GENERALIZED MISSILE GUIDANCE LAWS AGAINST MANEUVERING TARGETS

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Abstract

The proposed new guidance laws improve the effectiveness of the proportional navigation (PN) law against maneuvering targets. The new guidance laws utilize the same parameters as PN and APN (augmented PN) laws and can be easily realized in practice. Comparative analysis using the new guidance laws and the traditional PN and APN laws showed that the new guidance laws guarantee shorter homing time requirements and larger capture area. They are also very effective for midcourse guidance and enable us to guarantee performance comparable with Kappa optimal guidance. The new guidance laws are algorithmically simpler because they are not based on information about the intercept point and/or time-to-go.

1 Introduction

Maneuvers present the best strategy for missiles to achieve their goals. Sinusoidal or weave maneuvers of a target can make it difficult for a pursuing missile to obtain an intercept. Despite PN guidance has been studied in a detailed manner for non-maneuvering targets, it is widely utilized for maneuvering targets [1,2]. The analysis of PN guidance for the homing stage has usually been undertaken for non-maneuvering targets assuming a constant closing velocity [1,2]. The so-called augmented PN law and other modifications of the proportional navigation law have been obtained based mostly on the relationships established for non-maneuvering targets [1,2]. For maneuvering targets linear planar models enable researchers to obtain mostly qualitative results rather than the results that can be utilized directly in design procedures [1-4]. Moreover, guidance laws obtained from the analysis of the nonlinear planar models can not be directly applied for the three-dimensional case. As shown in [5], the guidance law obtained for a planar nonlinear model corresponds only to one coordinate of the three-dimensional guidance law. Guidance laws based on the results of control theory related to sliding modes and systems with variable structure (see, e.g., [6]) can not be considered as practical for missile guidance applications. The practical realization of systems with sliding mode is limited because of chatter, and related simplified control laws need rigorous justification and testing. Also, in the presence of a maneuvering target the sliding mode area depends on the target acceleration, and for small LOS derivatives the sliding mode can disappear. A variable structure (different from the ones considered, e.g., in [6]) that requires measurement of target acceleration is needed. Guidance laws obtained as a solution of an optimization problem (see e.g. [3,7]) assume that the trajectory of a maneuvering target as well as time-to-go and/or the intercept point are known. In practice, such information is unknown and can only be evaluated approximately. The accuracy of prediction influences significantly the accuracy of the intercept. The game approach to guidance laws (see e.g., [8]) deals mostly with models of engagement too simple to be recommended for practical applications. Taking into account that the PN law is a widely accepted guidance law and has been tested in practice (it was also justified as an optimal one corresponding to a certain quadratic performance index) we will develop a class of guidance laws that includes the PN law and has better performance than the PN and APN laws. This paper generalizes the results of [4, 5] for the case of missiles with thrust control. By considering missile radial and lateral motions, a class of guidance laws wider than in [4, 5] is obtained.

2 Problem Formulation and Derivation of Generalized Guidance Laws

For the three-dimensional case and the Earth-based coordinate system the target-to-missile range vector $\bar{r}(t)$ and its derivatives can be represented as

$$\bar{r}(t) = r(t)(\lambda_1(t)i + \lambda_2(t)j + \lambda_3(t)k) \quad (1)$$

$$\begin{aligned}\dot{\bar{r}}(t) &= (\dot{\lambda}_1(t)r + \dot{r}(t)\lambda_1(t))i \\ &+ (\dot{\lambda}_2(t)r + \dot{r}(t)\lambda_2(t))j \\ &+ (\dot{\lambda}_3(t)r + \dot{r}(t)\lambda_3(t))k \quad (2)\end{aligned}$$

$$\begin{aligned}\ddot{\bar{r}}(t) &= (\ddot{\lambda}_1(t)r(t) + 2\dot{r}(t)\dot{\lambda}_1(t) + \ddot{r}(t)\lambda_1(t))i \\ &+ (\ddot{\lambda}_2(t)r(t) + 2\dot{r}(t)\dot{\lambda}_2(t) + \ddot{r}(t)\lambda_2(t))j \\ &+ (\ddot{\lambda}_3(t)r(t) + 2\dot{r}(t)\dot{\lambda}_3(t) + \ddot{r}(t)\lambda_3(t))k \quad (3)\end{aligned}$$

where $r(t)$ is a range, $\lambda_s(t)$ (s = 1,2,3) are the line of sight (LOS) coordinates and i, j and k are unit vectors along the north, up and east Cartesian coordinate axes, respectively,

$$\lambda_s(t) = \frac{R_s}{r} \qquad (s = 1,2,3) \qquad (4)$$

R_s (s = 1,2,3) are the *RTM*-vector coordinates (*RTM* means range r between a target and a missile).

