OPTIMISATION OF COMPOSITE AIRCRAFT PANELS USING EFFICIENCY ENHANCING EVOLUTIONARY ALGORITHMS

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Abstract

The use of Evolutionary Algorithms is a promising approach to solve complex optimisation problems such as the minimum weight design of composite aircraft structures. The main drawback of these probabilistic algorithms is the large number of design configurations that have to be analysed. Two methods are presented that enhance the efficiency of Evolutionary Algorithms: an adaptive adjustment of optimisation parameters as well as an approximate evaluation of design alternatives using a support vector machine classification. It is shown that both have the capability to reduce the computational effort considerably.

1 Introduction

One of the most important issues in the development of commercial transport aircraft is the reduction of the structural weight. To achieve this aim, both the structural design configuration as well as the selection of materials have to be considered in the structural optimisation process in addition to geometrical data. This problem becomes even more complex with composite structures, where laminate stacking sequences have to be taken into account. Thus, the design problem involves a combination of continuous and discrete variables. Typical examples of discrete variables are the ply thickness and ply orientations. Practical constraints mean that the ply angles usually have to be selected from a discrete set such as 0, 90 and ±45 degrees. Other discrete variables are different design configurations as well as the number and type of stiffeners, whereas geometrical dimensions usually are continuous design parameters. To handle this kind of problems for composite fuselage design, an optimisation code named GEOpS/F (*Genetic and Evolutionary Optimisation of Structures / Fuselage*) has been developed at the Institute of Aerospace Engineering of TU Dresden in collaboration with Airbus. In this code both closed form solutions as well as finite element analyses are employed to evaluate the strength and buckling behaviour of fuselage panels.

Considering different design configurations as well as the material selection in the optimisation requires the handling of discrete variables, resulting in a discrete design space. Therefore, the calculation of derivatives is not possible. Additionally, a highly non-linear and non-convex problem with a global and several local optima has to be solved. The result of this optimisation is often not simply a single point in the design space but more a complete set of different design alternatives. Also, from a practical view quite often, not only the optimal solutions but also near optimal ones are of interest. All these aspects have to be considered when selecting the appropriate optimisation algorithms. To solve such problems the class of population based *evolutionary algorithms* (EA) has been shown to be well suited.

Several investigations described in the literature which consider EA deal with the discrete optimisation of laminate stacking sequences, e.g. [1][2][4][5][6][7][8][9][11]. Furthermore in the majority of the above given papers ([1][2][4][5][6][8][9][11]) the discrete stacking sequence is optimised in combination with continuous geometrical design variables. In the optimisation code GEOpS/F different types of EA (*genetic algorithms*, *evolution strategies* and *differential evolution*) are combined and used in parallel in a single optimisation analysis [6]. Because of this combined application, the number of parameters which are required to control the algorithms increases considerably. In addition, it has to be considered that the appropriateness of the parameter settings changes during the optimisation process. At the beginning of the optimisation run, completely different parameter values are required for an extensive search in the design space than in the final precise, adjustment stage. However, the suggested parameter adaptation approach benefits from the advantages of the different algorithm types during the several stages of the whole optimisation process and avoids their drawbacks.

The main drawback of EA is the large number of design alternatives which have to be evaluated, particularly if the finite element method is employed. Therefore, in addition to parallel computing on multiprocessor computers and workstation clusters, other approaches are required to reduce the computational effort. In [1] [2] [7] binary trees are used to avoid a replicated evaluation of the same design alternatives. But this approach is only applicable to discrete design variables, because continuous design spaces contain an unlimited number of different alternatives. If there are, alongside the discrete design variables, only a few continuous ones, it is advantageous to use a combination of a binary tree and a spline approximation [1] or a response surface approach [2]. However, these methods are also restricted, because, for every set of the discrete variables, a sufficient number of design points for each continuous variable is required.

In this paper an approximation method is presented which involves discrete and continuous variables in parallel. Using this approach, finite element analyses are completely avoided during intermediate calculations. The approach is based on a data classification between feasible and infeasible design regions. Afterwards, in both regions, a separate approximation of the objective function is used. Appropriate classification methods are *neuronal networks* [12] or *support vector machines* (SVM) [8]. An SVM approach with the special capability to classify also laminated composite design alternatives is implemented in the GEOpS toolbox [6]. The basic investigations in [5] are extended in GEOpS/F to find the optimal design of composite fuselage panels involving a complete design configuration, geometry and laminate stacking sequence optimisation.

2 Structural optimisation

2.1 Fundamentals

Structural optimisation involves the task of finding the best design or design alternatives, taking into account given restrictions. In order to achieve this aim, the design has to be changed. The design vector $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ contains the design variables x_i such as geometrical and material data. Depending on the type of problem, design parameters are expressed either as continuous or discrete variables. Discrete variables are often merely an identification number, describing the design configuration or the used material. The n design variables constitute the design space Ω:

$$x_i \in D_i \subset R^1, i = 1(1)n$$

$$\Omega := D_1 \times D_2 \times \ldots \times D_n \subset R^n.$$

To evaluate the different design alternatives, the objective function f:$\Omega \rightarrow R$ is introduced, which has to be minimised:

$$\min f(\mathbf{x}) \text{ and } \mathbf{x} \in \Omega. \qquad (1)$$

In case of the fuselage panels considered here the aim is to find a design with a minimal structural weight $w(\mathbf{x})$. Furthermore, the design space Ω is restricted by inequality and equality constraints:

$$\mathbf{g}(\mathbf{x}) \geq \mathbf{0}, \mathbf{g}(\mathbf{x}) = \left(g_1(\mathbf{x}), g_2(\mathbf{x}), \ldots, g_m(\mathbf{x})\right)^T, \mathbf{g}: \Omega \rightarrow R^m$$

$$\mathbf{h}(\mathbf{x}) = \mathbf{0}, \mathbf{h}(\mathbf{x}) = \left(h_1(\mathbf{x}), h_2(\mathbf{x}), \ldots, h_l(\mathbf{x})\right)^T, \mathbf{h}: \Omega \rightarrow R^l.$$

