AERODYNAMIC SHAPE OPTIMIZATION OF TRANSONIC AIRFOIL AND WING USING RESPONSE SURFACE METHODOLOGY

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Keywords: *response surface methodology, optimization, transonic*

Abstract

A Response Surface Methodology (RSM) for Aerodynamic Shape Optimization (ASO) using the compressible Reynolds-Averaged Navier-Stokes (RANS) equations is implemented and tested. The quadratic polynomials which cancel the second-order cross items are employed to construct RS model. This approach improves on the efficiency of using the RSM for high-dimensional design optimization problems. By using present method the aerodynamic performance of the transonic airfoil and wing are greatly improved. Successful design results confirm validity and efficiency of the present design method.

1 Introduction

Aerodynamic Shape Optimization (ASO) technology based on Computational Fluid Dynamics (CFD) becomes a very active object in the CFD field. Among several design optimization methods applicable to aerodynamic design problems, the approximation optimization methods have been widely used due to their advantages. First, these methods can be effectively applied to Multidisciplinary Design Optimization (MDO), and the computational costs are acceptable in spite of the high-fidelity analysis tools are used. A second advantage of using these methods during optimization process is that they can be used with optimization algorithms which do not rely on the computation of sensitivity derivatives and can find a solution near the global optimum.

One of the most common methods for building an approximate model is Response Surface Methodology (RSM) in which a polynomial function of varying order (usually a quadratic function) is fitted to a number of sample data points using least squares regression. This method has achieved popularity since it provides an explicit function representation of the sampled data, and is both computationally cheap to run and easy to use. [1, 2, 3]

However, if the full quadratic polynomials are employed to construct RS models for ASO problem, the number of function evaluations (CFD analysis, in our cases) required for a RS model increase with the square of the number of design variables, seriously preventing their use in high-dimensional design optimization especially using high-fidelity analysis tools. And for ASO problem of transonic airfoil and wing, only small number design variables are not enough. How to solve this dilemma? A modified RSM that requires fewer functional evaluations must be developed if the approximation method is to be used in an ASO problem.

In this research, a modified RSM that propose modification to the RS model which cancel the second-order cross items of the full quadratic is developed to greatly reduce the computational cost, and approximate the original function without significantly sacrificing the accuracy of the approximation when the design spaces are carefully selected.

In common, during the preliminary design phase, numerical optimization starts with an existing design, and the goal is to redesign and improve aerodynamic performance. The geometric change between initial and optimized shape are very small [4], but the difference in the performance can be substantial. So small design space is enough for ASO problem. And the accuracy of linear RS approximation model can be insured. Some numerical optimization examples including drag minimization for transonic airfoil and wing are performed to verify the effectiveness of present method.

2 Flow Analysis

In flow field calculation the compressible Reynolds-Averaged Navier-Stokes equations are used as governing equations. The 3-D N-S equations in Cartesian coordinates (x_1, x_2, x_3) can be written in the conservation form as

$$\frac{\partial \boldsymbol{w}}{\partial t} + \frac{\partial \boldsymbol{f}_i}{\partial x_i} = \frac{\partial \boldsymbol{f}_{vi}}{\partial x_i} \tag{1}$$

where

$$\boldsymbol{w} = \begin{Bmatrix} \rho \\ \rho u_1 \\ \rho u_2 \\ \rho u_3 \\ \rho E \end{Bmatrix}, \quad \boldsymbol{f}_i = \begin{Bmatrix} \rho u_i \\ \rho u_i u_1 + p\delta_{i1} \\ \rho u_i u_2 + p\delta_{i2} \\ \rho u_i u_3 + p\delta_{i3} \\ \rho u_i H \end{Bmatrix}, \quad \boldsymbol{f}_{vi} = \begin{Bmatrix} 0 \\ \tau_{ij}\delta_{j1} \\ \tau_{ij}\delta_{j2} \\ \tau_{ij}\delta_{j3} \\ u_j \tau_{ij} + k\dfrac{\partial T}{\partial x_i} \end{Bmatrix} \tag{2}$$

and ρ is the density, (u_1, u_2, u_3) are the Cartesian velocity components, p , E , H is the pressure, total energy and total enthalpy respectively, δ_{ij} is the Kronecker delta function. and σ_{ij} is the component of stress tensor.