The dynamic equations of the three-dimensional engagement can be presented in the form

$$\begin{aligned}\ddot{\bar{r}}(t) &= \bar{a}_T(t) - \bar{a}_M(t) \\ &= \bar{a}_{Tr}(t) + \bar{a}_{Tt}(t) - \bar{a}_{Mr}(t) - \bar{a}_{Mr}(t) \quad (5)\end{aligned}$$

where missile $\bar{a}_M(t)$ and target $\bar{a}_T(t)$ accelerations consist of two components – longitudinal (along the line of sight) and lateral (perpendicular to the line of sight), i.e.,

$$\begin{aligned}\bar{a}_M(t) &= \bar{a}_{Mr}(t) + \bar{a}_{Mt}(t), \\ \bar{a}_T(t) &= \bar{a}_{Tr}(t) + \bar{a}_{Tt}(t), \quad (6)\end{aligned}$$

$\bar{a}_{Tr}(t)$, $\bar{a}_{Mr}(t)$, $\bar{a}_{Tt}(t)$, and $\bar{a}_{Mt}(t)$ are the target and missile longitudinal (radial) and lateral (tangential) accelerations with the coordinates $a_{Trs}(t)$, $a_{Mrs}(t)$, $a_{Tts}(t)$, and $a_{Mts}(t)$ (s=1,2,3) , respectively.

Unlike missiles without throttlable engines, missiles with axial control can employ thrust control as a part of guidance. Because of their superior guidance ability, for the purpose of the detailed analysis of this type of missiles, we consider separately the longitudinal and lateral motions.

Combining (3) and (5),we obtain the following system of equations describing the three-dimensional engagement

$$\begin{aligned}&\ddot{\lambda}_s(t)r(t) + 2\dot{r}(t)\dot{\lambda}_s(t) + \ddot{r}(t)\lambda_s(t) \\ &= \bar{a}_T(t) - \bar{a}_M(t) \quad (s = 1,2,3) \quad (7)\end{aligned}$$

The last term of the left part of (7) corresponds to the vector directed along the LOS. The components $q\lambda_s$ of $h_s = \ddot{\lambda}_s(t)r(t) + 2\dot{r}(t)\dot{\lambda}_s(t)$ (s =1,2,3) that correspond to the vector directed along the LOS are determined from the orthogonality of radial and tangential vectors, i.e., $\sum_{s=1}^{3}(h_s - q\lambda_s)q\lambda_s = 0$. Using the equalities obtained from the sequential differentiation of $\sum_{s=1}^{3}\lambda_s^2 = 1$, the following expression for the factor q can be obtained

$$q = r(t)\sum_{s=1}^{3}\ddot{\lambda}_s(t)\lambda_s(t) = -r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t) \quad (8)$$

The expressions for the missile radial and lateral motions follow from (7) and (8).

We analyze these motions in the Cartesian frame of coordinates of an inertial reference coordinate system, in contrast to the well-known presentation of the three-dimensional kinematics of guidance (see, e.g., [2]) describing the radial and lateral motions using a rotating frame of coordinates with axes along the unit vectors 1_r directed along $\bar{r}$, 1_w directed along $\bar{r}\times\dot{\bar{r}}$, and $1_t = 1_r \times 1_w$.

For the radial motion we have

$$\ddot{r}(t)\lambda_s(t) - r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t)\lambda_s = \bar{a}_{Tr}(t) - \bar{a}_{Mr}(t) \quad (s=1,2,3) \qquad (9)$$

By presenting the radial vectors $\bar{a}_{Tr}(t)$ and $\bar{a}_{Mr}(t)$ in the form

$$a_{Trs}(t) = a_{Tr}(t)\lambda_s(t)\,,\; a_{Mrs}(t) = a_{Mr}(t)\lambda_s(t) \quad (s=1,2,3) \qquad (10)$$

where $a_{Tr}(t)$ and $a_{Mr}(t)$ are the target and missile radial accelerations, respectively, the equation (9) can be reduced to

$$\ddot{r}(t) - r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t) = a_{Tr}(t) - a_{Mr}(t) \qquad (11)$$

For the lateral motion we have

$$\ddot{\lambda}_s(t)r(t) + 2\dot{r}(t)\dot{\lambda}_s(t) + r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t)\lambda_s(t) = a_{Tts}(t) - a_{Mts}(t) \quad (\mathrm{s}=1,2,3) \qquad (12)$$

The system (11) and (12) is equivalent to the system (7). The analysis of its specifics will enable us to simplify the analysis of the original system (7).

