The expression of the correlation degree for lateral stick displacement and the roll angle is defined as

$$R(b) = \frac{\sum_{a_{\inf}}^{a_{\sup}} \left|W_{X_r}(a,b)\right| \left|W_{\phi}(a,b)\right|}{\sqrt{\sum_{a_{\inf}}^{a_{\sup}} \left|W_{X_r}(a,b)\right|^2 \left(\sum_{a_{\inf}}^{a_{\sup}} \left|W_{\phi}(a,b)\right|^2\right)}} \quad (1)$$

Where $a_{\sup}$ is the maximum scale of wavelet transform, and $a_{\inf}$ is the minimum scale of wavelet transform. $\left|W_{Xr}(a,b)\right|$ is the element of the magnitude matrix of wavelet transform of the lateral stick displacement $X_r(t)$. $\left|W_{\phi}(a,b)\right|$ is the element of the magnitude matrix of wavelet transform of the roll angle $\phi(t)$.

The correlation degree for the lateral stick displacement and the roll rate is expressed as

$$R(b) = \frac{\sum_{a_{\inf}}^{a_{\sup}} \left|W_{X_r}(a,b)\right| \left|W_{p}(a,b)\right|}{\sqrt{\sum_{a_{\inf}}^{a_{\sup}} \left|W_{X_r}(a,b)\right|^2 \sum_{a_{\inf}}^{a_{\sup}} \left|W_{p}(a,b)\right|^2}} \quad (2)$$

In the formula (2), $\left|W_p(a,b)\right|$ represents the element of the magnitude matrix of the wavelet transform of the roll rate $p(t)$.

3 Calculation Methods for Time Frequency Plane of PIO oscillation Signals

3.1 Description of General Procedure

The results of complex wavelet transform of the control input and the aircraft response are complex matrices. The row elements of these matrices relate to the frequency at the maximum of the wavelet spectrum corresponding to different scale a, and their column elements relate to different time point b, so these matrices are called as time-frequency matrices. For APC identification, some time-frequency matrices are derived from the complex wavelet transforms of the control input and the aircraft response. The magnitude matrices take absolute values of the complex wavelet transform of the control input and aircraft response as their elements. The phase difference matrices take the phase differences of the complex wavelet transform of the control input and aircraft response as their elements. The magnitude change rate matrices are obtained through numerically differentiate each rows of the magnitude matrices, both for the control input and the aircraft response. The correlation degree matrices are constructed by the correlation degree of the wavelet transform magnitudes of the control input and the aircraft response. Multiplying all the relative elements of these matrices, the resulting time-frequency matrix can be used for APC identification. Thus a time-frequency plane for APC identification can be drawn.

3.2 The Calculation Methods for the Lateral Stick Displacement and Roll Angle

The calculation formula of an element of the time-frequency matrix for PIO identification from the lateral stick displacement and roll angle is as follows:

$$W_c(f_c,b)=\left|W_{X_r}(f_c,b)\right|\cdot\left|W_\phi(f_c,b)\right|\cdot C_{ph}(f_c,b)\cdot C_{dW_{X_r}}(f_c,b)\cdot C_{dW\phi}(f_c,b)\cdot R(f_c,b)$$

Where f_c is the frequency at the maximum of the wavelet spectrum with scale a, and indicates the longitudinal coordinate instead of a.

$\left|W_{X_r}(f_c,b)\right|$, $\left|W_\phi(f_c,b)\right|$ are magnitudes of the complex wavelet transform of the lateral stick displacement $X_r(t)$ and the roll angle $\phi(t)$ respectively.

$$C_{\Delta\varphi}(f_c,b)=\begin{cases}\left|\Delta\varphi\right|-\dfrac{\pi}{2} & \left|\Delta\varphi\right|>\dfrac{\pi}{2}\\ 0 & \left|\Delta\varphi\right|\le\dfrac{\pi}{2}\end{cases}$$

Where $\Delta\varphi=\angle W_\phi(f_c,b)-\angle W_{X_r}(f_c,b)$ and the variation range is from -π to +π. $C_{\Delta\varphi}(f_c,b)$ is the criterion of phase difference.

