A NEW METHOD FOR OPTIMUM ALLOCATION OF DESIGN REQUIREMENTS IN AIRCRAFT CONCEPTUAL DESIGN

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Abstract

A new method, Collaborative Allocation (CA), is proposed to solve large-scale optimum allocation problem in aircraft conceptual design. According to the characteristic of optimum allocation in aircraft conceptual design. the principle and mathematical model of CA is established. The optimum allocation problem is decomposed into one main optimization problem and several sub-optimization problems. A group of design requirements for subsystems are provided by the main system respectively, and the subsystems execute their own optimizations or further provide detailed design requirements to bottom components of aircraft, such as spars, ribs and skins, etc. The subsystems minimize the discrepancy between their own local variables and corresponding allocated value, and then return optimization results to main optimization. Main optimization is performed to reallocate design requirements for improving integration performance and progressing toward compatibility between subsystems. CA provides general optimum allocation architecture and is easy to be carried out. Furthermore concurrent computation can also be realized. Two numerical examples of optimum reliability allocation are used to describe the implementation procedure of CA for two-level allocation and three-level allocation respectively, and to preliminarily validate its correctness and effectiveness. Then an engineering problem further proves our method is applicable for engineering design. It is shown that the developed method can be successfully used in optimum allocation of design requirements. Then taking weight requirement allocation as example, the mathematical model and solution procedure for collaborative allocation of design requirement in aircraft conceptual design is briefly depicted.

1 Introduction

Optimum allocation of design requirements (reliability, weight, cost, etc.) has been and is an important problem in aircraft conceptual design. A good allocation of design requirements can shorten design cycle, improve performance and reduce cost, etc. Since optimum allocation is to acquire best integration performance by allocating design requirements reasonably, it is an optimization problem in essence. Optimum allocation in aircraft conceptual design is a complicated large-scale problem. Apparently the conventional allocation depending on experience and statistics can hardly provides the best design results. Direct Method (DM) [1,2] and Decomposition Coordination Method (DCM) [3,4] are two conventional methods for optimum allocation. DM is problem dependent and cannot reflect comparatively independence of subsystems [3]. DCM is frequently used for large-scale engineering optimization. It transforms an all-at-once optimum allocation problem into many small-scale optimization problems in multi-level nested optimization architecture. Each sub-optimization shares in the duty of optimizing original objective function by minimizing or maximizing part of it. Father optimization requires optimum sensitivity provided by its daughter optimization. More levels the system is decomposed, more complicated the nested optimization of DCM goes and worse convergence appears. It is proven by practice that DCM is very sensitive to step size, which indicates it is not so well in robustness. In addition, like DM, DCM cannot also provide a general allocation framework. For disadvantages mentioned above, DCM is still not so appropriate for aircraft conceptual design. This study is motivated by developing a new method with general allocation framework, better robustness and easy to be carried out, which is appropriate for large-scale optimum allocation problem in aircraft conceptual design.

According to our experience [5-8], it is found that Collaborative Optimization (CO) has a few features that are applicable to optimum allocation. Firstly, CO is designed for multidisciplinary complex problems. Secondly, CO provides a general optimization framework. Thirdly, system level providing disciplinary level with targets of variables is similar to allocation of design requirements, which deserves attention mostly. And lastly, coordination for variables of different disciplinary can easily be associated with repeated coordination for design requirements allocation.

In this study, a new method, Collaborative Allocation (CA), is proposed to solve large-scale optimum allocation problem in aircraft conceptual design. CA is of similar solution procedure with CO. CA provides general optimum allocation architecture and is easy to be carried out. And concurrent computation can also be realized. Two numerical examples of reliability optimum allocation are used to describe the implementation procedure of CA for two-level allocation and three-level optimum allocation, respectively, and to preliminarily validate its correctness and effectiveness. Then an engineering problem is to further prove our method is applicable for engineering design. And in last part of this paper, weight requirement allocation is taken as example to briefly describe the mathematical model and solution procedure for collaborative allocation of design requirement in aircraft conceptual design.

2 Optimum Allocation in Aircraft Conceptual Design

In aircraft design process, before detail design begins, design requirements must be assured to indicate some design constraints, such as reliability and weight constraints for each part of aircraft. The problem, which is how to allocate design requirements can make the system (such as an aircraft) achieving best integration performance, is defined as optimum allocation of design requirements or optimum allocation as abbreviation. For aircraft design, conventional design requirements need to be defined include reliability, cost and weight requirements. They are usually allocated according to topology structure of aircraft, which is characteristic of hierarchy and decomposition. Aircraft can be hierarchically decomposed into wing, fuselage, horizontal tail and vertical tail, etc, or further decomposed into spars, ribs, skins and frames, etc, as shown in Fig.1. In this way, design requirements may be allocated to large-scale parts (such as wing and fuselage), or to medium-scale components (such as wing box and spar) in more detail. It is apparent that the former belongs to two-level allocation problem and the latter belongs to three-level one. Ref.9 suggests that the bottom level of decomposed aircraft had better be medium-scale components. It can be concluded that the approach of three-level allocation architecture is sufficient for optimum allocation problem in aircraft conceptual design.