In the structural optimisation such constraints are stress and strain allowables, deformation limits and buckling loads. Thus, we have to formulate the constraint optimisation problem:

$$\min w(\mathbf{x}) \text{ and } \mathbf{x} \in \Omega, \mathbf{g}(\mathbf{x}) \geq \mathbf{0}, \mathbf{h}(\mathbf{x}) = \mathbf{0}. \qquad (2)$$

Regarding the constraints, the whole design space Ω is divided into two disjoint sets, a feasible region Ω_f and an infeasible one Ω_{inf}:

$$\Omega_f := \{\mathbf{x} \in \Omega \mid \mathbf{g}(\mathbf{x}) \geq \mathbf{0}, \mathbf{h}(\mathbf{x}) = \mathbf{0}\} \subseteq \Omega$$

$$\Omega_{inf} := \Omega \setminus \Omega_f.$$

For the problem considered here the constraints are not available in a closed form. Therefore, a penalty approach [3] is applied. A positive penalty term $P(\mathbf{g}(\mathbf{x}),\mathbf{h}(\mathbf{x}))$ is added to the structural weight $w(\mathbf{x})$ to get the objective function $f(\mathbf{x})$, if constraints are violated:

$$f(\mathbf{x}) := \begin{cases} w(\mathbf{x}) & \forall \mathbf{x} \in \Omega_f \\ w(\mathbf{x}) + P(\mathbf{g}(\mathbf{x}), \mathbf{h}(\mathbf{x})) & \forall \mathbf{x} \in \Omega_{inf}. \end{cases} \qquad (3)$$

2.2 Evolutionary algorithms

Evolutionary algorithms mimic the principles of the biological evolution process. The optimisation starts from a population of different design solutions. Based on the design information from the parent individuals, an offspring population is created by using several evolutionary operators. The selection of the better individuals leads to a progress in the optimisation run. The selection is based only on the computed values of the objective function. Therefore, no derivative information is required. This makes it possible to find optimal solutions in discontinuous design spaces with combined discrete and continuous design variables.

Nevertheless, the basic principle is the same for all types of evolutionary algorithms, they differ considerably in the coding of the design variables and the way operators are working. *Genetic algorithms* (GA) are based on binary coded design variables, which are combined in a so-called chromosome. These strings are modified by using special operators in order to find better solutions. Applying crossover, the main operator of the GA, string parts of different individuals are changed between each other. The swapping of single binary bits is done by the bit mutation operator. The newly generated design alternatives are considered in the following selection process in which the new parent population is formed. The creation of new individuals and the selection process alternate until a stop criterion terminates the optimisation run. Because of the binary coding of the design variables, the GA are well suited to discrete and combinatorial problems.

To search in design spaces with combined continuous and discrete design variables, the application of *evolution strategies* (ES) is more preferable. These methods are based on a real valued coding. The mutation operator uses a Gaussian distribution centred at the point of the original design alternative. The step size is then evaluated to create a new individual. Small step sizes are very common. Large ones are rare, but possible. The so-called recombination operator exchanges design information between several individuals. Single design values from specific individuals are taken over or mean values are computed. Afterwards, a selection operation follows works in the same way as with *genetic algorithms*.

Another type of evolutionary algorithms is the *differential evolution* (DE) which is closely related to the ES. The operators of the DE are also based on real coded design variables. The determination of the step size in the mutation process involves the computation of differential vectors between the design points of the parent individuals. An increasing homogeneity in the population causes a reduction of the step size and finally enforces a precise adjustment of the optimised individuals in the final phase of the optimisation. So the DE represents an intermediate state between the stochastic evolutionary algorithm types such as GA and ES and purely deterministic mathematical algorithms. The DE is well suited for non-convex continuous problems and offers advantages for local search.

With the aim to combine the advantages of the different evolutionary algorithm types, the three above mentioned algorithms (GA, ES and DE) are used in parallel in the developed optimisation code GEOpS/F.

3 Adjustment of the algorithm parameters

The parallel use of different types of *evolutionary algorithms* results in the problem that a large number of parameters is required to control the algorithms. These parameters have a considerable influence on the course of the optimisation. Hence, the parameter selection affects strongly the number of generations required to achieve good results. Typical examples for control parameters are the number of parent and offspring individuals or the number of offspring individuals that are created by each type of operator. In case of the crossover and recombination operators it has to be determined, how many offspring alternatives are created using a specified number of parent individuals. In the ES mutation process the standard deviation of each design variable controls the step size. For the bit mutation in the GA a probability has to be given, with which the bits are switched. In case of the DE the number of differential vectors and diverse factors and constants have to be defined. Not only the operators that change design information of the individuals need quite a number of parameter settings. Also the selection operators are controlled by parameters. For example, it has to be chosen between an objective function based and a ranking based selection. Additionally, tournament selection methods are also implemented in GEOpS/F, involving a smaller group of pre-selected individuals which compete with each other. A further optimisation parameter defines, whether only the offspring individuals or both the parent and the offspring ones are used during the selection process. Furthermore, it has to be recognised, that the optimal parameter settings are changing during the optimisation process.

