PARAMETER ESTIMATION AND FLIGHT PATH RECONSTRUCTION USING OUTPUT-ERROR METHOD

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Abstract

This work describes the application of the output-error method using the Levenberg-Marquardt optimization algorithm to the Flight Path Reconstruction problem, which constitutes an important preliminary step towards the aircraft parameter identification. This method is also applied to obtain the aerodynamic and control derivatives of a regional jet aircraft from flight test data with measurement noise and bias. Experimental results are reported, employing an EMBRAER aircraft, with flight test data acquired by smart probes, inertial sensors (gyrometers and accelerometers) and GPS receivers.

1 Introduction

Modeling and simulation has become an integral part of the aeronautical industry design and evaluation processes. One of its major parts is system identification and parameter estimation, applied to complex aerodynamic systems such as airplane. System Identification is a general procedure to match the observed input-output response of a dynamic system by a proper choice of an input-output model and its physical parameters. From this point of view, the aircraft system identification or inverse modelling comprises proper choice of aerodynamic models, the development of parameter estimation techniques by optimization of the mismatch error between predicted and real aircraft response and the development of proper tools for integration of the equations of motion within the system simulation and correlated activities [3].

The problem of Flight Path Reconstruction (FPR) arises naturally when the main goal is an accurate identification of the aircraft parameters, because, in this case, the proper characterization of the sensors constitutes a fundamental preliminary step. For example, if the bias of a certain sensor is not adequately estimated, the accuracy of the ensuing parameter identification may be degraded.

The flight path reconstruction is specially useful in the validation of the instruments applied in a prototype. The interpretation of the results can furnish important information with respect to sources of problems. Additionally, it decreases the uncertainties about the quality of data, which is one of the main causes of poor flight tests results.

One of the first approaches for FPR may be found in [5]. The authors employ the kinematic model of an aircraft, with 6 degrees of freedom, and then consider an augmented state vector, incorporating the parameters to be identified. This procedure leads to a general problem of state estimation with nonlinear dynamics, solved by the extended Kalman filter approach. A more detailed investigation of the problem is conducted by [9]. Experimental results are reported for the approach based on extended Kalman filter, but the GPS readings are not used.

In this work, the FPR problem is investigated by parametric identification of a nonlinear model, based on output-error method and Levenberg-Marquardt algorithm. The results are reported for an EMBRAER aircraft, considering 28 parameters and 6 outputs, and comparing the calibration results obtained with those determined by traditional methods used by EMBRAER. The identification method used in this work, based on the optimization algorithm of Levenberg-Marquardt [6], is of the output-error type, which is susceptible to process noise. However, this approach is justified in the present case, since the noise is not large and because the methods presented in [5] and [9] could lead to incorrect results: the extended Kalman filter could mask instrumentation errors (when operating under weak exciting signals), mainly those arising from inadequate compatibilization between the INS and GPS coordinates.

The Levenberg-Marquardt algorithm is also used here to determine the stationary aerodynamic derivatives of the aircraft, using a linearized lateral-directional model. The effectiveness of the implemented parameter estimation method was tested by matching real flight test data with the predicted response of the aircraft.

This work is structured as follows: in the first part, the kinematic model for FPR and the lateral-directional model for parameter estimation are presented. In section 3, the parametric estimation method is described with special attention to the Gauss-Newton and Levenberg-Marquardt algorithms. The experimental results obtained in the FPR problem and parameter estimation are analyzed in section 4.

2 Aircraft Models

2.1 Kinematic model for FPR

The equations that constitute the kinematic model of an aircraft can be grouped in 3 sets of first-order differential equations, providing translational velocities, angular velocities and attitude angles. Using the standard body-fixed reference frame F_B, the equations for the components u, v e w of true air speed V along the body axes X_B, Y_B and Z_B are:

$$\begin{aligned} X &= m(\dot{u} + qw - rv) + mgsen\theta \\ Y &= m(\dot{v} + ru - pw) - mg\cos\theta sen\phi \\ Z &= m(\dot{w} + pv - qu) - mg\cos\theta\cos\phi \end{aligned} \tag{1}$$

where p, q e r denote the rates of rotation about the axes of F_B; θ and ϕ denote pitch and roll angle, respectively; m denotes aircraft mass and g denotes the local acceleration due to gravity. X, Y and Z represent the components of the *total aerodynamic force*, including the aerodynamic effects of propulsion systems.