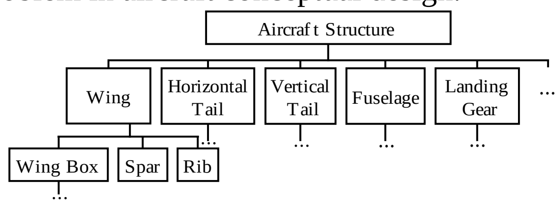


Fig. 1 Hierarchical decomposition framework of aircraft

3 The Principle and Mathematical Model of Collaborative Allocation

3.1 Principle of CA

For CA, the optimum allocation problem is decomposed into one main optimization problem and several sub-optimization problems. Main optimization provides subsystems with design requirements. Sub-optimization is to minimize the discrepancy between allocation value and its own corresponding variables. Sub-optimization optimizes its local variables, such as structure size, or provides bottom components, such as wing box, with detailed design requirements. The results of sub-optimization are returned to main optimization to construct compatibility constraints. Then main optimization is performed to reallocate design requirements for improving integration performance and progressing toward compatibility between subsystems.

CA is of two-level optimization architecture, as shown in Fig.2. Compared with DCM, CA owns general allocation framework and really realizes separating main optimization from sub-optimization. The allocation procedure of CA is almost same as optimization procedure of CO. Therefore most CO algorithms [5-8], such as response surface based CO [6], Subspace Optimization Algorithm (SAO)[7], can be applied in CA.

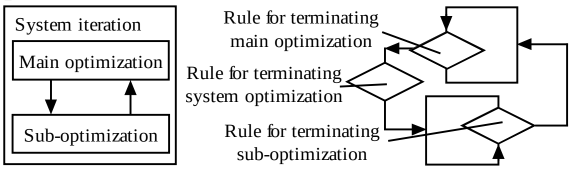
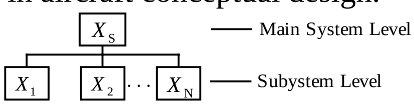
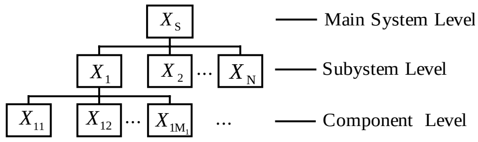


Fig. 2 Optimization architecture of CA

The allocation framework of CA is illustrated in Fig.3. Where, N is the number of subsystems, M_i is the number of components in subsystem i, X is design requirement, and subscript '$_S$', '$_i$' ($i=1,2,\cdots,N$) and '$_{ij}$' ($j=1,2,\cdots,M_i$) indicates corresponding value of system, subsystem i and component j in subsystem i, respectively. For case 1 in Fig.3, main system provides subsystems with design requirements and subsystems optimize their local variables. And for case 2 in Fig.3, main system provides subsystems in medium level with design requirements and subsystems gives detailed design requirements for components in bottom level and optimizes local variables of components. Accordingly, in the aspect of allocation architecture, our method is appropriate for conventional optimum allocation problem in aircraft conceptual design.



(1) Framework of two-level allocation



(2) Framework of three-level allocation

Fig. 3 Allocation framework of CA

3.2 Mathematical Model of CA

According to the principle defined in section 2.1, mathematical model of CA is established in Eq.1 and Eq.2.

$$\begin{aligned}
\text{Min } & J_i = \left(X_i - P_{i1}^{sys}\right)^2 + \left(\boldsymbol{Y}_i - \boldsymbol{P}_{i2}^{sys}\right)^2 \\
\text{s.t. } & \boldsymbol{Y}_i = \boldsymbol{g}_i(\boldsymbol{T}_i) \in [\boldsymbol{p}_i, \boldsymbol{q}_i] \\
& X_i = h_i(\boldsymbol{T}_i) \in [a_i, b_i] \\
& \boldsymbol{T}_i \in [\boldsymbol{m}_i, \boldsymbol{n}_i]
\end{aligned} \tag{1}$$

$$\begin{aligned}
\text{Min } & F_S(\boldsymbol{P}) = F_S(\boldsymbol{P}_{12}, \cdots, \boldsymbol{P}_{N2}) \\
\text{s.t. } & P_{S1} = G_{S1}(P_{11}, \cdots, P_{N1}) \le P_{S1}^0 \\
& J_1 = \left(X_1^* - P_{11}\right)^2 + \left(\boldsymbol{Y}_1^* - \boldsymbol{P}_{12}\right)^2 = 0 \quad \text{(a)} \\
& J_2 = \left(X_2^* - P_{21}\right)^2 + \left(\boldsymbol{Y}_2^* - \boldsymbol{P}_{22}\right)^2 = 0 \quad \text{(b)} \\
& \vdots \\
& J_N = \left(X_N^* - P_{N1}\right)^2 + \left(\boldsymbol{Y}_N^* - \boldsymbol{P}_{N2}\right)^2 = 0 \quad \text{(c)} \\
& P_{11} \in [a_1, b_1], \cdots, P_{N1} \in [a_N, b_N] \\
& \boldsymbol{P}_{12} \in [\boldsymbol{p}_1, \boldsymbol{q}_1], \cdots, \boldsymbol{P}_{N2} \in [\boldsymbol{p}_N, \boldsymbol{q}_N]
\end{aligned} \tag{2}$$

Eq.1 is sub-optimization model. Where, P_{i1}^{sys} and $\boldsymbol{P}_{i2}^{sys}$, are design requirement and auxiliary variables for subsystem i provided by main system, respectively. X_i and $\boldsymbol{Y}_i$, as variables in subsystem i, correspond to allocated value above. If the system is decomposed in two-level, $\boldsymbol{T}_i$ are local variables in subsystem level, by which X_i and $\boldsymbol{Y}_i$ can be calculated. And for three-level decomposition, $\boldsymbol{T}_i$ are local variables in component level. In this condition, X_i is acquired through calculation of design requirements of components in subsystem i, and $\boldsymbol{Y}_i$ can be gotten in the similar way. Eq.2 is main-optimization model. $\boldsymbol{P}$ are design variables. The first inequality constraint shows that the prescribed design requirement cannot be exceeded. (a)-(b) are compatibility constraints, which indicates compatibility between allocation value prescribed by main system and expected value for subsystem. Superscript '^{sys}' and '*' indicates allocated value and expected value, respectively.