$$C_{dW_{X_r}}(f_c,b)=\begin{cases}1 & \dfrac{d\left|W_{X_r}(f_c,b)\right|}{dt\cdot k}>0.8\\ \dfrac{d\left|W_{X_r}(f_c,b)\right|}{dt\cdot 1.6k}+0.5 & otherwise\\ 0 & \dfrac{d\left|W_{X_r}(f_c,b)\right|}{dt\cdot k}<-0.8\end{cases}$$

Formula $C_{dW_{X_r}}(f_c,b)$ is the criterion of the magnitude change rate of the wavelet transform for lateral stick displacement.

$$C_{dW_\phi}(f_c,b)=\begin{cases}1 & \dfrac{d\left|W_\phi(f_c,b)\right|}{dt\cdot k}>0.8\\ \dfrac{d\left|W_\phi(f_c,b)\right|}{dt\cdot 1.6k}+0.5 & otherwise\\ 0 & \dfrac{d\left|W_\phi(f_c,b)\right|}{dt\cdot k}<-0.8\end{cases}$$

Formula $C_{dW_\phi}(f_c,b)$ is the criterion of the magnitude change rate of the complex wavelet transform for the roll angle. Formula $R(f_c,b)$ is the correlation degree matrix of the complex wavelet transform of lateral stick displacement and the roll angle. All the rows of the matrix are the same. The value of $R(b)$ is calculated by formula (1).

3.3 The Calculation Methods for the Lateral Stick Displacement and Roll Rate

The calculation formula of an element of the time-frequency matrix for PIO identification from the lateral stick displacement and roll Rate is as follows:

$$W_c(f_c,b)=F_{W_{X_r}}(f_c,b)\cdot F_{W_p}(f_c,b)\cdot F_{\Delta\varphi}(f_c,b)\cdot F_{dW_{X_r}}(f_c,b)\cdot F_{dW\phi}(f_c,b)\cdot F_R(f_c,b)$$

Where f_c is the frequency at the maximum of the wavelet spectrum with scale a, and indicates the longitudinal coordinate instead of a.

The expression of the judging criterion of lateral stick displacement is as follows:

$$F_{W_{X_r}}(f_c,b)=\begin{cases}\left|W_{X_r}(f_c,b)\right|-T_{W_{X_r}} & \left|W_{X_r}(f_c,b)\right|>T_{W_{X_r}}\\ 0 & \left|W_{X_r}(f_c,b)\right|\le T_{W_{X_r}}\end{cases}$$

Item $\left|W_{X_r}(f_c,b)\right|$ is the magnitude of wavelet transform of the lateral stick displacement $X_r(t)$. $T_{W_{X_r}}$ is the threshold of stick displacement signal and determined by the simulation tests.

The expression of roll rate and judging criterion is as follows:

$$F_{W_p}(f_c,b)=\begin{cases}\left|W_p(f_c,b)\right|-T_{W_p} & \left|W_p(f_c,b)\right|>T_{W_p}\\ 0 & \left|W_p(f_c,b)\right|\le T_{W_p}\end{cases}$$

Formula $\left|W_p(f_c,b)\right|$ is the magnitude of wavelet transform of the roll rate $p(t)$. T_{W_p} is the threshold of signal and chosen by the simulation tests.

The phase difference of the wavelet transform of roll rate and lateral stick displacement is expressed as $\Delta\varphi = \angle W_{\phi}(f_c, b) - \angle W_{X_r}(f_c, b)$, $\Delta\varphi$ varies from 0 to 2π. The judging criteria expression is as follows

$$F_{\Delta\varphi}(f_c, b) = \begin{cases} 1 & \frac{5\pi}{4} < |\Delta\varphi| > \frac{7\pi}{4} \\ 0 & otherwise \end{cases}$$

The slope of small window recursion regression straight line of wavelet transform magnitude is used to approximate $\frac{d|W_{X_r}(f_c, b)|}{dt}$ and $\frac{d|W_p(f_c, b)|}{dt\cdot}$, which are the magnitude change rate of wavelet transform of roll rate and lateral stick displacement. The window widths are 1/5 of which of the wavelets. The expression of judging criteria of wavelet transform magnitude change rate of lateral stick displacement is as follows:

$$F_{dW_{X_r}}(f_c, b) = \begin{cases} 1 & \frac{d|W_{X_r}(f_c, b)|}{dt} > T_{dW_{X_r}} \\ 0.5 & otherwise \\ 0 & \frac{d|W_{X_r}(f_c, b)|}{dt} < -T_{dW_{X_r}} \end{cases}$$

$T_{dW_{X_r}}$ is the signal threshold set by the simulation tests.