Taking these aspects into consideration a parameter adaptation dependent on the optimisation progress seems to be a promising approach to enhance the efficiency of *evolutionary algorithms*.

3.1 Predefined adjustment

In the case of a predefined adjustment in GEOpS/F, a list of parameter sets containing the above mentioned optimisation parameters has to be provided. The sets are arranged so, that the optimisation run can start with an extensive search in the global design space. Subsequently, a fast reduction of the objective function values is enforced and the last set controls a precise adjustment of the final solution. After a predefined number of generations the next parameter set is chosen without any relation to the optimisation progress. Furthermore, before starting the optimisation run the user has to know, what are the effects of the different parameter sets. This knowledge is necessary in order to find the right order in the list.

3.2 Adaptive adjustment

Another approach in GEOpS/F is an adaptive adjustment of the parameter sets. In comparison to the predefined method, here the exchange of the parameter set is done in accordance with the optimisation progress. After a specified number of generations the forecasted and the actually reached status are compared and then the optimisation program itself decides if it goes upwards or downwards in the parameter list in order to chose the next set. Only the optimisation start is an exception to this rule. To ensure at the beginning a wide and extensive search in whole the design space, the optimisation starts with the first parameter set. Then immediately a set in the middle of the list is chosen in order to provide enough potential for the parameter adjustment in all directions. To evaluate the optimisation progress, appropriate criteria have to be defined. For this purpose in GEOpS/F the distribution of the minimal objective function values related to the generation number and the heterogeneity of the population is used. The heterogeneity $het(G)$ of the generation G involves the differences of all design variables x_k comparing the individuals i and j:

$$het(G) = \frac{2 \sum_{i=1}^{\#ind-1} \sum_{j=i+1}^{\#ind} \sum_{k=1}^{\#\mathrm{var}} \Delta x_k^{ij}}{\#ind \cdot (\#ind - 1) \cdot \#\mathrm{var}} \quad (4)$$

To get an independent criterion, the sum of the differences is related to the number of comparisons (with the population size *#ind* including the parent and offspring individuals) and to the number of variables *#var*.

If a composite design has to be optimised, the heterogeneity involves the laminate stacking sequence of the population members including the number of plies and the orientation of each ply. In the modified formulation of equation (4)

$$het(G) = \frac{2 \sum_{i=1}^{\#ind-1} \sum_{j=i+1}^{\#ind} \left(\Delta\#S_{ij} + \sum_{k=1}^{\min(\#S_i,\#S_j)} O_{ijk} \right)}{\#ind \cdot (\#ind - 1) \cdot (1 + \#S_{\max})}$$

with

$$O_{ik} = O_{jk} \rightarrow O_{ijk} = 0$$
$$O_{ik} \neq O_{jk} \rightarrow O_{ijk} = 1 \quad (5)$$

the stacking sequences of two individuals i and j are compared. $\Delta\#S_{ij}$ denotes the difference in the ply number, $\#S_{max}$ the maximum number of predefined plies and O_{ij} the orientation of a specific ply. Furthermore, the plies with differing orientations are summed up using the variable O_{ijk}. The number of orientations which can be compared is limited by the number of plies of the thinner laminate.

Equation (5) considers only the heterogeneity of the laminate stacking sequences of the individuals in the current population. If the design space contains both variables defining the laminate stacking sequence as well as geometrical data, the equations (4) and (5) have to be used in combination.

3.2.1 *Progression of the objective function values*

To evaluate the progression of the optimisation in terms of the minimal objective function values $f_{min}(G)$ of generation G a regression function is determined using the last exact values. Based on this regression $f_{reg}(G,C_i)$ with the constant regression factors C_i the minimal objective function value is predicted for a specified number of generations. Afterwards, the exact and predicted values are compared in order to assess the appropriateness of the used optimisation parameter set.

As a first approach the following regression function is introduced:

$$f_{reg}(G, C_i, i=1,2) = C_1 \cdot G^{-1} + C_2 \quad (6)$$

This function enforces a strong reduction of the objective function values $f_{min}(G)$ in the starting phase of the optimisation process. In the final phase only a precise adjustment is allowed. Fig. 1 shows the basic shape of the function and the influence of the parameters C_i.

To start the optimisation run with an extensive search in the design space another approach is more appropriate. A sigmoid function based on the hyperbolic tangent is used in GEOpS/F:

$$f_{reg}(G, C_i, i=1(1)4) = -C_4 \cdot \tanh\left(C_2\left(G - C_1\right)\right) + C_3$$

$$= C_4\left[\frac{\exp\left(-C_2\left(G-C_1\right)\right) - \exp\left(C_2\left(G-C_1\right)\right)}{\exp\left(-C_2\left(G-C_1\right)\right) + \exp\left(C_2\left(G-C_1\right)\right)}\right] + C_3$$

with

$$\forall i : C_i \geq 0. \quad (7)$$

In a similar manner to the previous approach, the influence of the regression parameters C_i is shown in Fig. 2.