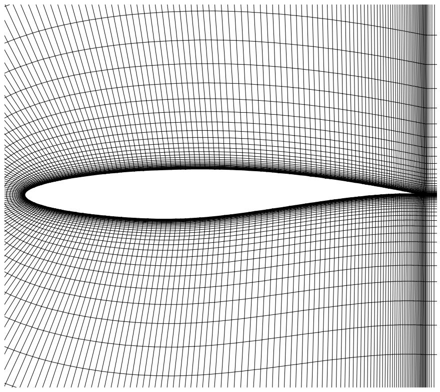
In computational space, the Navier-Stokes equations can be written as

$$\frac{\partial (J\boldsymbol{w})}{\partial \mathrm{t}} + \frac{\partial (\boldsymbol{F}_i - \boldsymbol{F}_{vi})}{\partial \xi_i} = 0 \tag{3}$$

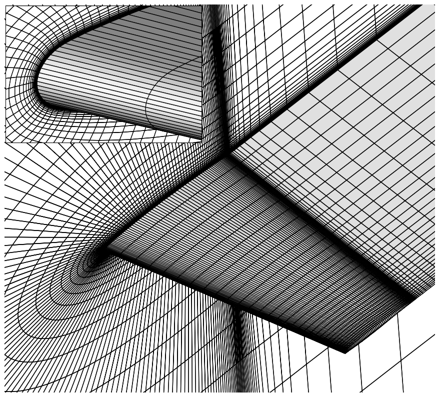
where

$\boldsymbol{F}_i = \boldsymbol{S}_{ij} \boldsymbol{f}_j$, $\boldsymbol{F}_{vi} = \boldsymbol{S}_{ij} \boldsymbol{f}_{vj}$, $\boldsymbol{S}_{ij} = \boldsymbol{J}\dfrac{\partial \xi_i}{\partial x_j}$ and $\boldsymbol{J}$ is the transformation Jacobian.

In optimization process the wing section shape varies continuously and a lot of iteration of field solution is required. So a fast, robust and high qualitative grid generation method is essential. An improved transfinite interpolation grid generation method, in which the orthogonality control and elliptic smooth are added, is used. The grid topology for flow field calculation is C grids for 2-D and C-H grids for 3-D. Fig.1 gives the 2D and 3D grids.



(a) Two dimension C-grid (321×65)

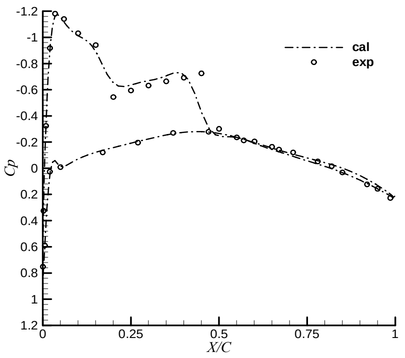


(b) Three dimension C-H grid （209×49×49）

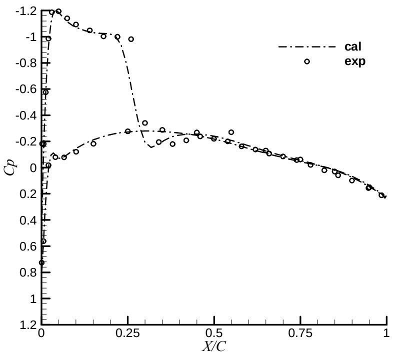
Fig. 1 Computation grid

Jameson's cell central finite volume method [5] is used for the space discretization in the inviscid flux term $\boldsymbol{F}_i$; A central difference method is adopted for viscous flux term $\boldsymbol{F}_{vi}$, and Baldwin-Lomax turbulence model [6] is used to calculate the turbulent viscosity, and artificial dissipation terms are added to prevent oscillation. For temporal discretization, a five step Runge-Kutta explicit time stepping scheme is used. In order to accelerate the convergence, local time stepping, implicit residual smooth and multigrid technical are used.