The expression of judging criterion of wavelet transform magnitude change rate of roll rate is as follows:

$$F_{dW_p}(f_c, b) = \begin{cases} 1 & \frac{d|W_p(f_c, b)|}{dt} > T_{dW_p} \\ 0.5 & otherwise \\ 0 & \frac{d|W_p(f_c, b)|}{dt} < -T_{dW_p} \end{cases}$$

T_{dW_p} is the threshold and determined by the simulation tests.

The Formula of the correlation judging criterion of wavelet transform magnitude of lateral stick displacement and roll rate is as follows:

$$F_R(f_c, b) = \begin{cases} 1 & R(b) > 0.85 \\ 0 & R(b) \le 0.85 \end{cases}$$

Each row of the judging logic matrix is the same. $R(b)$ is calculated by formula (2).

4. Application Example

4.1 Explanation of PIO Event

As an example of the application of above methods, consider the information presented in Figure 3 and Figure 4. The example is a typical of the PIO experienced in a fighter flight test of approach and landing phase. The time history of its control response is shown in Figure 3 and Figure 4. It can be seen that the first cycle of the lateral constant oscillation was

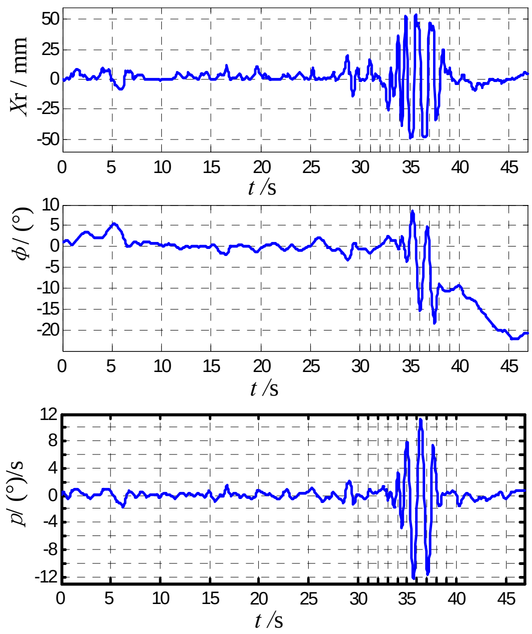

Fig.3. Time History of Lateral Stick Position, Roll Angle and Roll Rate

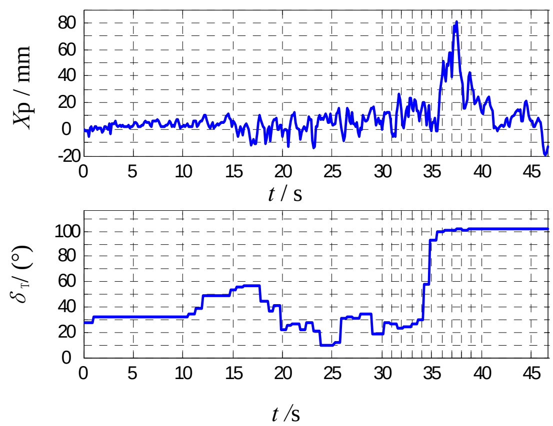

Fig.4. Time Histories of Positions of Pitch Stick and Throttle position

approximately from 32.85 to 33.45 second, the second cycle of divergent oscillation from 33.45 to 34.2 second and the third cycle of divergent oscillation from 34.2 to 35.3 second. The pilot then pushed the throttle forward at about 34.3 second. The fourth cycle of the Constant oscillation was approximately from 35.3 to 36.8second while the stick Displacement reached its right limit position. The pilot pulled the stick at 35.6 second and then started to push forward at 37.5 second. The fifth cycle of oscillation started to converge at 36.8 second and meanwhile the pilot started to reduce the lateral control amplitude. The oscillation almost stopped at 38.5 second. The pilot took the right action to control aircraft, recovering from PIO, to go around and then approached and landed successfully.