To determine the parameters C_i a least square error approach has been used to minimise the differences between the forecasted $f_{reg}(G)$ and the actually reached objective function values $f_{min}(G)$:

$$\sum_{G=1}^{G_c-1} \left(f_{\min}(G) - f_{reg}(G)\right)^2 \rightarrow \min. \quad (8)$$

Thereby, the objective function value $f_{min}(G_c)$ of the current generation G_c could not be involved in the sum, because this value is unknown until this time. In addition, the regression function should contain the last available function value $f_{min}(G_c\text{-}1)$. So we have a restricted quadratic optimisation problem with one equality constraint:

$$\sum_{G=1}^{G_c-1} \left(f_{\min}(G) - f_{reg}(G)\right)^2 \rightarrow \min \quad (9)$$

s.t.

$$f_{\min}(G_c - 1) - f_{reg}(G_c - 1) = 0.$$

After a specified number of generations ΔG the values $f_{min}(G_c\text{-}1)$ and $f_{reg}(G_c\text{-}1)$ are compared. If $f_{min}(G_c\text{-}1) > f_{reg}(G_c\text{-}1)$, a stagnation in the optimisation process is expected. Now a precise adjustment is necessary and the next parameter set from the list will be chosen. Otherwise, if we have the relation $f_{min}(G_c\text{-}1) < f_{reg}(G_c\text{-}1)$, it is assumed, that this solution is far away from the optimum. Thus, a more extensive search in the design space using the previous parameter set will probably result in a faster progress during the following generations. Only if the values $f_{min}(G_c\text{-}1)$ and $f_{reg}(G_c\text{-}1)$ are the same, regarding a predefined tolerance, then the parameter set is kept up to the next check.

In parallel with the previously stated comparison, the parameters C_i are updated for the next regression of the minimal objective function values using the information of the new evaluated design alternatives from the previous generations.

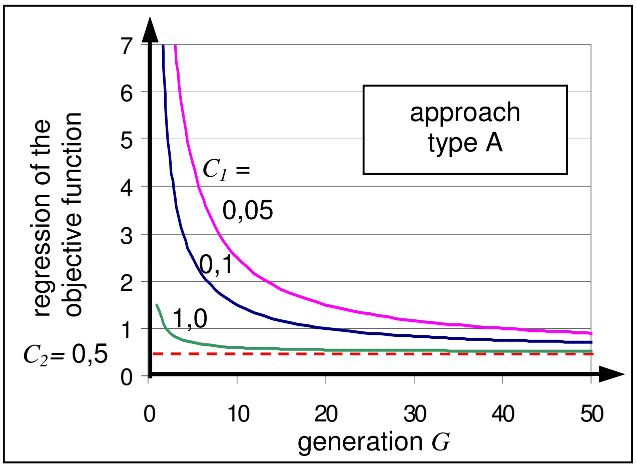

Fig. 1. Objective function regression, type A

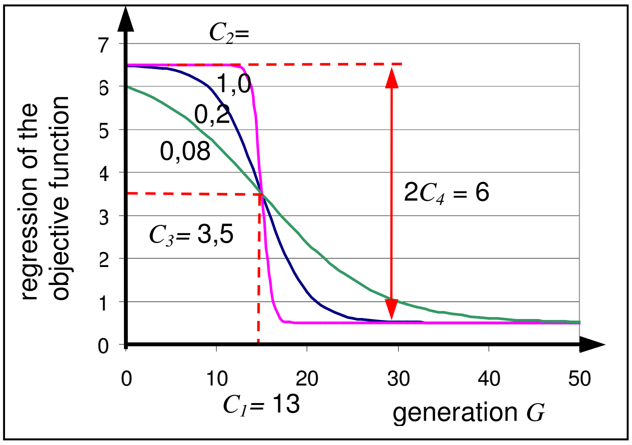

Fig. 2. Objective function regression, type B

3.2.2 Heterogeneity of the population

To avoid a premature stagnation in the optimisation process we have to keep on a specified level of heterogeneity of the several individuals in the population. But in this case the approaches used for the regression of the minimal objective function values are not useful. During the whole optimisation run we enforce a constant value of the heterogeneity using the equations (4) and (5). In fact the comparison involves the actually reached value *het*(*G*) and the mean value *mhet*(*G*) computed on the information basis of all the previous generations:

$$mhet(G) = \frac{1}{G_c - 1}\left(\sum_{G=1}^{G_c-1} het(G)\right). \tag{10}$$

After a specified number of generations ΔG we compare the values $mhet(G\text{-}\Delta G)$ and $het(G\text{-}1)$. If the value $het(G\text{-}1)$ is greater than $mhet(G\text{-}\Delta G)$, it indicates that the heterogeneity is too high. Hence, the next parameter set from the list is chosen in order to enforce a precise adjustment. Otherwise, to avoid a strong reduction of the heterogeneity and finally an increasing danger of stagnation of the whole optimisation process, the previous set from the parameter list is used. In the remaining case the heterogeneity reaches a value that lies in a specified tolerance of the forecasted mean value. Thus, the parameter set is kept.