Missiles without axial control are able to control only the lateral motion using information about a missile thrust, drag and a target acceleration and considering them as external factors with respect to control actions. The basic widespread philosophy behind controlling the lateral motion is that a missile acceleration should nullify the line of site (LOS) rate, i.e., the lateral acceleration as control is aimed at implementing parallel navigation. In the ideal case $\dot{\lambda}_s(t) = 0$ (s=1,2,3) the system (11) and (12) is reduced to

$$\ddot{r}(t) = a_{Tr}(t) - a_{Mr}(t) \qquad (13)$$

It can be easily observed from (11) and (12) that the dynamics of radial and lateral motions can be decoupled by using a pseudo-acceleration $a_{Mr1}(t)$ in the radial direction

$$a_{Mr1}(t) = a_{Mr}(t) - r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t) \qquad (14)$$

so that instead (11) we can analyze (13) where $a_{Mr}(t)$ is changed for $a_{Mr1}(t)$.

The terms lateral acceleration and lateral motion were used above to characterize the motion in a plane orthogonal to the LOS. The TPN (true proportional navigation) law $a_{Mts}(t) = -N\dot{r}(t)\dot{\lambda}_s(t)$, N>2 characterizes the motion belonging to this plane. However, the class of guidance laws implementing parallel navigation does not necessarily satisfy (12) because the acceleration vector required by the guidance law doesn't lie in this plane. For example, in the PPN (pure PN) law the commanded acceleration is applied normal to the missile velocity vector; in the GPN (generalized PN) the commanded acceleration forms a constant angle with the normal to the LOS [2]. Because these laws have a dominant tangential component of acceleration, we will use the term lateral acceleration to characterize them and the term radial acceleration to characterize the motion satisfying (13).

Instead of (11) and (12) we will consider the system

$$\ddot{\lambda}_s(t)r(t) + 2\dot{r}(t)\dot{\lambda}_s(t) = a_{Tts}(t) - a_{Mts}(t) \quad (s=1,2,3) \qquad (15)$$

and (13).

The guidance problem can be formulated as the problem of choosing controls $a_{Mr}(t)$ and $a_{Mts}(t)$ $(s=1,2,3)$ to guarantee $\dot{r}(t)<0$ and the asymptotic stability of the system (15) with respect to $\dot{\lambda}_s(t)$ $(s=1,2,3)$ [4,5,9]. Because in reality we deal with a finite problem, for simplicity and a more rigorous utilization of the term 'asymptotic stability' we assume disturbance (target acceleration) to be a vanishing function , i.e. contains a factor $e^{-\varepsilon t}$, ε is an infinitely small positive number; moreover, if t_f is the time of intercept then $\lim_{t\to t_f} r(t)\to 0$ and $a_T(t)=0$ for $t>t_f$.

From (13) the conditions $\dot{r}(t)<0$ and $\lim_{t\to t_f} r(t)\to 0$ can be achieved by choosing $a_{Mr1}(t)>a_{Tr}(t)$ for $t\le t_f$ and $a_{Mr1}(t)=0$ for $t>t_f$, i.e.,

$$a_{Mr1}(t)=k_1(t)a_{Tr}(t),\ \ k_1(t)\ge 1 \qquad (16)$$

This follows from the condition of negative definiteness of the derivative of the Lyapunov function $r^2(t)$ along any trajectory of (13), i.e., $r(t)\,\dot{r}(t)<0$ where $\dot{r}(t)=\dot{r}(t_0)+\int_{t_0}^{t}(a_{Tr}(t)-a_{Mr1}(t))dt<0$; t_0 is the initial moment of guidance .

The system described by the equation (13) has been examined thoroughly in the literature; various optimal problems have been considered and solved (see, e.g., [3,7]). Without considering here concrete optimal problems (their practical application is limited because of lack of information about future values of a target acceleration), we indicate only that a pseudo-acceleration $a_{Mr1}(t)$ in the radial direction should exceed the radial target acceleration, so that the larger their difference the faster the decrease in range.