4.2 Post-event Analysis of Time Frequency Characteristics with Gaussian Complex Wavelet Transforms

Figure 5 is a time-frequency plane generated from the calculation of whole PIO time history data with the Gaussian complex wavelet transform.

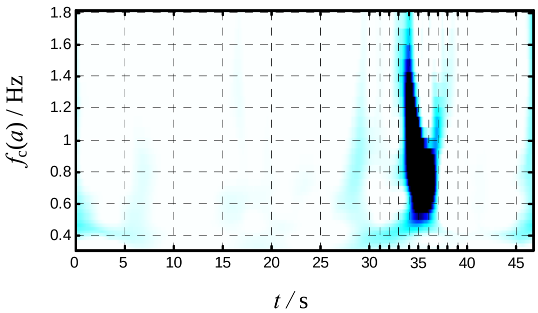

Fig. 5. Gaussian Complex Wavelet Detection of Roll Angle

It is showed that PIO oscillation indication signal is sent out at 33.5 second (the second oscillation cycle), and the indication signal ceases at 36.8 second (the fifth oscillation cycle). In addition, the frequencies at the maximum of the wavelets spectrum corresponding to scale *a*, are approximately from 0.5Hz to 1.6 Hz. According to the analysis of the time histories, the first cycle time of oscillation is approximately 0.6 second (which equals to around 1.7 Hz), the second one 0.75 second (1.33 Hz), the third one 1.1 second (0.9 Hz), the fourth one 1.5 second (0.6 Hz) and the fifth one 1.2 second (0.83 Hz). It is concluded from above analysis that the time-frequency characters identification of PIO oscillation with the Gaussian complex wavelet transform is effective and exact, but the above calculation and analysis are the off-line and non-real time process. For on-line timely detection of PIO, the hyperbola-Gaussian complex wavelet FIR filter was combined with PIO judging rules to generate the PIO indication of time-frequency plane. The thresholds of the judging rules should be defined reasonably; otherwise, the algorithm for PIO detection can not operate effectively.

4.3 On-line Detection of Onset of PIO from Roll Angle with Hyperbola-Gaussian Complex Wavelet Transforms

Figure 6 illustrates the PIO indication of time-frequency plane obtained from hyperbola-Gaussian complex wavelet FIR filter with PIO judging rules for the stick displacement and the roll angle.

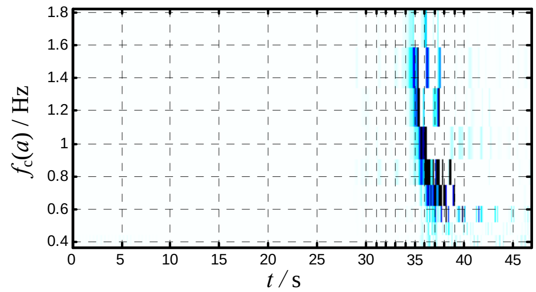

Fig. 6. Hyperbola-Gaussian Complex Wavelet FIR Detection of Roll Angle

It is found that the indication signal of PIO start sending out at 34.4 second (slightly over the second cycle of the oscillation), with 1.5 Hz frequency at the maximum of wavelet spectrum. Meanwhile, the roll angle of the aircraft is approximately 2.5° at 34.2 second and - 3.5° at 34.7 second, it is not so large that pilot can control aircraft to recover from PIO. The results show that the time-frequency plane can exactly reflect PIO characteristic for detecting. It is further indicated that the time of sending out PIO signal is almost identical to the actual moment for pilot starting to take control action. During the above mentioned event of PIO, the test pilot, at right time, took the right action to control aircraft to go around and then landed successfully. It can be said that the detection of PIO occurring is effective and timely by use of the hyperbola-Gaussian complex wavelet FIR filter with PIO judging rules for the stick displacement and the roll angle, although there exists some false signal in the time-frequency plane at low frequency (about 0.5Hz) after PIO disappeared, and it can be removed by improving the algorithm of PIO features extraction and identification criteria.