4 Applications of CA in Reliability Optimum Allocation

4.1 A Numerical Example of Two-level Allocation Architecture

Reliability optimum allocation problem in Eq.3 is used to explain how to apply CA for two-level optimum allocation and preliminarily validate it.

$$\begin{aligned}
&\text{Find } \boldsymbol{R} = [R_1, R_2] \\
&\min \ C_S = C_1 + C_2 \\
&\text{s.t.} \ R_S = R_1 R_2 \geq 0.9 \\
&\quad C_1 = 0.8(1 - \ln(1 - R_1)/10) \leq 1.1 \\
&\quad C_2 = 0.7(1 - \ln(1 - R_2)/8) \leq 1.0 \\
&\quad R_1, R_2 \in [0.5, 0.99]
\end{aligned} \tag{3}$$

Where, R is reliability requirement and C is cost. Subscript '$_S$' and '$_i$' indicates corresponding value of main system and subsystem i, respectively.

According to CA, Sub-optimization in Eq.4 and 5 and main optimization in Eq.6 is established.

$$\begin{aligned}
&\text{Find } T = R_1 \\
&\min \ f = (R_1 - R_1^{sys})^2 + (C_1 - C_1^{sys})^2 \\
&\text{s.t.} \ C_1 = 0.8(1 - \ln(1 - R_1)/10) \leq 1.1 \\
&\quad R_1 \in [0.5, 0.99]
\end{aligned} \tag{4}$$

$$\begin{aligned}
&\text{Find } T = R_2 \\
&\min \ f = (R_2 - R_2^{sys})^2 + (C_2 - C_2^{sys})^2 \\
&\text{s.t.} \ C_2 = 0.7(1 - \ln(1 - R_2)/8) \leq 1.0 \\
&\quad R_2 \in [0.5, 0.99]
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\text{Find } \boldsymbol{P} = [R_1, R_2, C_1, C_2] \\
&\min \ C_S = C_1 + C_2 \\
&\text{s.t.} \ R_S = R_1 R_2 \geq 0.9 \\
&\quad \text{Con1} = 0, \ \text{Con2} = 0 \\
&\quad R_1, R_2 \in [0.5, 0.99]
\end{aligned} \tag{6}$$

Where, Con$i$ is compatibility constraint corresponding to subsystem i, superscript '^{sys}' indicates value allocated by main system.

SAO [7] is transplanted into CA to solve this problem, the flowchart of which is shown in Fig.4. Initial allocation is provided experientially. Auxiliary variables including cost of subsystems are introduced to calculate total cost. Sub-optimization is to minimize the discrepancy between allocation value and corresponding value in subsystems. After that linear approximation constraints representing sub-optimization are established and return to main optimization to replace initial compatibility constraints. Then main optimization is carried out with reliability and cost of subsystems as design variables, the results of which are reallocated to subsystems. As the iteration going on, the linear approximation constrains provided by subsystems are continuously appended in main optimization. All these linear constraints gradually approach initial constraints, until convergence is achieved.

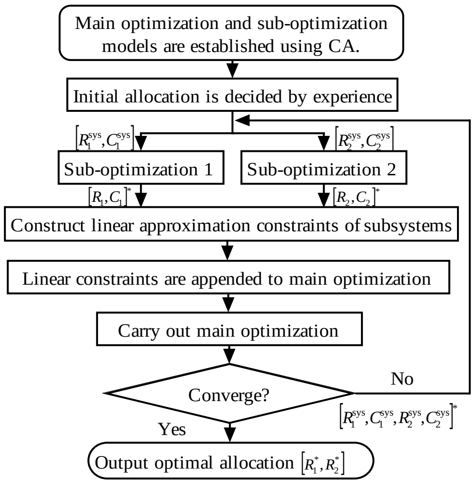


Fig. 4 Flowchart of optimum allocation in Eq.3 using CA

DM and CA are both used to solve problem in Eq.3 and results are listed in Table 1 for comparison. And iteration histories for main optimization and sub-optimizations using CA are shown in Fig.5 and 6, respectively. Table 1 shows that, with constraint of $R_S \geq 0.9$, $C_1 \leq 1.1$ and $C_2 \leq 1.0$, the lowest cost of 1.9973 and 1.9974 is acquired using CA and DM, respectively, which preliminarily validate our method. Fig.5 and 6 indicates that CA is of better convergence performance, and compatibility constraints finally achieve ideal value zero.