In order to validate the code of RANS flow solver, the flow field of ONERA M6 wing is calculated at Ma=0.839, Re=11.72×10^6, α=3.06°. The grid number is 209×49×49. Fig.2 gives the comparison of the calculated and the experimental pressure distributions on two span-wise positions. The agreement is very good.



(a) $\eta = 0.65$



(b) $\eta = 0.90$

Fig. 2 Pressure distribution of ONERA M6 wing

3 Design Variables

In optimization design the shape geometry is modified adding a linear combination of Hicks and Henne shape functions [1] as follows:

$$y = y_{base} + \sum_{k=1}^{n_v} \delta_k b_k \qquad (4)$$

$$b_k = sin^3(\pi x^{e(k)}), \qquad (5)$$

$$e(k) = \frac{\ln(0.5)}{\ln(x_k)} \qquad (6)$$

where x is the chordwise coordinate and $0 \le x \le 1$, x_k represents the location of the maximum b_k, Fig.3 shows the shape of these functions. They are added to an initial airfoil shape to form a new shape. The weight b_k of these shape functions are then the design variables. For 3-D problem, along the wing span-wise direction 5 sections are chosen as control sections in which the tip section is fixed. In each of them shape functions are added to form new wing.

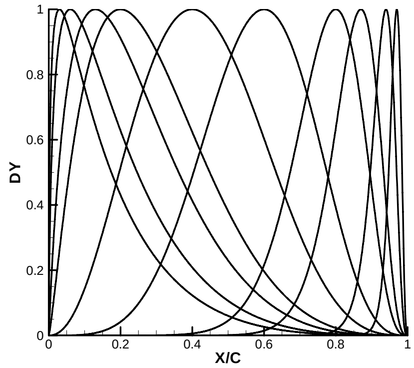


Fig. 3 The Hicks-Henne shape functions

4 Response Surface Methodology

Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques for using to obtain a relationship between a specified dependent variable (the response) and a number of independent variables (the predictor variables) [7]. The model used to describe the relationship between the response and the predictor variables is known as the response model and may be written in general as follows:

$$y = f(x_1, x_2, \cdots, x_{n_v}) + \varepsilon \qquad (7)$$

In common ε is treated as statistical error, often assuming it to have a normal distribution with mean zero. x_1, x_2, x_3, …, x_{n_v} are the design variables. The form of the true response function f is unknown and perhaps very complicated, usually assumed as a second-order polynomial:

$$y^{(p)} = c_0 + \sum_{i=1}^{n_v} c_i x_i + \sum_{i=1}^{n_v} c_{ii} x_i^2 + \sum_{j=1}^{n_v-1} \sum_{i=j+1}^{n_v} c_{ij} x_j x_i \qquad p = 1, \ldots, n_s \qquad (8)$$

where n_v is the number of the design variables, n_s is the number of the sample points. The number of the terms in second-order polynomial is $n_{rc} = (n_v + 1)(n_v + 2)/2$.

The model in terms of the observations, equation (8) may be written in matrix notation as:

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{c} + \boldsymbol{\varepsilon} \qquad (9)$$

where

$\boldsymbol{y}=[y^1,y^2,y^3,\ldots,y^{n_s}]^{\mathrm{T}}$, $\boldsymbol{c}=[c_1,c_2,\ldots,c_{n_{rc}}]^{\mathrm{T}}$

and $\boldsymbol{X}$ is the $n_s\times n_{rc}$ matric, as follow:

$$\boldsymbol{X}=\begin{bmatrix}\varphi_1(x^{(1)}) & \cdots & \varphi_{n_{rc}}(x^{(1)})\\ \vdots & \ddots & \vdots\\ \varphi_1(x^{(n_s)}) & & \varphi_{n_{rc}}(x^{(n_s)})\end{bmatrix}$$