3.2.3 Combined evaluation

To determine the used parameter set considering several control parameters like the distribution of the minimal objective function values in accordance to the generation and the heterogeneity of the population we have to introduce a further evaluation function $E(G_c)$. The value of this function indicates the number of the parameter set from the existing list, *pset*, that has to be chosen. Involving weighting factors $w_i \geq 0$ to balance the influence of the several control parameters in GEOpS/F the following approach is used:

$$\begin{aligned} E(G_c) = 0.5\{&w_1(f_{\min}(G_c-1) - f_{reg}(G_c-1)) \\ &+ w_2(het(G_c-1) - mhet(G_c - \Delta G))\} \end{aligned} \tag{11}$$

$$\begin{aligned} E(G_c) < -\varepsilon \quad &\rightarrow pset = pset - 1 \\ -\varepsilon \leq E(G_c) \leq +\varepsilon \rightarrow \quad &pset = const. \\ E(G_c) \leq +\varepsilon \quad &\rightarrow pset = pset + 1. \end{aligned}$$

The scalar $\varepsilon \geq 0$ denotes the predefined tolerance to keep the parameter set. After each ΔG generations the appropriateness of the current parameter set is checked and, if necessary, a better suited set is chosen.

4 Approximate evaluation

4.1 Initiation

Using the information of the exact evaluated design configurations of a predefined number of initial generations, the first approximation of design evaluation is established. Afterwards, generation blocks with exact and with approximated evaluation alternatives are produced until a stop criterion is reached. Of course, if new designs with a complete structural analysis are available, the approximation is updated.

In case of structural optimisation, the solution often lies on the separating surface between feasibility and infeasibility (Ω_f and Ω_{inf}). Thus, establishing the approximation in this region has to be considered very carefully. Commonly, the formulation describing the separating surface is unknown. So it is useful to give an approximation of this surface in a first approach and afterwards to approximate the objective function in the feasible and infeasible regions separately. In order to determine if a new, unevaluated point in the design space is feasible or not, a data classification approach is used. In the GEOpS/F implementation this is a so called *support vector machine* (SVM) [7].

4.2 SVM based design classification

A function $c(\mathbf{x})$: $\Omega \rightarrow \{-1,1\}$ divides the design space into two disjoint sets Ω_f and Ω_{inf}:

$$\Omega_f := \{\mathbf{x} \in \Omega \mid c(\mathbf{x}) = 1\} \quad \text{and} \quad \Omega_{inf} := \{\mathbf{x} \in \Omega \mid c(\mathbf{x}) = -1\}. \tag{12}$$

Regarding m exact evaluated design points in the training data set $\mathbf{X}=(\mathbf{x}^1,\mathbf{x}^2,\ldots,\mathbf{x}^m)$, $\mathbf{X} \in R^{n\times m}$ the classification of these points into Ω_f and Ω_{inf} is undertaken and thus the function values $c(\mathbf{x})$ are established. Based on this information, an approximation of $c(\mathbf{x})$ over the whole design space is determined. The feasible points lie on one side of a computed hyperplane and the infeasible ones on the other side. Unfortunately, it is commonly not possible to divide Ω_f and Ω_{inf} by a separating hyperplane. Thus we use a transformation of the original design space R^n into a higher dimensional feature space R^N, such that a separating hyperplane exists:

$$\mathbf{x} \in R^n \mid \mathbf{K}(\mathbf{x}^T, \mathbf{X})\mathbf{du}^* - \gamma^* = 0. \tag{13}$$

The components of the transformation matrix $\mathbf{K}(\mathbf{X_A},\mathbf{X_B}):=(k_{ij}(\mathbf{x_A}^{\mathbf{i}},\mathbf{x_B}^{\mathbf{j}}))$ are functions of the two data sets $\mathbf{X_A}=(\mathbf{x_A}^{\mathbf{1}},\ldots,\mathbf{x_A}^{\mathbf{l}})^T$, $\mathbf{X_A}\in R^{l\times n}$ and $\mathbf{X_B}=(\mathbf{x_B}^{\mathbf{1}},\ldots,\mathbf{x_B}^{\mathbf{k}})$, $\mathbf{X_B}\in R^{n\times k}$. $\mathbf{d}$ denotes a diagonal matrix $\mathbf{d}=diag(c(\mathbf{x}^{\mathbf{i}}))$, $i=1(1)m$, $\mathbf{d}\in R^{m\times m}$. To determine k: $R^n\times R^n\rightarrow R$ in this investigation a *Gaussian kernel*

$$k_{ij}\left(\mathbf{x_A}^{\mathbf{i}},\mathbf{x_B}^{\mathbf{j}}\right):=\exp\left(-\mu\left\|\mathbf{x_A}^{\mathbf{i}}-\mathbf{x_B}^{\mathbf{j}}\right\|_2^2\right). \qquad (14)$$

with the predefined parameter $\mu>0$ is used.

The parameters $(\mathbf{u}^*,\gamma^*)$, $\mathbf{u}^*\in R^m$, $\gamma^*\in R$ in equation (13) are the solution of the SVM approach:

$$\eta\|\mathbf{y}\|_2^2+\gamma^2+\|\mathbf{u}\|_2^2\rightarrow\min \qquad (15)$$
$$\text{s.t. } \mathbf{d}\left(\mathbf{K}\left(\mathbf{X}^T,\mathbf{X}\right)\mathbf{d}\mathbf{u}-\gamma\mathbf{e}\right)+\mathbf{y}\geq\mathbf{e}$$

with the vector $\mathbf{e}=(1,1,\ldots,1)^T$, $\mathbf{e}\in R^m$. The equation of the hyperplane is computed in such a way, that the existing design points of the training data set $\mathbf{X}$ are positioned as far as possible away from this plane. This approach provides the best classification of new unevaluated points. Additionally, with $\eta>0$ a weighted slack variable vector $\mathbf{y}\in R^m$ permits a small misclassification, and that also has to be minimised. The aim by doing this is to get a better classification of the remaining training data points.