The asymptotic stability of (12) with respect to $\dot{\lambda}_s(t)$ $(s=1,2,3)$ is guaranteed by the guidance law [5]

$$a_{Mts}(t)=Nv_{cl}\dot{\lambda}_s(t)+\sum_{k=1}^{2}u_{sk}(t) \qquad (17)$$

where $v_{cl}=-\dot{r}(t)$,

$$u_{s1}(t)=N_{1s}\dot{\lambda}_s^3(t), \qquad N_{1s}>0 \qquad (18)$$

$$u_{s2}(t)=N_{2s}a_{Tts}(t) \qquad (19)$$

$$N_{2s}\ \begin{matrix}\le 1\\ \ge 1\end{matrix} \quad \text{if} \quad sign(a_{Tts}(t)\dot{\lambda}_s(t))\ \begin{matrix}\le 0\\ \ge 0\end{matrix} \qquad (20)$$

$(s=1, 2, 3)$

The above equations (17)-(20) follow immediately from the procedure based on the Lyapunov approach described in [4,5]. The Lyapunov function is chosen as the sum of squares of the LOS derivative components which corresponds to the nature of the parallel navigation.

$$Q=\frac{1}{2}\sum_{s=1}^{3}d_s\dot{\lambda}_s^2(t) \qquad (21)$$

where d_s are positive coefficients. The law (17) was obtained from the condition of negative definiteness of the Lyapunov function along any trajectory of (12).

Based on (15)-(20) the guidance law can be presented in the following form

$$a_{Ms}(t)=Nv_{cl}\dot{\lambda}_s(t)+r(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t)+N_1\dot{\lambda}_s^3(t)$$
$$+k_1(t)a_{Trs}(t)+N_{2s}a_{Tts}(t)\ \ (s=1, 2, 3) \qquad (22)$$

The first term of (22) corresponds to the traditional PN law. The PN law reacts almost identically to various changes of the LOS rate (assuming that the closing velocity v_{cl} doesn't vary drastically), i.e., small and fast changes of the LOS result in proportional changes of acceleration. As mentioned in [4], we can decrease the LOS rate faster by increasing *N* in the PN law. But this will increase the level of noise when the LOS rate becomes small and, hence, the accuracy of guidance is deceased. From a pure physical consideration we can assume that the system with a variable gain which is bigger when LOS rate is big and smaller when LOS rate is small will act better than the traditional PN system. The third component of (22) with a properly chosen N_1 serves this purpose. The coefficient $k_1(t)$ is chosen to guarantee fast decrease of $r(t)$. It can be constant or time-varying depending on available information about a target. The term $N_{2s}a_{Tts}(t)$ is different from the corresponding term in the augmented proportional navigation (APN) law because the parameter N_{2s} is time-varying. The $sign(a_{Tts}(t)\dot{\lambda}_s(t))$ factor reflects the dependence of the correction on the target behavior.

The coefficients N_1, N_2 and k_1 (constant or time-varying) can be determined based on simulation results of the whole missile system taking into account the autopilot limits on a missile acceleration, airframe dynamics and some other factors, i.e., the same way as the most appropriate values N= 3.5-4 were established.

The guidance law (22) assumes a missile is able to control all three-dimensional space. It is important to mention that for many types of existing missiles (e.g., without throttlable engines) radial acceleration can not be utilized as a control action. Such missiles are not able to use thrust control as a part of a guidance law. Radial component induced by the lateral acceleration (see (14) and (16)) can influence the missile trajectory only by decelerating its motion.

For these types of missiles only the components of the lateral missile acceleration are available controls and instead of (13) and (15) the initial equation (7) should be examined. By using the Lyapunov function (21) and its derivative along any trajectory of (7)

$$2\dot{Q} = \sum_{s=1}^{3} d_s\left(-\frac{\ddot{r}(t)}{r(t)}\lambda_s(t)\dot{\lambda}_s(t) - \frac{2\dot{r}(t)}{r(t)}\dot{\lambda}_s^2(t) + \frac{1}{r(t)}\dot{\lambda}_s(t)(a_{Ts}(t) - a_{Ms}(t))\right) \quad (23)$$

analogous to [4,5], the controls $a_{Ms}(t)$ that guarantee $\lim\|\bar{\lambda}\| \to 0$, $t \to \infty$ can be presented as