The regression coefficient $\boldsymbol{c}$ can be determined using the least-square fitting.

$$\boldsymbol{c}=(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{Y} \qquad (10)$$

As a selection technique of sample points, the D-optimality condition is used. The D-optimality criterion states that the n_s sample points to be chosen are that maximize the determinant $\left|\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X}\right|$, therefore minimize the determinant of the covariance matric of $\boldsymbol{c}$:

$$\operatorname{cov}(c_i,c_j)=\sigma^2(\mathbf{X}^T\mathbf{X})^{-1} \qquad (11)$$

It is known to be sufficient to construct a response model with n_s of 1.5~3 times n_{rc} [1]. In Ref.1 121 experiments are selected when n_v is 10, and 201 sample points are selected when n_v is 12. The number of sample evaluations required for a full quadratic polynomials RS model increase with the square of the number of design variables, seriously preventing their use in high-dimensional design optimization especially using RANS equations for complex 3-D problem. In this study, the design variables is 26 (2-D) and 24 (3-D), so about 500 sample points are need. The computational cost is can't be affordable in a single PC.

In contrast, the propose modification to the RS models which cancel the second-order cross items of the full quadratic polynomials as follow:

$$y^{(p)}=c_0+\sum_{j=1}^{n_v}c_jx_j+\sum_{j=1}^{n_v}c_{jj}x_j^2+\varepsilon \qquad p=1,\ldots,n_s \qquad (12)$$

Then the relation of the n_s and n_v become linearity. The computational cost for construct the RS models can greatly be reduced and acceptable, and can approximate the original function, without significantly sacrificing the accuracy of the approximation in small design space. If the design space is large, the step-wise technology can be used until the design requirement is achieved.

The final models determined are evaluated by calculating some statistic measures such as: the coefficient of multiple determination R^2 , the adjusted R-square R_a^2 and root mean square error $\%RMSE$. These terms may be defined as:

$$R^2=1-\frac{SS_E}{SS_T} \qquad (13)$$

$$R_a^2=\frac{SS_E/(n_s-n_{rc}-1)}{SS_T/(n_s-1)} \qquad (14)$$

$$\%RMSE=100\sqrt{\frac{1}{n_s}\sum_{i=1}^{n_s}(y_i-y_i^{(p)})^2}\Bigg/\frac{1}{n_s}\sum_{i=1}^{n_s}y_i \qquad (15)$$

$$SS_E=\sum_{i=1}^{n_s}(y_i-y_i^{(p)})^2 \qquad (16)$$

$$SS_T=\sum_{i=1}^{n_s}y_i^2-(\sum_{i}^{n_s}y_i^{(p)})^2\Big/n_s \qquad (17)$$

After construct the RS models, an appropriate optimization algorithm should be chosen. In this study, the BOX complex shape method [10] is used. Fig.4 shows the design cycle using RSM.

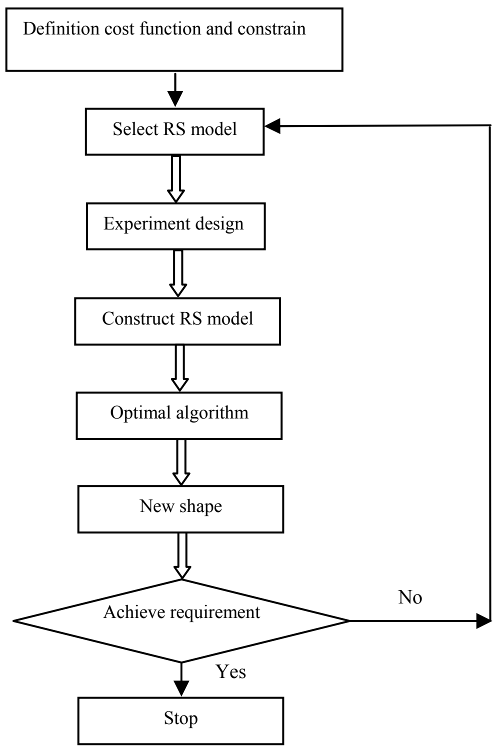


Fig. 4 Design cycle

5 Numerical result and discussion

The present method is applied to a drag minimization problem of 2-D and 3-D geometrics under transonic flow conditions.

