A PRACTICAL MULTIDISCIPLINARY DESIGN OPTIMIZATION ALGORITHM BASED ON THE UNIFORM DESIGN THEORY

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Abstract:

Collaborative optimization (CO) and concurrent sub-space optimization (CSSO) are two typical multidisciplinary optimization (MDO) algorithms. Both of them are bi-level approaches including one system-level optimization and several discipline-level optimizations, which have been proved to be effective by some examples. However, this paper reveals the low computational efficiency of these two algorithms caused by discipline-level optimizations. To solve this problem, the computational framework of CSSO is analyzed, which indicates that the purpose of the discipline-level optimizations is only to provide good sample points for constructing response surface (RS) models. According to this important conclusion, the uniform design (UD) theory was introduced into CSSO to develop a more practical algorithm named discipline-level analysis system-level optimization (DASO). In this algorithm, discipline-level optimizations are replaced by UD to provide good sample points, which dramatically reduces the computational amount. In the last section, an analytical example and a transporter wing design problem are successfully solved by DASO.

1 Introduction

In conventional optimization strategy, an optimizer directly deals with a system analysis, that is, at each iteration, a system analysis is to be performed at least one time. This strategy is suitable for a simple system whose analysis is also simple. However, for a complex system, this strategy will cause low computational efficiency. Take a wing structural design for example. For a complex aircraft wing structure, the simplified beam theory is no longer applicable; instead, the finite element method (FEM) has to be utilized. It normally takes several hours to perform a structural FEM analysis for a wing even after adopting many simplifications. In the wing structural optimization design, at each iteration, if we have to do such an analysis at least one time until convergence is achieved, we can imagine how time-consuming and inefficient this optimization process is. For a multidisciplinary problem, the condition is even worse because some other complex disciplinary analyses such CFD also have to be performed at each iteration. Moreover, complex information exchanging exits between different disciplines [1].

Multidisciplinary optimization (MDO) is a promising strategy for solving complex optimization design problems. Collaborative optimization (CO) and Concurrent subspace optimization (CSSO) are two typical multidisciplinary optimization algorithms, which have been proved to be effective by some examples [2-4]. However, in this paper it will be shown that these two algorithms still have some shortcomings in solving a complex design problem due to discipline-level optimizations. By analyzing the iterative process of CSSO, a new computational framework based on the uniform design (UD) theory is put forward. In the last section an analytical example and a transporter wing design shows the effectiveness of this new method.

2 Computational Problems

Both CO and CSSO are bi-level approaches including one system-level optimization and several subsystem optimizations. In CO and CSSO, an original complex optimization problem is decomposed into several relatively simple problems. This bi-level strategy has proved to be effective by some demonstrated examples. But for a complex design, the problem of low computational efficiency still exists in CO and CSSO. The analysis is as follows.

Consider a wing design with two disciplines of aerodynamics and structure. In CSSO, the original design problem is decomposed into an aerodynamic optimization, a structural optimization and a system-level optimization. In the system-level optimization, all disciplinary constraints are replaced by response surface (RS) model constraints, that is to say, in the system-level optimization no complex disciplinary analysis is necessary. However, in the structural optimization, although the aerodynamic analysis is replaced by RS model, the optimizer still directly deals with full FEM analyses. Similarly, in the aerodynamic optimization, the optimizer still directly deals with full CFD analyses. In CO, it is the same condition that although no complex disciplinary analysis is necessary in system-level optimization, in aerodynamic optimization and structural optimization, optimizer still directly deals with CFD and FEM analyses respectively. As a result, the above-mentioned problem of low computational efficiency still exists in CO and CSSO.

2 Analysis of the CSSO Computational Framework

It has been concluded from the above analysis that in CO and CSSO, the problem of low computational efficiency still exists due to discipline-level optimizations. In this section, we will reveal the purpose of the discipline-level optimizations in CSSO by examining the iterative process of CSSO, shown in Fig.1 [4].