Table 1 Two-level reliability optimum allocation results using DM and CA

	Subsystem 1		Subsystem 2		Main System	
	R_1	C_1	R_2	C_2	R_S	C_S
IA	0.94	0.85	0.96	0.9		
DM	0.9497	1.0392	0.9477	0.9582	0.9000	1.9974
CA	0.9510	1.0413	0.9464	0.9560	0.9000	1.9973

IA=Initial Allocation

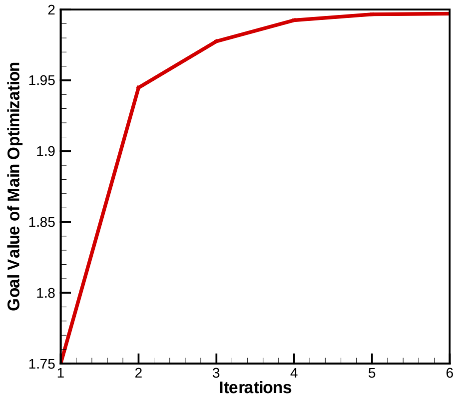


Fig. 5 Iteration history of main optimization

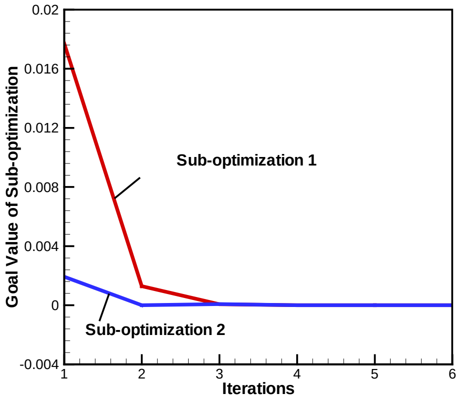


Fig. 6 Iteration history of sub-optimization

4.2 A Numerical Example of Three-level Allocation Architecture

The reliability optimum allocation problem in Eq.7 is used to validate CA for three-level optimum allocation. Through Fig.7 it is apparent that system is composed of five subsystems and each subsystem encompasses two components.

$$
\begin{aligned}
&\text{Find } R=\left[R_{11},R_{12},\cdots,R_{51},R_{52}\right]\\
&\min\ C_S=\sum_{i=1}^{5}\sum_{j=1}^{2}C_{ij}\left(R_{ij}\right)\\
&\text{s.t.}\quad R_S\ge 0.999\\
&\qquad R_S=R_5+R_1\left(1-R_5\right)\left(R_2R_3+R_4-R_2R_3R_4\right)\\
&\qquad 0.5\le R_{ij}\le 0.98, i=1,2;\ j=1,2\\
&\qquad 0.2\le R_{ij}\le 0.99, i=3,4,5;\ j=1,2\\
&\qquad R_i=R_{i1}R_{i2}\ge 0.5, i=1,2\\
&\qquad 0.5\le R_i\le 0.998, i=3,4,5\\
&\qquad R_i=1-\left(1-R_{i1}\right)\left(1-R_{i2}\right), i=3,4,5\\
&\qquad C_{i1}\left(R_{i1}\right)=R_{i1}^2/3,\quad C_{i2}\left(R_{i2}\right)=R_{i2}^2/2, i=1,2\\
&\qquad C_{i1}\left(R_{i1}\right)=\left[\ln\left(1-R_{i1}\right)\right]^2/100, i=3,4,5\\
&\qquad C_{i2}\left(R_{i2}\right)=\left[\ln\left(1-R_{i2}\right)\right]^2/60, i=3,4,5
\end{aligned}
\tag{7}
$$

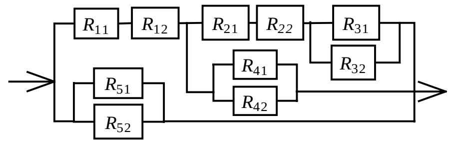


Fig. 7 Topology of system

Where, R is reliability requirement and C is cost. Subscript '$_S$', '$_i$' and '$_{ij}$'indicates corresponding value of main system, subsystem i and component j in subsystem i, respectively.

According to CA, sub-optimization in Eq.8, Eq.9 and main optimization in Eq.10 is established. Eq.8 is the optimization model for subsystem 1 and 2, while Eq.9 shows that for subsystem 3-5. Main optimization takes the duty of allocating reliability requirements for subsystems, and sub-optimization defines those for components. Auxiliary variables, $[C_1^{sys},\cdots,C_5^{sys}]$, are also transmitted to subsystems in addition to reliability requirements to calculate total cost.

$$
\begin{aligned}
&\text{Find } \boldsymbol{T}=\left[R_{i1},R_{i2}\right],\ i=1,2\\
&\min\ f=\left(R_i-R_i^{sys}\right)^2+\left(C_i-C_i^{sys}\right)^2\\
&\text{s.t.}\quad C_i=C_{i1}+C_{i2},\qquad R_i=R_{i1}R_{i2}\le 0.5\\
&\qquad C_{i1}\left(R_{i1}\right)=R_{i1}^2/3,\quad C_{i2}\left(R_{i2}\right)=R_{i2}^2/2\\
&\qquad 0.5\le R_{ij}\le 0.98,\quad j=1,2
\end{aligned}
\tag{8}
$$

$$
\begin{aligned}
&\text{Find } \boldsymbol{T}=\left[R_{i1},R_{i2}\right],\ i=3,4,5\\
&\min\ f=\left(R_i-R_i^{sys}\right)^2+\left(C_i-C_i^{sys}\right)^2\\
&\text{s.t.}\quad C_i=C_{i1}+C_{i2}\\
&\qquad 0.5\le R_i=1-\left(1-R_{i1}\right)\left(1-R_{i2}\right)\le 0.998\\
&\qquad C_{i1}\left(R_{i1}\right)=\left[\ln\left(1-R_{i1}\right)\right]^2/100\\
&\qquad C_{i2}\left(R_{i2}\right)=\left[\ln\left(1-R_{i2}\right)\right]^2/60\\
&\qquad 0.2\le R_{ij}\le 0.99,\quad j=1,2
\end{aligned}
\tag{9}
$$

$$
\begin{aligned}
&\text{Find } \boldsymbol{X}=\left[R_1,\cdots,R_5,\mathrm{C}_1,\cdots,\mathrm{C}_5\right]\\
&\min\ C_S=\sum_{i=1}^{5}C_i\\
&\text{s.t.}\quad R_S\ge 0.999\\
&\qquad R_S=R_5+R_1\left(1-R_5\right)\left(R_2R_3+R_4-R_2R_3R_4\right)\\
&\qquad \mathrm{Con1}=0,\ \cdots,\mathrm{Con5}=0\\
&\qquad R_i\ge 0.5,\quad i=1,2\\
&\qquad 0.5\le R_i\le 0.998,\quad i=3,4,5
\end{aligned}
\tag{10}
$$