$$a_{Ms}(t) = Nv_{cl}\dot{\lambda}_s(t) + \sum_{k=1}^{3} u_{sk}(t) \quad (24)$$

where

$$u_{s2}(t) = N_{2s}a_{Ts}(t) \quad (25)$$

$$N_{2s} \begin{matrix} \le 1 \\ \ge 1 \end{matrix} \quad \text{if} \quad sign(a_{Ts}(t)\dot{\lambda}_s(t)) \begin{matrix} \le 0 \\ \ge 0 \end{matrix} \quad (26)$$

$$u_{3s}(t) = N_{3s}\lambda_s(t)\ddot{r}(t) \quad (27)$$

$$N_{3s} \begin{matrix} \ge 1 \\ \le 1 \end{matrix} \quad \text{if} \quad sign(\ddot{r}(t)\dot{\lambda}_s(t)\lambda_s(t)) \begin{matrix} \le 0 \\ \ge 0 \end{matrix} \quad (28)$$

(s=1, 2, 3)

Comparison of (22) and (24) shows that in the case of missiles with uncontrollable thrust the $u_{2s}(t)$ terms depend upon the total target acceleration rather than its tangential component and that instead of the radial components $k_1(t)a_{Trs}(t) + r(t)\lambda_s(t)\sum_{s=1}^{3}\dot{\lambda}_s^2(t)$ the guidance law contains the radial component $u_{3s}(t)$ (s=1, 2, 3). By substituting $u_{3s}(t)$ in (24) it is easy to conclude that these components influence the derivative $\dot{Q}$ only in a case of unequal coefficients d_s (s=1, 2, 3). Moreover, only negative $u_{3s}(t)$ (s=1, 2, 3), i.e., deceleration, can be realized in practice.

Remark: In our simplified model of engagement we consider the radial acceleration acting along the LOS. In reality, the radial acceleration acts along a missile's body and the tangential acceleration acts in the orthogonal direction, so that the real tangential acceleration obtained by projecting the acceleration of (24) on the axis perpendicular to a missile's body axis may reflect the influence of the $u_{3s}(t)$ components (s=1, 2, 3).

The obtained guidance laws assume that current information about a target acceleration is available. Usually, we operate only with the estimated target acceleration, so that a result worse than in the ideal estimation case can be expected. Many missiles are unable to measure a target acceleration and use it in a guidance law. In this case, the components $u_{2s}(t)$ (s=1, 2, 3) are not present in the guidance law and its performance is worse compared to the case when a target acceleration can be measured.

The new developed laws can be utilized for midcourse and terminal guidance [4,5]. As indicated in [5], the test showed that the guidance law (24) guaranteed performance comparable with Kappa optimal guidance and even enabled us to obtain smaller time of intercept without loss of terminal velocity. During the midcourse stage the components of the LOS are obtained from (4). For the terminal stage these components are usually calculated based on measurements of azimuth and elevation angles (see e.g., [10]). The vectors $\bar{\lambda}(t)$ and $\dot{\bar{\lambda}}(t)$ can be presented as

$$\bar{\lambda} = \begin{bmatrix} \cos\alpha\cos\beta \\ \cos\alpha\sin\beta \\ \sin\alpha \end{bmatrix},$$

$$\dot{\bar{\lambda}} = \begin{bmatrix} -\sin\alpha\cos\beta \\ -\sin\alpha\sin\beta \\ \cos\alpha \end{bmatrix}\dot{\alpha} + \begin{bmatrix} -\cos\alpha\sin\beta \\ \cos\alpha\cos\beta \\ 0 \end{bmatrix}\dot{\beta} \quad (29)$$

where α and β are elevation and azimuth angles, respectively.

3 Numerical Simulation

The guidance laws are tested on an example of the engagement model with the parameters close to those considered in [6]: the effective navigation ratio N=3; a target initial conditions R_{T1}=4500 m, R_{T2}= 2500 m, R_{T3}=0; V_{T1}= -350 m/s, V_{T2}=30 m/s; V_{T3}=0; a missile initial conditions R_{M1}=R_{M2}=R_{T3}=0; V_{M1}=-165 m/s, V_{M2}=475 m/s; V_{T3}=0; a target acceleration $a_{T1}=0$, a_{T2} =3g sin 1.31t, a_{T3}=0; a missile acceleration limit $5g$ (R_i, V_i, i=1-3, are distance and velocity coordinates; lower indices "M" and "T" indicate a missile and target, respectively). In contrast to [6], a missile dynamics are taken into consideration: the missile flight control system right half-plane airframe zero frequency ω_z =30 rad/s, damping ζ =0.7, natural frequency ω_M=20 rad/s , and time constant τ=0.5s. A target weaving frequency is chosen according to [11].