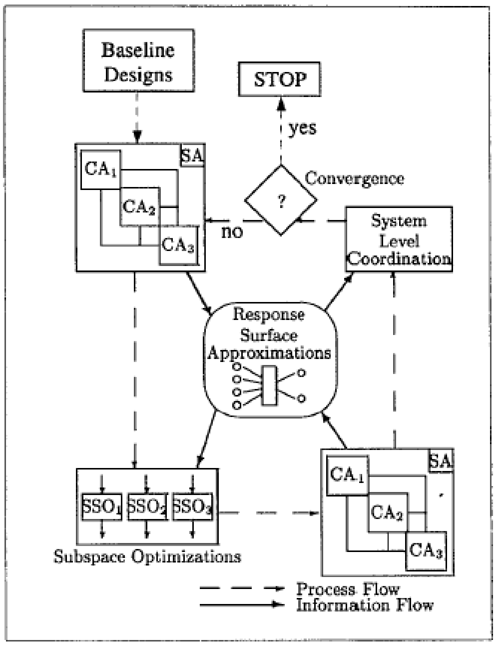

Fig.1. Iterative Process of CSSO

There are several notable considerations in the CSSO computational framework:

- The convergence criterion is set for system-level optimization and the results are also provided by system level
- No discipline-level optimization (also called subspace optimization) but RS model directly links with system-level optimization.

From the above two points we can find an interesting problem that since the computational results are provided by system-level optimization and no discipline-level optimization relates to system-level optimization, what is the purpose of discipline-level optimizations?

To answer this question, let's investigate the computational process of CSSO shown in Fig.1. After the discipline-level optimizations are finished, system analyses are performed at the discipline-level optimum design points and the results of these analyses are added to the system database. Then a refined RS model is constructed based on the expanded database. Next, a system-level optimization is performed by using the refined RS models. Thus, it is clear that the purpose of each discipline-level optimization is just to provide a sample point to expand the system database. Compared with random selected sample points, the sample points provided by the discipline-level optimizations are generally closer to the real optimum design point. Using these sample points to construct RS model is helpful to accelerate the convergence. However, it is an expensive cost to obtain such good sample points because we have to perform many discipline-level optimizations.

Thus, if we can obtain good sample points without discipline-level optimization, computational cost will be dramatically reduced. Based on such a concept, an improved CSSO computational framework named discipline-level analysis system-level optimization (DASO) is put forward. DASO exploits the uniform design theory and the main difference between DASO and CSSO is that in DASO what all optimizers deal with are only RS models instead of disciplinary analyses. Details of this method will be further discussed in the next section.

3 DASO Method

3.1 Uniform Design Theory

It is easy to understand that for the same number of sample points, a set of sample points which are uniformly scattered in the design space can more comprehensively represent the original system. Therefore, it is reasonable to utilize the uniformly distributed sample points to construct a global approximate model.

For a one-dimensional design space, it is intuitive to understand the uniformity of distribution and it is easy to determine a set of uniformly scattered sample points. However, for a general multi-dimensional problem, how to measure the uniformity of distribution and how to obtain a set of uniformly scattered sample points are two complex mathematical problems. Uniform design theory is a powerful tool to solve these problems.

A brief description of UD theory is as follows.

Suppose $\{X_k, k=1\cdots n\}$ are n points in an s-dimensional design space C^s, where $X_k=(x_1\cdots x_s)_k$. For any X in C^s, let $v(X)=x_1*x_2*\cdots*x_s$ be the volume of a hyper-rectangle $[0,X]$. n_x is the number of points of $\{X_k, k=1\cdots n\}$ which lies in $[0,X]$. Then

$$D(\{X_k, k = 1 \cdots n\}) = \sup_{X \in C^s} \left| \frac{n_x}{n} - v(X) \right| \qquad (1)$$

is called the discrepancy of $\{X_k, k = 1 \cdots n\}$ in C^s. A uniform design requires that the absolute value of ratio of the number of points lying in the hyper-rectangle $[0, X]$ and the total number of points of the set minus the volume of the hyper-rectangle should be small.

To determine a set of uniform design points is also an optimization process, which involves some complex mathematical theories. Fortunately, like orthogonal design tables, tables of uniform design have been constructed, which is convenient for engineering application. Table 1 illustrates a 3-factor 4-level and 8-experiment UD table $U_8(4^3)$. For the details of UD theory, references [5,6] are recommended.