Where, Con$i$ is compatibility constraint corresponding to subsystem i, superscript '^{sys}' indicates value allocated by main system.

Response surface based collaborative optimization [6] is transplanted into CA to solve this problem, the flowchart of which is shown in Fig.8.

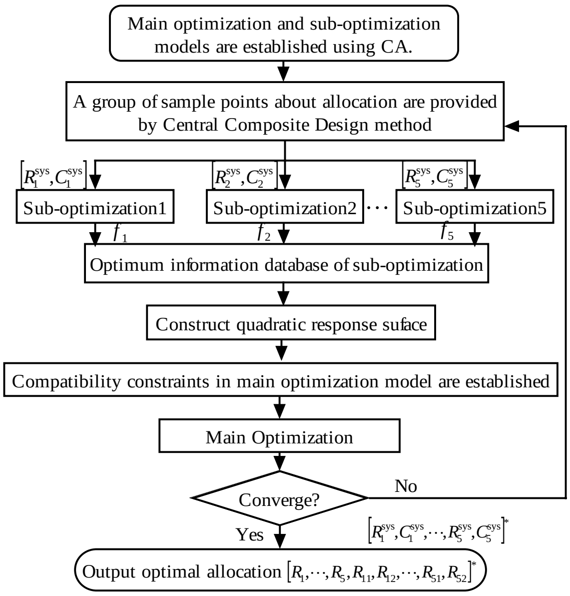


Fig. 8 Flowchart of CA solving allocation problem in Eq.7

DM, DCM and CA are all adopted to solve optimum allocation problem in Eq.7 and results are listed in Table 2 for comparison. Where, S_{ij} ($i=1,2,\cdots,5$, $j=1,2$) represents component j in subsystem i. And Fig.5 and 6 shows iteration histories for main optimization and sub-optimizations of CA, respectively. Table 2 shows that, with constraint satisfaction, DM, DCM and CA provide best allocation of the lowest cost of 1.1266, 1.1533 and 1.1397. The solution of DM is a little better than that of CA, which is due to compatibility constraints in main optimization of CA are approximated by quadratic response surface method. Nevertheless, for problem of multiple variables and complicated analysis, such as aircraft conceptual design, CA is easier to be realized. While compare to DCM, the solution of CA is better, which may be caused by DCM being sensitive to step size. And Fig.5 and 6 shows CA's better convergence performance.

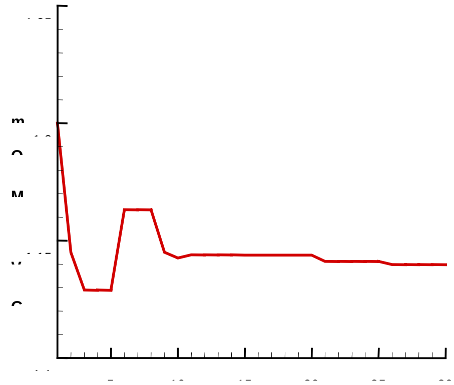


Fig. 9 Iteration history of main optimization

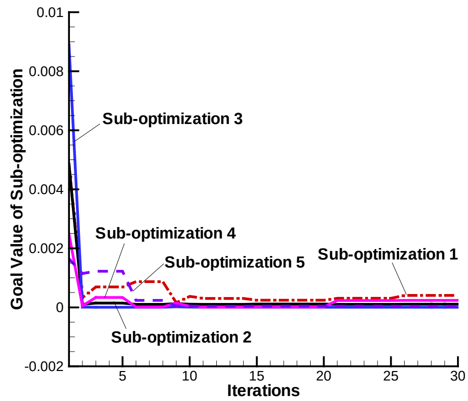


Fig. 10 Iteration history of sub-optimization

Table 2 Three-level reliability optimum allocation results using DM, DCM and CA

		Subsystem (S1)		Subsystem (S2)		Subsystem (S3)		Subsystem (S4)		Subsystem (S5)	
		S_{11}	S_{12}	S_{21}	S_{22}	S_{31}	S_{32}	S_{41}	S_{42}	S_{51}	S_{52}
DM	R_{ij}	0.8093	0.6607	0.7815	0.6398	0.3536	0.2687	0.7746	0.6082	0.9792	0.9040
	C_{ij}	0.2183	0.2183	0.2036	0.2046	0.0019	0.0016	0.0222	0.0146	0.1499	0.0915
	R_i	0.5348		0.5000		0.5273		0.9117		0.9980	
		$R_S = 0.9990$ $C_S = 1.1266$									
DCM	R_{ij}	0.8472	0.6917	0.7826	0.6389	0.5572	0.3870	0.6159	0.4363	0.9795	0.9027
	C_{ij}	0.2392	0.2392	0.2042	0.2041	0.0066	0.0040	0.0092	0.0055	0.1510	0.0904
	R_i	0.5860		0.5000		0.7286		0.7835		0.9980	
		$R_S = 0.9990$ $C_S = 1.1533$									
CA	R_{ij}	0.8102	0.6608	0.7830	0.6386	0.3757	0.2001	0.8465	0.2039	0.9900	0.8000
	C_{ij}	0.2188	0.2183	0.2044	0.2039	0.0022	0.0008	0.0351	0.0009	0.2121	0.0432
	R_i	0.5354		0.5000		0.5006		0.8778		0.998	
		$R_S = 0.9990$ $C_S = 1.1397$									