5.1 2-D Case

RAE2822 is selected as the baseline airfoil of the design study. The design condition is imposed as Mach number 0.73 at an angle of attack of 2.70 degrees and the Reynolds number is 6.5 million. The computational grid system is 321×65 C-type, as showing in Fig.1 (a). The cost function for the airfoil optimization design of drag minimization under the aerodynamic character such as lift and moment coefficients and geometric constraints as shown:

$$I = C_d \tag{18}$$

$$\begin{gathered} C_l / C_{l_0} \geq 0.99 \\ \left|C_m\right| \leq \left|C_{m_0}\right| \\ A / A_0 \geq 0.99 \end{gathered} \tag{19}$$

where C_{l0}, C_{d0}, C_{m0}, A_0 are the initial lift, drag, moment coefficient and area. 13 Hick-Henne functions are respectively used to modify the upper and lower surface of airfoil. The total number of design variables is 26.

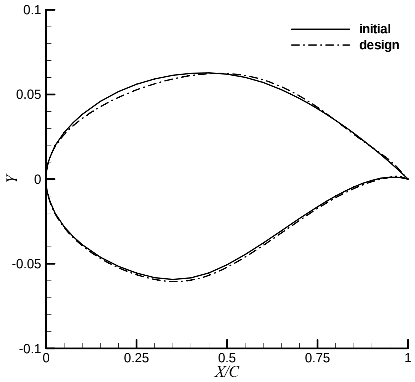
80 sample points are selected using D-optimality criterion. After two design cycle the drag is reduced to 80% of initial value. The aerodynamic performances and geometric parameters are show in Table.1. The *Cp* distributions of RAE 2822 and designed airfoil are displayed in Fig.4. A shock-free airfoil can be obtained after the design optimization and constraints can be satisfied. The fitting quality of RS model is show in Table.2. These statistic measures R^2 and R_a^2 for C_l, C_d, C_m models are larger than 0.95 and RMSE% are less than 1.2%, the response surface models which construct using equation (12) are fitted successfully. Therefore the models are sufficient to model the cost function and constraints for this 2-D transonic viscous flow problem.

Table.1 Airfoil performances comparison

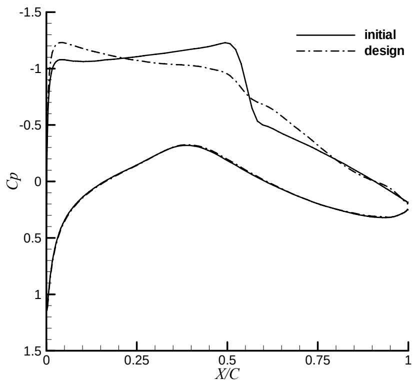
	C_l	C_d	C_m	A
RAE2822	0.7638	0.01780	-0.09313	0.0777
Design (Predict)	0.7630	0.01401	-0.08995	
Design (CFD)	0.7613	0.01423	-0.09004	0.0781
Change(%)	-0.3	-20.1	+3.3	+0.5
Error(%)	0.2	1.5	0.0	

Table2 Fitting quality

	R^2	R_a^2	$RMSE\%$
C_l	0. 986	0. 959	0. 86
C_d	0. 983	0. 951	1. 17
C_m	0. 999	0. 995	0. 11



(a) Airfoil shape



(b) Computation pressure

Fig. 4 The comparison of airfoil shape and pressure distribution for RAE2822 airfoil (initial) and design airfoil

5.2 3-D Case

The test problem is a drag minimization of ONEAR M6 wing with lift coefficient and geometric constraints. The flow condition is

Mach number 0.839 at an angle of attack of 3.09 degrees and the Reynolds number is 11.72 million. The computational grid system is 209×49×49 CH-type, as showing in Fig.1 (b). For this case, the aerodynamic cost function and constraints are equation (18) and (19).