Table1 UD Table $U_8(4^3)$

No.	x_1	X_2	x_3
1	1	1	3
2	1	3	2
3	2	2	1
4	2	4	4
5	3	2	4
6	3	4	1
7	4	1	2
8	4	3	3

3.2 Framework of DASO

As shown in Fig.2, in this method, the sample points are firstly selected by using uniform design sampling; then the system analyses at these points are performed and the results are used to build the database of the object of design; next, the RS model is constructed and the optimization is performed based on the RS model. If it is not satisfy the convergence criterion, system analysis is performed at the optimum point and the results are added to the database. Then a refined RS model is constructed based on the expanded database and a system-level optimization is performed based on the refined RS model. The above-described iteration is repeated until convergence is achieved.

Compared with the framework in Fig.1, in Fig.2, optimizers are completely separated from disciplinary analysis; that is to say, during each optimization process only RS model evaluation is needed instead of complex disciplinary analyses, which dramatically decrease the computational amount. Moreover there is another advantage for this strategy that it avoids the complex interfaces between optimizer and different disciplinary analyses, which facilitates programming.

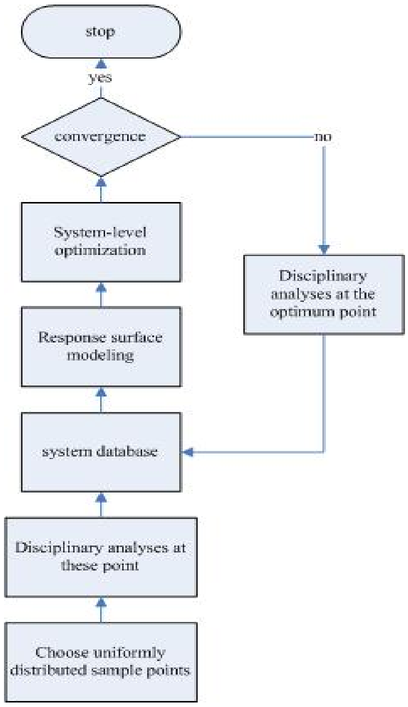

Fig.2. Framework of DASO

4 Examples

4.1 An Analytical Example

This is an analytical example taken from [4]

$$\min \quad f = x_2^2 + x_3 + y_1 + e^{-y_2} \qquad (2)$$

$$s.t. \quad c_1 = \frac{y_1}{8} - 1 \geq 0 \qquad (3)$$

$$c_2 = 1 - \frac{y_2}{10} \geq 0 \qquad (4)$$

$$y_1 = x_1^2 + x_2 + x_3 - 0.2 y_2 \qquad (5)$$

$$y_2 = \sqrt{y_1} + x_1 + x_3 \qquad (6)$$

$$-10 \leq x_1 \leq 10, \quad 0 \leq x_2 \leq 10, \quad 0 \leq x_3 \leq 10$$

where (5) and (6) represent two different disciplinary analyses respectively. The provided optimum design point is (3.03 0 0)

A set of 8 uniformly distributed Sample points is chosen, shown in table 1. RS model is constructed by neural network. Fig.3 is the convergence history of DASO and the optimum point is (3.0024 0 0). Compared with CSSO, the computational amount of DASO is much less because of the elimination of discipline-level optimizations. 184 disciplinary analyses are needed in CSSO, while in DASO only 82.

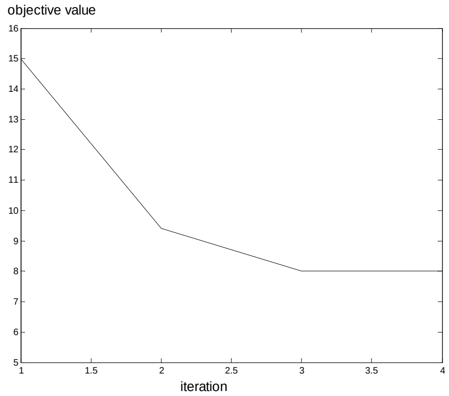

Fig.3 Convergence History

4.2 Transporter Wing Design

In this example, the DASO method is applied to a transporter wing design problem with two disciplines of aerodynamics and structure. The design point is the cruise condition: altitude 10000m, velocity 0.76M

The optimization problem is stated as follows

$$\min \ W(\boldsymbol{X})$$

$$\text{s.t.} \quad 1 - \frac{\sigma_1(\boldsymbol{X})}{\sigma_b} > 0 \quad 1 - \frac{\sigma_2(\boldsymbol{X})}{\sigma_b} > 0$$

$$1 - \frac{\sigma_3(\boldsymbol{X})}{\sigma_b} > 0 \quad 1 - \frac{\sigma_4(\boldsymbol{X})}{\sigma_b} > 0$$

$$1 - \frac{\delta(\boldsymbol{X})}{\delta_{\text{al}}} > 0 \quad 1 - \frac{r(\boldsymbol{X})}{r_{\text{al}}} < 0 \qquad (7)$$