4.3 Reliability Optimum Allocation for An Engineering Truss System

In this section, CA is applied in the reliability optimum allocation problem for an engineering truss in Fig.11. We aim to define reliability requirement for each bar in it.

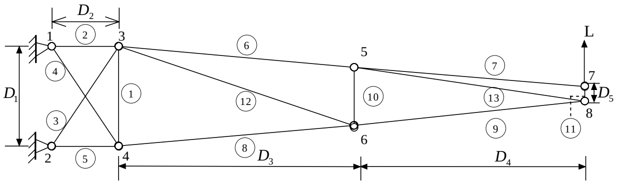


Fig.11 Topology of the engineering truss

The dimension and applied force of the truss is listed in Table 3. The material attributes are listed in Table 4.

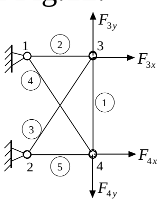
Table 3 Data of dimension and applied force

Parameter/Unit	D_1/m	D_2/m	D_3/m	D_4/m	D_5/m	L/kN
Value	3	2	8	8	0.5	200

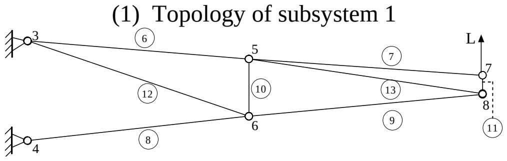
Table 4 The material attributes

Parameter / Unit	Symbol	Value
Density / kg/m^3	ρ	2.68×10^3
Admissible pulling stress / GN/m^2	$[\bar{S}_d]$	0.1724
Admissible crushing stress/ GN/m^2	$[\bar{S}_c]$	0.1724
Modulus of elasticity / GPa	E	69
Intensity variability	V_R	0.1
Load variability	V_L	0.2

Because D_3+D_4 is much larger than D_2, the displacement at node 3 or 4 must be much smaller than that at node 7 or 8. That is to say, comparing with the displacement at node 7 and 8, the displacement at node 3 and 4 is near to zero. Therefore, the truss in Fig.11 can be approximately treated as a system composed of two subsystems in Fig.12.



(1) Topology of subsystem 1



(2) Topology of subsystem 2

Fig.12 Topology of subsystems

The reliability optimum allocation problem is defined in Eq.11. This problem is carried out to allocate appropriate reliability requirement to each bar, on condition that the reliability requirement for system and subsystems no less than prescribed value.

$$\begin{aligned}
&\text{Find } R=[R_1,R_2,\cdots,R_{13}]\\
&\min\ W_S=\sum_{i=1}^{13}W_i\\
&\text{s.t.}\quad R_S = f_R(R_{S1},R_{S2})\ge 0.99\\
&\qquad\ R_{S1} = f_{R1}(R_1,R_2,\cdots,R_5)\ge 0.9999\\
&\qquad\ R_{S2} = f_{R2}(R_6,R_7,\cdots,R_{13})\ge 0.99
\end{aligned}\tag{11}$$

Where, R_S, R_{S1} and R_{S2} is reliability requirement for engineering truss system in Fig.11 and subsystems in Fig.12, respectively. R_i and W_i is reliability requirement and weight of No. i bar, respectively, $i=1,2,\cdots,13$.

According to failure rule, R_{S1} multiplied by $R(S_2|S_1)$ is R_S. Because the lower limit of R_{S1} is much larger than that of R_{S2} and is near to 1, R_S can be approximately calculated through multiplying R_{S1} directly by R_{S2}. That is to say, two subsystems in the engineering truss system are supposing to be series-wound. And due to $R_{S1}\cdot R_{S2}\le R_{S1}\cdot R(S_2|S_1)$, if $R_{S1}\cdot R_{S2}$ is no less than 0.99, $R_{S1}\cdot R(S_2|S_1)$ must also satisfy this constraint. In this condition, the supposition of subsystems being series-wound is credible.

CA is used to solve the reliability optimum allocation problem in Eq.11. Sub-optimization in Eq.12, Eq.13 and main optimization in Eq.14 is established.

$$\begin{aligned}
&\text{Find}\quad \boldsymbol{T}_1=[x_1,x_2,\dots,x_5]^{\mathrm{T}}\\
&\text{Min}\quad J_1=\left(R_{S1}-R_{S1}^{sys}\right)^2+\left(W_{S1}-W_{S1}^{sys}\right)^2\\
&\text{s.t.}\quad R_{S1}=1-P_{f1}\ge 0.9999\\
&\qquad\ \sigma_i\le[\sigma],\quad i=1,2,\cdots,5
\end{aligned}\tag{12}$$