Using the equation of the hyperplane, the classification of new unevaluated design points can be estimated. In case of $\mathbf{K}(\mathbf{x}^T,\mathbf{X})\mathbf{d}\mathbf{u}^*-\gamma^*\geq0$ the new point $\mathbf{x}^T$ is regarded as feasible, otherwise as infeasible.

4.3 Objective function approximation

Based on the before mentioned approximate design space classification between feasibility and infeasibility, it is now possible to formulate an objective function approximation. In the feasible design space region, the objective function value is equal to the structural weight, which is calculated with negligible computational effort. A more complicated approach is necessary in the infeasible region. Here, the level of constraint violations is involved in the objective function. Often in case of structural optimisation, the constraint computation includes finite element calculations. To avoid such a high computational effort, the complete penalty term is approximated in the approach applied in GEOpS/F:

$$P_{appr}(\mathbf{x})=\exp(\alpha\cdot b(\mathbf{x}))-1 \quad , \quad \alpha>0 \qquad (16)$$

with the introduced term

$$b(\mathbf{x})=\max\left\{0,-\left(\mathbf{K}\left(\mathbf{x}^T,\mathbf{X}\right)\mathbf{d}\mathbf{u}^*-\gamma^*\right)\right\} \qquad (17)$$
$$\forall\mathbf{x}\in\Omega.$$

Based on the SVM classification $b(\mathbf{x})$ gives a measure of the distance from the hyperplane in the assumed infeasible region. Using this formulation only in case of assumed infeasibility we get terms $b(\mathbf{x})>0$ and thus also $P_{appr}(\mathbf{x})>0$. Otherwise in the feasible classified design space there are $b(\mathbf{x})=0$ and $P_{appr}(\mathbf{x})=0$. So we are able to use the following expression as approximated objective function f_{appr} that is defined over the whole design space:

$$f_{appr}(\mathbf{x})=w(\mathbf{x})+P_{appr}(\mathbf{x}) \quad | \quad \forall\mathbf{x}\in\Omega. \qquad (18)$$

The coefficient α in (16) is determined using the least square error method and the information of the infeasible design points from which the exact evaluated penalty terms $P(\mathbf{x})$ and the values $b(\mathbf{x})$ is known.

In [6] the approximation approach implemented in GEOpS/F is described in more detail.

5 Optimisation problem

The code GEOpS/F has been particularly developed for the structural optimisation of composite aircraft fuselage structures. Several investigations have been performed in cooperation between Airbus and the Institute of Aerospace Engineering at TU Dresden. As an example the results for a compressive (n_x) and shear loaded (n_{xy}) fuselage side panel are given (see Fig. 3). The load ratio is $n_x/n_{xy}=0.5$. It is a carbon fibre reinforced plastics (CFRP) design composed of unidirectional plies known as prepregs. The objective is to find a minimal weight design. The following geometrical ratios of the panel are given: length/radius=0.75 and width/radius=0.52.

An overview of the considered design concepts is shown as Fig. 4. Stringer stiffened and sandwich design configurations are considered simultaneously in a single optimisation run. Design variables include geometrical parameters (core thickness or height and number of stiffeners), the stringer shape as well as the core material. Additionally, both the skin and stringer laminates of stiffened panels have variable stacking sequences. In case of sandwich concepts, the lay-up schemes of the skin laminates are additional design parameters. The feasible fibre angles in the stacking sequences are restricted to the discrete values 0, ±45 and 90 degrees. The thickness of a ply is kept to a constant 0.125mm, but the number of plies and thus the complete thickness of a laminate is variable.