For an aircraft with a geometrical plane of symmetry, the rotational dynamics are given by

$$\begin{aligned} L &= I_x\dot{p} - (I_x - I_z)\,qr - I_{xz}(\dot{r} + pq) \\ M &= I_y\dot{q} - (I_z - I_x)\,rp - I_{zx}(r^2 - p^2) \\ N &= I_z\dot{r} - (I_x - I_y)\,pq - I_{zx}(\dot{p} + qr) \end{aligned} \tag{2}$$

where L, M, N denote the *total aerodynamic moments*, including any aerodynamic effects of the propulsion system; I_x, I_y and I_z denote the moments of inertia and I_{xz} the only non-zero product of inertia in F_B (due to symmetry).

The orientation of F_B with respect to the earth-fixed vertical reference frame F_E is governed by the following equations for the Euler angles ϕ, θ and ψ,

$$\begin{aligned} \dot{\phi} &= p + qsen\phi\tan\theta + r\cos\phi\tan\theta \\ \dot{\theta} &= q\cos\phi - rsen\phi \\ \dot{\psi} &= qsen\phi\sec\theta + r\cos\phi\sec\theta \end{aligned} \tag{3}$$

To integrate the equations (1) and (3), it is necessary to determine X, Y, Z in (1). This is done assuming that these accelerations are measured, giving

$$\begin{aligned} X &= m\,a_x \\ Y &= m\,a_y \\ Z &= m\,a_z \end{aligned} \tag{4}$$

in which a_x, a_y e a_z denote the specific aerodynamic forces along the body axes X_B, Y_B and Z_B, respectively.

By replacing (4) into (1) and dividing by m leads to

$$\begin{aligned} \dot{u} &= a_x - (qw - rv) - gsen\theta \\ \dot{v} &= a_y - (ru - pw) + g\cos\theta sen\phi \\ \dot{w} &= a_z - (pv - qu) + g\cos\theta\cos\phi \end{aligned} \tag{5}$$

Once mass m (or any other physical properties) has been eliminated from equations (5) and (3), these equations can be integrated. More precisely, the solution of these equations can be obtained using the acceleration components (a_x, a_y, a_z) and the angular rates (p, q, r) as input variables, since they are measured by sensors installed on the aircraft, which are part of the inertial system. It is precisely the measure of these components that allow the realization of the FPR before the parameter identification of the aircraft is carried out.

Aiming to use GPS readings (geographical coordinates), we must characterize the position of the aircraft relative to the earth-fixed reference frame. To improve the quality of this signal, it was used the differential technique, namely the DGPS. This position is obtained from (5) and (3), through the relation

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = L_{EB} \begin{bmatrix} u \\ v \\ w \end{bmatrix} - \begin{bmatrix} W_{X_E} \\ W_{Y_E} \\ W_{Z_E} \end{bmatrix} \tag{6}$$

where L_{EB} denotes an orthogonal matrix of reference frame transformation, determined by the roll, pitch and yaw angles. The three vectors that form this matrix are

$$v_1 = \begin{bmatrix} \cos\theta\cos\psi \\ \cos\theta sen\psi \\ -sen\theta \end{bmatrix} \tag{7}$$

$$v_2 = \begin{bmatrix} sen\phi sen\theta\cos\psi - \cos\phi sen\psi \\ sen\phi sen\theta sen\psi + \cos\phi\cos\psi \\ sen\phi\cos\theta \end{bmatrix} \tag{8}$$

$$v_3 = \begin{bmatrix} \cos\phi sen\theta\cos\psi + sen\phi sen\psi \\ \cos\phi sen\theta sen\psi - sen\phi\cos\psi \\ \cos\phi\cos\theta \end{bmatrix} \tag{9}$$

W_{X_E}, W_{Y_E} e W_{Z_E} in (6) denotes the components of a constant atmospheric wind vector W_E along the axes of F_E.