$$\begin{aligned}
&\text{Find}\quad \boldsymbol{T}_2=[x_6,x_7,\dots,x_{13}]^{\mathrm{T}}\\
&\text{Min}\quad J_2=\left(R_{S2}-R_{S2}^{sys}\right)^2+\left(W_{S2}-W_{S2}^{sys}\right)^2\\
&\text{s.t.}\quad R_{S2}=1-P_{f2}\ge 0.99\\
&\qquad\ \sigma_i\le[\sigma],\quad i=6,7,\cdots,13
\end{aligned}\tag{13}$$

$$\begin{aligned}
&\text{Find}\quad \boldsymbol{P}=[R_{S1},R_{S2},W_{S1},W_{S2}]^{\mathrm{T}}\\
&\text{Min}\quad W_S=W_{S1}+W_{S2}\\
&\text{s.t.}\quad R_S=R_{S1}\cdot R_{S2}\ge R_S^0\\
&\qquad\ \text{Con1}=(R_{S1}-R_{S1}^*)^2+(W_{S1}-W_{S1}^*)^2=0\\
&\qquad\ \text{Con2}=(R_{S2}-R_{S2}^*)^2+(W_{S2}-W_{S2}^*)^2=0
\end{aligned}\tag{14}$$

Where, for No. i bar, x_i is section area and σ_i is pulling stress or crushing stress. P_{f1} and P_{f2} is probability of failure for subsystem 1 and 2, respectively. W_{Sk} is weight of subsystem k, and Con$k$ is compatibility constraint corresponding to subsystem k, $k=1,2$. Superscript '^{sys}' and '*' indicates value allocated by main system and expected value of subsystem, respectively. In order to calculate total weight, auxiliary variables, W_{S1}^{sys} and W_{S2}^{sys}, are also transmitted to subsystems in addition to reliability requirements.

Response surface based collaborative optimization [6] is transplanted into CA to solve this problem. In subsystem level, structural reliability optimization is carried out, using O.Ditlevsen's Narrow Reliability Bounds for Structural System[10]. The results are listed in Table 5. The results using DM are also listed in it for comparison. Table 5 shows that, with constraints satisfaction, DM and CA provide best allocation of the lowest weight of 1119.4 and 1119.6, which is almost equal. It indicates that CA is effective for reliability optimum allocation of engineering design. And the iteration history of main optimization in Fig.13 shows CA's better convergence performance.

Table 5 Results using DM and CA for the reliability optimum allocation problem in Eq.11

Variable		DM	CA
Subsystem 1	R_{S1}	0.9999	0.9999
	W_{S1}/kg	213.9947	210.0000
Subsystem 2	R_{S2}	0.9901	0.9901
	W_{S2}/kg	905.3687	909.5588
Main System	R_S	0.9900	0.9900
	W_S/kg	1119.4	1119.6
Optimal section area of bar	x_1/m^2	0.0005	0.0005
	x_2/m^2	0.0152	0.0149
	x_3/m^2	0.0024	0.0027
	x_4/m^2	0.0020	0.0025
	x_5/m^2	0.0147	0.0143
	x_6/m^2	0.0093	0.0093
	x_7/m^2	0.0003	0.0002
	x_8/m^2	0.0107	0.0106
	x_9/m^2	0.0093	0.0094
	x_{10}/m^2	0.0007	0.0007
	x_{11}/m^2	0.0023	0.0024
	x_{12}/m^2	0.0017	0.0018
	x_{13}/m^2	0.0090	0.0093

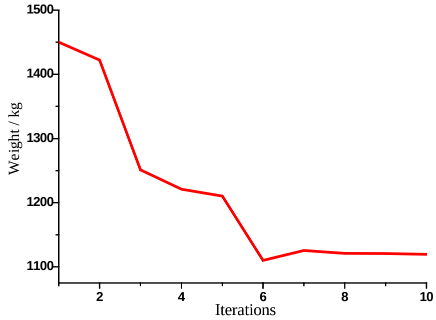


Fig.13 Iteration history of main optimization

5 Design Requirement Collaborative Allocation in Aircraft Conceptual Design

In aircraft conceptual design, designers care much about how to allocate weight requirements. In this section, how CA can be applied in this problem is briefly depicted. According to decomposition framework of aircraft in Fig.1, weight requirement is allocated, the allocation architecture of which is shown in Fig.14. Where, W is weight, R is reliability, subscript '$_S$', '$_w$', '$_f$', '$_{wb}$', '$_{ws}$', '$_{ff}$' and '$_{fs}$' indicates corresponding value of aircraft, wing, fuselage, wing box, spar, frame and crossbeam. Aircraft is composed of a great deal of large-scale parts and medium-scale components, most of which are omitted in Fig.14 for simplification.

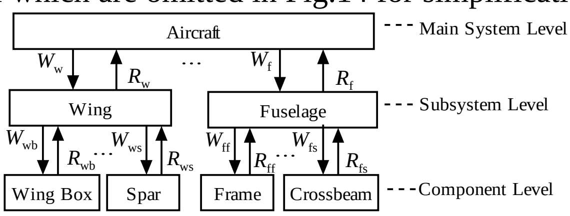


Fig. 14 Simplified weight requirement allocation architecture for aircraft structure

Here, total weight of aircraft, W_S, need to be reasonably allocated for components to achieve highest integration reliability R_S, with constraint that W_S is no more than prescribed weight requirement W_0. Mathematical models for weight requirement allocation problem in Fig.14 are listed in Fig.15. Where, $\boldsymbol{X}$ are local design variables in sub-optimization (such as structure size), by which reliability and weight of components can be expressed (such as $R_{wb}=R_{wb}(\boldsymbol{X}_{wb})$). Since optimum allocation must be finished in aircraft conceptual design, calculation of reliability and weight may rely on simplified analysis model. In Fig.15, constraints at last row in sub-optimization model are side constraints for local design variables, side constraints for reliability and weight of medium-scale components (such as wing box), and side constraints for reliability and weight of large- scale part (such as wing). Constraints at last row in main optimization model are side constraints for reliability and weight of large-scale part, and side constraints for integration reliability and total weight. According to mathematical model in Fig.15, and appropriate CA algorithm is adopted, it is expected that design requirement can be saved and integration performance can be improved.