Where $W(\boldsymbol{X})$ is the weight of the wing; $\sigma_1(\boldsymbol{X})$ is the maximum stress of the upper skin; $\sigma_2(\boldsymbol{X})$ is the maximum stress of the lower skin; $\sigma_3(\boldsymbol{X})$ is the maximum stress of the front spar; $\sigma_4(\boldsymbol{X})$ is the maximum stress of the rear spar; σ_{b} is the material allowable stress; $\delta(\boldsymbol{X})$ is the tip displacement; δ_{a} is the allowable displacement; $r(\boldsymbol{X})$ is the lift-drag ratio; r_{a} is the allowable lift-drag ratio.

After mathematical derivation, the objective is equivalent to the minimization of the material volume of the wing.

The design variables are shown in table 1.

Table 1 Design Variables

Design variables	Upper bounds	Lower bounds
Span l/m	18	12
Sweepback angle θ/rad	0.611	0.349
Taper ratio λ	0.4	0.2
Dihedral angle ϕ/rad	0.122	0
Thickness of upper skin d_1/mm	25	1
Thickness of lower skin d_2/mm	25	1
Thickness of front spar d_3/mm	25	1
Thickness of rear spar d_4/mm	25	1

Fig.4 and Fig.5 illustrate the aerodynamic analysis model and the structural analysis model respectively. The aerodynamic analysis is performed by the Quasi-Simultaneous

Viscous-Inviscid method; the structural analysis is performed under ANSYS environment.

The design problem is solved successfully by the DASO method. The iterative process is shown in Fig.6.

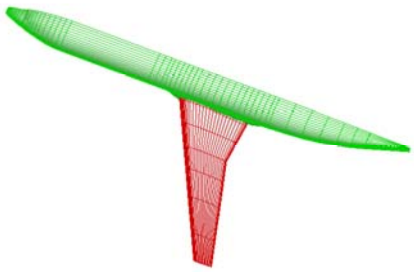

Fig.4. Aerodynamic Model

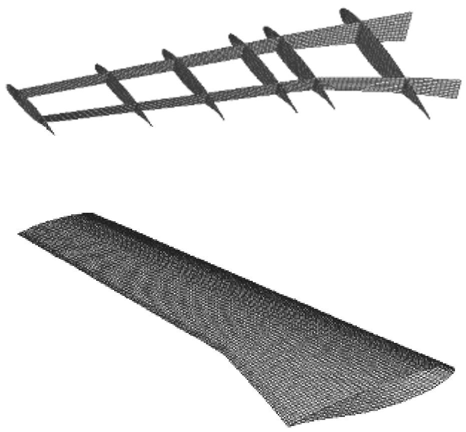

Fig.5. Structural Model

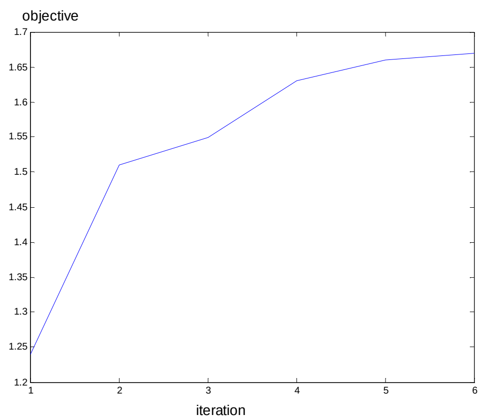

Fig.6. Convergence History

5 Conclusion

1 For a complex system design, optimizer directly dealing with disciplinary analysis will cause low computational efficiency.

2 In the disciplinary optimizations of CO and CSSO, optimizer still directly deals with disciplinary analysis, which makes CO and CSSO inefficient in solving complex problems

3 By analyzing the CSSO iterative process, it is concluded that the purpose of discipline-level optimizations in CSSO is only to provide good sample points.

4 Based on this conclusion, framework of DASO is put forward by using UD theory. In this framework, all disciplinary analyses are performed outside the optimization process, which dramatically lowers the computational cost.

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