Figures 1 corresponds to the guidance law (22) and the case when the missile dynamics are ignored. It shows the trajectories of the target (cross solid line) and missile for the PN law and the newly developed laws. Time of intercept for the APN and PN laws equals 8 s. The APN doesn't improve the PN guidance in this case. However, the newly developed laws enable us to improve the PN performance. Symbol "ATN" indicates the components $N_{2s}a_{Tts}(t)$ of (22) ($N_{21}=N_{22}=\{0.5;3.5\}$, N_{23}=0). The cubic term corresponds to $u_{s1}(t)$ components with gains $N_{11}=20000v_{cl}, N_{12}=2000v_{cl}, N_{13}=0$. The guidance law with all terms of (22) ($k_1(t)=7$) gives the best results. Time of intercept equals 7.35 s.

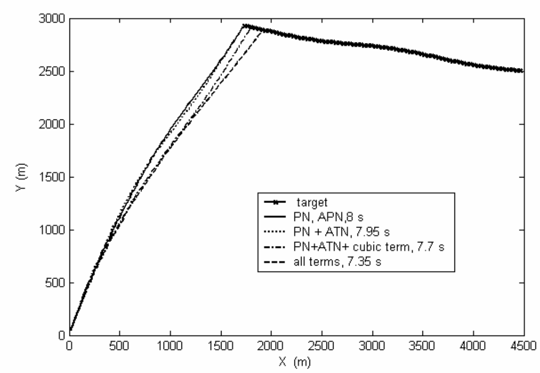

Fig. 1. Comparison of the New Guidance Laws with PN and APN Guidance (engagement model without missile dynamics)

As expected, inability to measure the target acceleration and absence of axial control decreases the missile performance. Time of intercept equals 7.8 sec for the guidance law (24), where $u_{s2}(t)=0, N_{31}=N_{32}=\{1;3.5\}$, $N_{33}=0$.

Figure 2 repeats the numerical simulations of Fig. 1 taking into account the missile dynamics. In Fig. 2 the miss distance and the time of intercept correspond to the moment of time when the closing velocity became positive. In the case of "PN +ATN" $N_{21} = \{0;1.5\}$, $N_{22} = 1$, N_{23}=0. In the case of "PN +ATN +cubic term" $N_{11} = 28000 v_{cl}, N_{12} = 40000 v_{cl}, N_{13} = 0$.

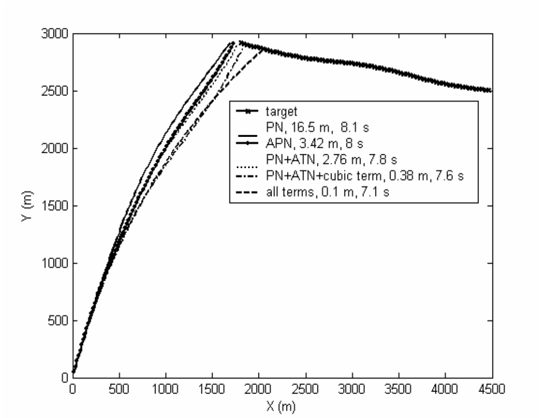

Fig. 2. Comparison of the New Guidance Laws with PN and APN Guidance (engagement model includes missile dynamics) .

As in the case when the missile dynamics were ignored, the guidance law with all terms of (22) gives the best results. The parameters of the guidance law are: $k_1(t) = 2.8$; $N_{11} = 400000 v_{cl}$, $N_{31} = 0$, $N_{12} = 19400 v_{cl}$, $N_{21} = \{0;1.5\}$, $N_{22} = 1$, N_{23}=0. The time of intercept and miss distance are significantly better than obtained under the PN and APN guidance laws.

4 Conclusion

Analytical expressions of new guidance laws were obtained for the generalized three-dimensional engagement model. It was shown the effectiveness of the proposed laws against maneuvering targets, their superiority to PN and APN guidance. In addition to showing better performance, the new developed guidance laws can be easily implemented in practice because they utilize the same parameters as the PN and APN laws.

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