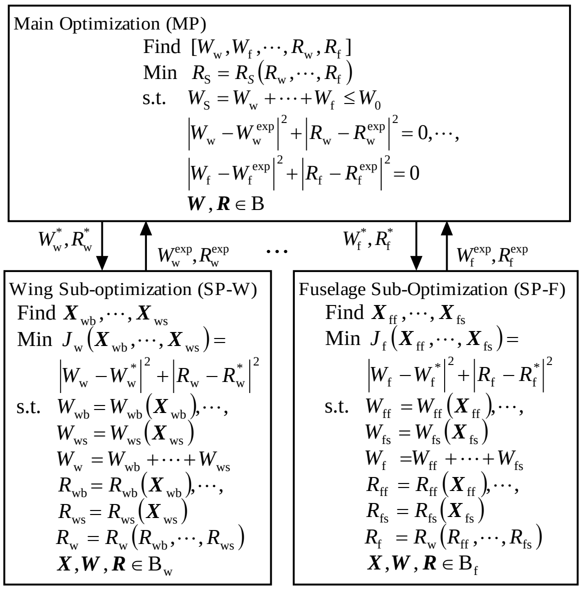


Fig. 15 Mathematical model of weight requirement collaborative allocation according to Fig.14

According to CA, weight optimum allocation problem above can be solved in steps below:

(1) Initial weight requirement allocation is provided experientially: $W \rightarrow W_w^*, \cdots, W_f^*$. Auxiliary variables are also initialized experientially: $R \rightarrow R_w^*, \cdots, R_f^*$.

(2) Concurrent sub-optimization is performed: SP-W is to provide expected value for wing subsystem: W_w^{exp}, R_w^{exp} ; SP-F is to provide those for fuselage subsystem: W_f^{exp}, R_f^{exp} 。

(3) Main optimization is carried out to provide a new allocation: $W \rightarrow W_w^*, \cdots, W_f^*$, $R \rightarrow R_w^*, \cdots,\ R_f^*$.

(4) If $\left|R_s^n - R_s^{n-1}\right| / R_s^n \le \varepsilon$, go to step 5. If not, back to step 2. Where, R_s^n is integration reliability in n iteration, ε is a user-defined small positive value.

Optimization is finished. The best weight requirements for components are $W_{wb}^*, W_{ws}^*, \cdots, W_{ff}^*, W_{fs}^*$.

6 Conclusion

A new method named Collaborative Allocation is developed for optimum allocation of design requirements in aircraft conceptual design. CA is preliminarily validated and it still needs to be further studied. Through our study, it is shown that:

Compare to DM and DCM, CA is of more general optimization architecture. For different allocation problem, main program may keep unchanged except little modification of optimization model. So CA is better in program inheritance.

(1) Compare to DM, the dimension of design variables is reduced through decomposition of optimization in CA. In this way, complicated analysis of subsystem may be performed inside its respective sub-optimization. So optimization is easier and concurrent computation can be realized.

(2) Compare to DCM, main optimization is really departed from sub-optimization in CA. Sub-optimization need not to be performed in the process of main optimization. Accordingly optimization is easier and robust is better.

The main difficulty in CA is how to construct compatibility constraint can make main optimization easier to be solved.

References

[1] Mettas, A. Reliability Allocation and Optimization for Complex Systems. 2000 Proceedings Annual Reliability and Maintainability Symposium, Los Angeles, California, USA, January 24-27, 2000.

[2] Bao, L.P, Zhao, R.Q, Ren, J.F. Optimal Allocation of Large-system Reliability with Genetic Algorithms. Journal of Ordnance Engineering College, 2001, 13(2): 73-76.

[3] Wang, G.Y, Chen, S.X. Theory and Applications of Soft Design in Engineering Structure and System. Beijing: National Defence Industry Press, 1996.2: 87-145.

[4] Li, D, Haimes, Y.Y. A Decomposition Method for Optimization of Large-System Reliability. IEEE Transactions on Reliability, 1992, 41(2): 183-189.

[5] Braun, R, Gage, P, Kroo, I, et al. Implementation and Performance Issues in Collaborative Optimization. AIAA 96-4017.

[6] Sobieski, I.P, Kroo, I.M. Collaborative Optimization Using Response Surface Estimation. AIAA 98-0915.

[7] Hu, Y, Li, W.J. A New Effective MDO Algorithm. Proceedings of The 4th World Congress of Structural and Multidisciplinary Optimization, Dalian, China, June, 2001.

[8] Li, X, Li, W.J. Collaborative Optimization Based-on Inter-disciplinary Inconsistency Information and Its Application to Mechanical System Design. Journal of Northwestern Polytechnical University, 2003, 21(4): 461-464.

[9] Song, B.F, Li, W.J, Ji, G.M, et al. A Reliability-based Multi-level Optimization Method for Large-scale Structure. Advances in Mechanics, 2000, 30(1):29-36.

[10] An, W.G. Structural systems reliability and optimization design based on reliability. Bejing: National Defense Industry Press, 1997.