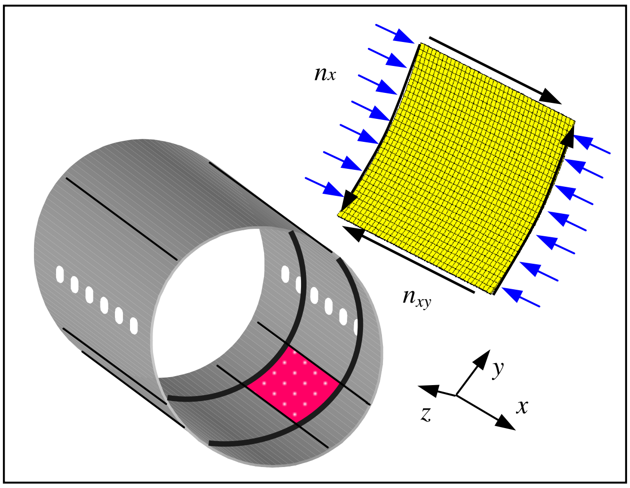

Fig. 3. Fuselage panel

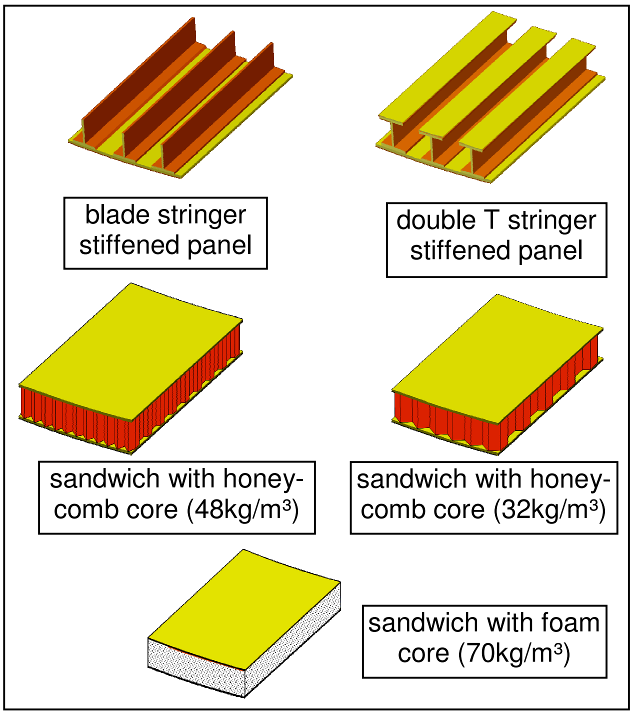

Fig. 4. Design concepts

In order to avoid matrix cracking the maximal number of adjacent plies with the same orientation is restricted to two. A further restriction enforces the laminates to be symmetric and balanced. A lower bound of the core thickness ensures a sufficient impact protection. Overall, the considered design space encloses a variety of about $1.08 \cdot 10^{13}$ different discrete design alternatives in combination with a continuous design variable, the stringer and core height respectively.

Strength and buckling restrictions are considered. To avoid the predefined material strain allowables at ultimate load (UL) being exceeded, the following criterion is applied:

$$\left(\frac{\varepsilon_x}{\varepsilon_x^{UL}}\right)^2 + \left(\frac{\varepsilon_y}{\varepsilon_y^{UL}}\right)^2 + \left(\frac{\gamma_{xy}}{\gamma_{xy}^{UL}}\right)^2 \leq 1. \tag{19}$$

The calculation of the strains ε and the shear deformation γ is based on the classical lamination theory. In case of stringer stiffened panels, local buckling of skin segments and stringer elements is not allowed below limit load (LL=0.67·UL). Regarding this restriction the criteria

$$\frac{1}{\lambda_{loc}} = \frac{1}{\lambda_{loc}(n_x, n_y)} + \frac{1}{\left(\lambda_{loc}(n_{xy})\right)^2}\ , \tag{20}$$

$$\lambda_{loc} \geq 0.67$$

is used to determine the local buckling load factor λ_{loc}. This equation considers the interaction of buckling caused by normal forces (n_x, n_y) and shear buckling (n_{xy}). Furthermore, wrinkling of the sandwich skins must not appear before UL is reached. The critical wrinkling load factor λ_{wr} is determined by using the equation

$$\lambda_{wr} = \min_{\varphi} \frac{1.5 \cdot \sqrt[3]{2D_{11}(\varphi) E_z^{core} G^{core}(\varphi)}}{n_x \cos^2\varphi + n_y \sin^2\varphi - 2n_{xy}\cos\varphi \sin\varphi}$$

$$\lambda_{wr} \geq 1.0 \tag{21}$$

in which the in plane load distribution is analysed regarding different angles φ (φ=0°...180°) in the laminate plane [11].

In addition to the closed formulations described above, a numerical finite element analysis is performed to compute the load factor for global buckling λ_{gl}. Global buckling is not allowed below ultimate load. The finite element models of the stringer stiffened panels consist of 8 node layered shell elements, whereas the sandwich panels are modelled by using 8 node layered volume elements.

6 Results

Regarding this special optimisation problem, the best calculated design configurations satisfying all restrictions are sandwich designs. They all have a honeycomb core with a density of 32kg/m³ and the same thickness at the lower bound of 20mm. Also, the number of plies in the inside and the outside skin is equivalent in all cases. Only the stacking sequences show differences. A summary of the sequences of three obtained optimal design configurations is given in table 1.

Design configuration A outside skin: $[0/90/\pm45]_S$ inside skin: $[\pm45/90/0/\pm45_2/90/\pm45_2/0/\pm45_2]_S$
Design configuration B outside skin: $[90/\pm45/0]_S$ inside skin: $[90/0/\pm45_4/90/\pm45_3/0]_S$
Design configuration C outside skin: $[0/\pm45/90]_S$ inside skin: $[\pm45_3/90/0/\pm45/0/90/\pm45_3]_S$

Tab. 1. Laminate stacking sequence of the optimal design configurations

The following comparison of several optimisation runs shows the effect of the efficiency enhancement. Results obtained by using only the standard operators of ES and GA without any enhancement are plotted in Fig. 5. They serve as basis for the comparison. In the following two sets of optimisation runs (Fig. 6 and Fig. 7) all the before described methods to enhance the efficiency are implemented. That means, (a) a combination of the operators of ES, GA and DE, (b) an adaptive adjustment of the optimisation parameters involving the distribution of the objective function values and the heterogeneity of the population members and (c) an approximate evaluation of design alternatives during intermediate generations using the SVM approach described in Chapter 4. The difference between the runs shown in Fig. 6 and Fig. 7 is the regression approach for the objective function values. In Fig. 6 the type A approach (see Fig. 1) and in Fig. 7 the sigmoid type B (see Fig. 2) is used. In all diagrams the distribution of the relative objective function values is plotted. That means, for each generation the best objective function value in the population is related to the best known objective function value of the problem.