Along the wing span-wise direction 5 sections are selected as control sections in which tip section is fixed. In each of them six design variables are used to modify the upper surface while the lower surface vary in the same manner [8] and the section area can be kept constant.

80 sample points are selected using D-optimality criterion. After two design cycle the drag is reduced to 90% of initial value. The aerodynamic performances and geometric parameters are show in Table.3. Fig.5 illustrates the contour comparison of pressure on wing upper surface for design and initial case. Fig.6 illustrates the comparison of section pressure of design and initial wing. The strong λ-shape shock waves are smeared through the design optimization and constraints can be satisfied. But the change of the shape is very small. The fitting quality of RS model is show in Table.4. These statistic measures R^2 and R_a^2 for C_l, C_d models are larger than 0.96 and RMSE% are less than 1.2%, the response surface models which construct using equation (12) are fitted successfully. Therefore the models are sufficient to model the cost function and constraints for this 3-D transonic viscous flow problem.

Table.3 Wing performances comparison

	C_L	C_D
ONEAR M6	0.2674	0.01855
Design (Predict)	0.2672	0.01658
Design (CFD)	0.2668	0.01668
Change(%)	-0.2	-10.0
Error(%)	0.1	0.6

Table4 Fitting quality

	R^2	R_a^2	$RMSE\%$
C_L	0. 994	0. 976	0. 74
C_D	0. 986	0. 967	1. 14

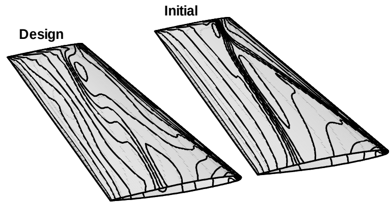
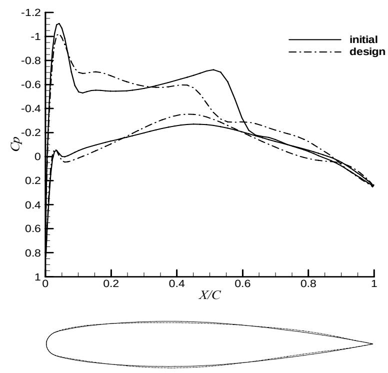
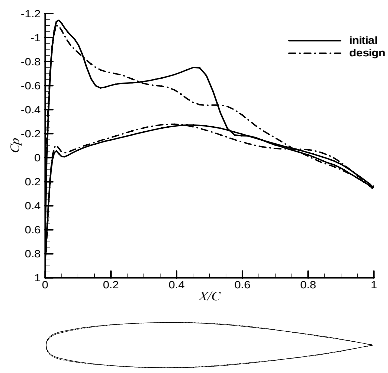


Fig. 5 Contour comparison of pressure on wing upper surface for optimum and initial case



(a) $\eta = 0.31$



(b) $\eta = 0.51$

Fig. 6 The comparison of section shape and pressure distribution for ONERA M6 wing (initial) and design wing

6 Conclusions

In this study, a way to improve the efficientcy of RSM by using quadratics without second-order cross items as RS models are proposed and its applicability in high-dimensional 2-D and 3-D transonic ASO problem successfully. It is demonstrated that:

1、The aerodynamic performance of transonic airfoil and wing can be greatly improved with multi-constrains by using the present method.
2、The accuracy of RS models are good enough for design variables as many as about 26 in small design space ASO problem.
3、The computation costs are acceptable even in a single PC. So it is suitable for high-dimensional MDO problem using high-fidelity analysis tools.

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