4.4 On-line Detection of Onset of PIO from Roll Rate with Hyperbola-Gaussian Complex Wavelet Transforms

It is shown in Figure 7 that the time-frequency characteristic is calculated by hyperbola-Gaussian complex wavelet FIR filter with PIO judging rules for the stick displacement and the roll rate.

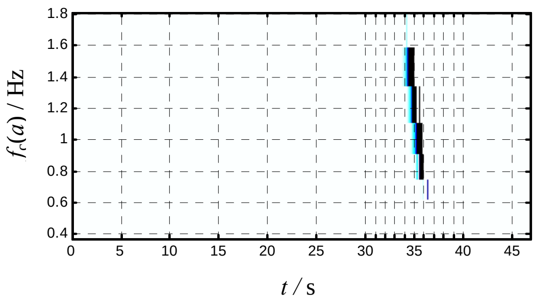

Fig. 7 Hyperbola-Gaussian Complex Wavelet FIR Detection of Roll Rate

The indication signal of PIO oscillation comes out at 33.8 second (about 1.5 oscillation cycle of roll angle), with 1.5 Hz frequency at the maximum of wavelet spectrum, just 0.4 second before the pilot started to take measuring to control aircraft. The PIO indication signal vanishes at approximate 36.4 second. There is no false signal at a low frequency for PIO detecting with the stick displacement and the roll rate, while it exists with the stick displacement and the roll angle. This might be attributed to the phase leading of the roll rate to the roll angle by $\pi/2$. In addition, the steady value of the roll rate is zero, which can avoid the false alarm of PIO due to the low frequency spectrum of the wavelet transform from the non-zero value of the steady roll angle. In MATLAB environment, it takes only several milliseconds for the PIO detection calculation of one time scale.

With the PIO judging rules for the signals of roll rate and roll angle respectively, hyperbola-Gaussian complex wavelet FIR filter is applied to process the time history record of the second successful approach and landing after going round. The result shows that there is no any indication of PIO oscillation.

5 Conclusions

Despite the great efforts of aircraft designers, especially flight control law designers, even the significant progress in the development of PIO design criteria, the predictive evaluation test technique of pilot in the loop and PIO suppression & prevents techniques, it is still quite difficult to eliminate PIO. Therefore, the detection and compensation schemes might have been an effective way to cope with the problem of PIO. The purpose of this paper was to make a tentative research on the real time identification of PIO. The main conclusions are as follows:

1) In the post-event analysis, the Gaussian complex wavelet transform have proven effective at identifying PIO features, the onset of PIO, as well as the frequency, phase and magnitude characteristic of the event.
2) The hyperbola-Gaussian complex wavelet transform, combining with the defined PIO identification criteria, is better in timely detection of PIO than Gaussian one. The identification with the roll rate and the lateral stick displacement is better than the roll angle and the lateral stick displacement for the real time PIO detection.
3) The effect of PIO detection can be further improved by properly upgrading the wavelet transform algorithm, the definition of the PIO features and the design of identification criteria.
4) It is suggested that the further researches be carried out on the structures for the classifier and recognizer of PIO features, PIO fuzzy logic classifier model, PIO detecting neural network and other techniques, so that an on-board device can be developed for the feasible and effective PIO detection as soon as possible.

REFERENCES

[1] Geffrey J Jeram, J V R Prasad. Fuzzy Logic Detector for Pilot Induced Oscillation [R]. *Proceedings of the American Helicopter Society 59th Annual Forum* [C]. Phoenix, Arizona, USA May, 2003.

[2] David G. Mitchell, David H. Klyde. Recommended Practices for Exposing Pilot – Induced Oscillations or Tendencies in the Development Process. AIAA 2004-6810. 2004.

[3] Qin Qianaing, Yang Zongkai. *Utility Wavelet Analysis*. Xi'an Electronic Technique.University, 1994.

Writers' introduction:

Tian Fu-li, male, born in 1962, is a senior researcher with doctor degree, specialized in flight dynamics and control.

Gao Zheng-hong, female, born in 1959, is a professor with doctor degree, specialized in flight dynamics.

Yu Zhi-gang, male, born in 1945, is a senior engineer with master degree, specialized in flight simulation.