A reduction of the relative objective function below a value of 1.0 is only possible in combination with the approximation approach. At least during the starting phase of the optimisation, the accuracy of the approximation is not very high. Therefore, it is possible, that the infeasible design points could be classified incorrectly as feasible ones. Using additional exactly evaluated design points in the further optimisation progress, such configurations will be rejected.

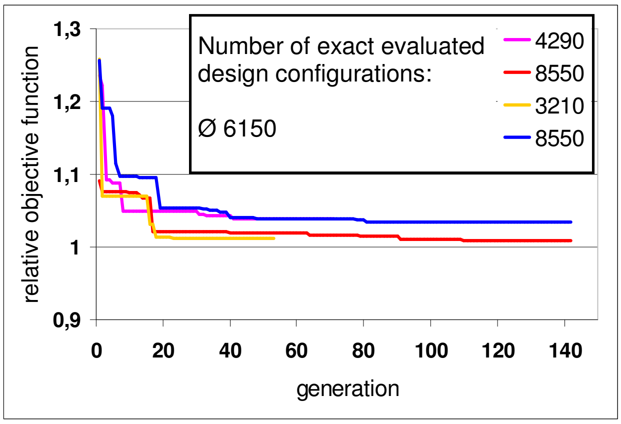

Fig. 5. Optimisation runs without efficiency enhancement

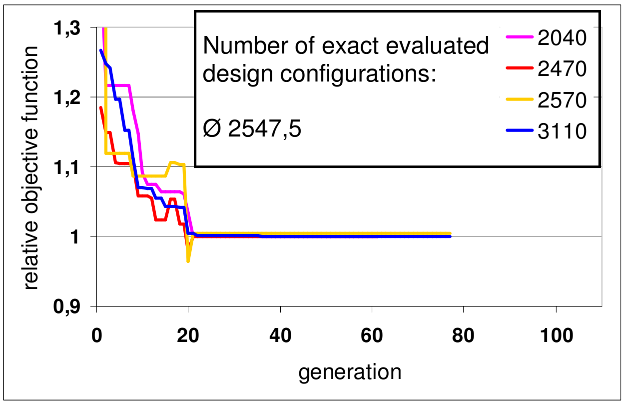

Fig. 6. Optimisation runs with efficiency enhancement, objective function regression type A

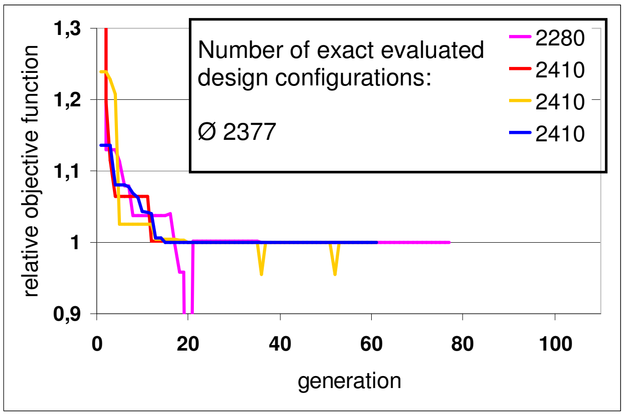

Fig. 7. Optimisation runs with efficiency enhancement, objective function regression type B

In comparison of Fig. 5 and Fig. 6 the applied efficiency enhancement reduces the average number of exact evaluated design configurations to a fraction of 40%. That means a considerable reduction in the computational effort, particularly in the case of finite element analyses being involved in the calculation. As well, the efficiency enhancement leads to a substantial increase in the reliability of single optimisation runs. Regarding the basic algorithm, no run leads to the best known objective function value with the minimal weight. Otherwise, using the efficiency enhanced approach, the optimum has been obtained in 4 of 5 runs. Only one run results in an insignificantly higher value of 0.4%. From Fig. 6 and 7 it becomes obvious, that the sigmoid objective function regression tends to better results in comparison to the type A approach. Using the sigmoid approach, all the exemplary runs lead to a minimal weight design. Additionally, the number of exact configuration evaluations slightly decreases. For the type A approach the runs show a higher risk of premature stagnation because of the strongly enforced reduction of the objective function values especially in the starting phase of the optimisation. Otherwise, starting the optimisation with a wide search in the design space, the sigmoid regression approach leads in general to a continuous reduction of the objective function values over the whole optimisation process.

7 Conclusion

The design optimisation of composite fuselage structures is a very complex problem with combined discrete and continuous design variables and several local and global optima. *Evolutionary algorithms* are well suited to handle such kind of problems, but with the drawback of needing to evaluate a large number of design configurations. That causes an enormous computational effort, particularly, if finite element analyses are involved. In this paper several methods to enhance the efficiency of such algorithms are presented. In detail these are a combined application of different types of evolutionary operators, an adaptation of the optimisation parameters itself and an approximate evaluation of design configurations during intermediate generations. The implemented approximation is based on a *support vector machine* classification approach. By the use of these methods the reliability of *evolutionary algorithms* to get the optimal solution in a single run is increased considerably. Furthermore, the number of exact evaluated design configurations is reduced by 60%.

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