To summarize, the aircraft motion can be described by the nonlinear model (5), (3) and (6), which can be rewritten in the form

$$\dot{x}(t) = f(x(t), u(t)) \tag{10}$$

with state and input vectors given by

$$\begin{aligned} \mathbf{x} &= \begin{bmatrix} u & v & w & \phi & \theta & \psi & x_E & y_E & z_E \end{bmatrix}^T \in \Re^9 \\ \mathbf{u} &= \begin{bmatrix} a_x & a_y & a_z & p & q & r \end{bmatrix}^T \in \Re^6 \end{aligned} \tag{11}$$

Basically, the observation models take the form of nonlinear algebraic relations between the observed variables and the state and input vector components. In this work the models are derived for observations of true air speed V, angle of attack α, side slip angle β and geographical position measurements.

True air speed V_T can be derived from differential and absolute barometric and temperature transducers, resulting

$$V_{Tm} = K_v V_T + \Delta V_T \tag{12}$$

where K_v is a scale factor and ΔV_T the bias term. By definition, V_T is the absolute value of the resultant of the air velocity components u, v e w along the axes of F_B,i.e.,

$$V_T = \sqrt{u^2 + v^2 + w^2} \tag{13}$$

Also by definition, the angle of attack and the side slip angle are given by, respectively,

$$\begin{aligned} \alpha &= \arctan\left(\frac{w}{u}\right) \\ \beta &= \arctan\left(\frac{v}{\sqrt{u^2+w^2}}\right) \end{aligned} \tag{14}$$

These values differ from the measured angles, due to many effects, like velocities induced by aircraft rotational motion and modification of the air flow due to disturbs of the air near the aircraft, resulting the following measurement equations

$$\begin{aligned} \alpha_m &= \arctan\left(\frac{w - x_\alpha q + y_\alpha p}{u}\right) + K_\alpha \alpha + K_\alpha^\beta \beta + \Delta\alpha \\ \beta_m &= \arctan\left(\frac{v + x_\beta r - z_\beta p}{u}\right) + K_\beta \beta + \Delta\beta \end{aligned} \tag{15}$$

where K_α, K_α^β and K_β are scale factors, $\Delta\alpha$ and $\Delta\beta$ denote bias terms and the parameters x_α, x_β, y_α and z_β denote the position of the sensors. More details can be found in [9].

Finally, the geographical coordinates are obtained by DGPS. Therefore, the observation model takes the form

$$\begin{aligned} x_{Em} &= x_E + \Delta x_E \\ y_{Em} &= y_E + \Delta y_E \\ h_{Em} &= z_E + \Delta z_E \end{aligned} \tag{16}$$

where Δx_E, Δy_E and Δz_E denote bias terms.

Based in (12), (15) and (16), the observation vector is defined as

$$\mathbf{y} = \begin{bmatrix} \alpha_m & \beta_m & V_{Tm} & x_{Em} & y_{Em} & z_{Em} \end{bmatrix} \in \Re^6 \tag{17}$$

From equations (5), (3), (6), (12), (15)-(17), and adding bias terms in the measurements of accelerations and angular velocities, Δa_x, Δa_y, Δa_z, Δp, Δq and Δr, the following dynamic model is obtained:

state equations:

$$\begin{aligned}
\dot{u} &= (a_{xm}-\Delta a_x)-(q_m-\Delta q)w+(r_m-\Delta r)v-g sen\theta \\
\dot{v} &= \left(a_{ym}-\Delta a_y\right)-(r_m-\Delta r)u+(p_m-\Delta p)w+g\cos\theta sen\phi \\
\dot{w} &= (a_{zm}-\Delta a_z)-(p_m-\Delta p)v+(q_m-\Delta q)u+g\cos\theta\cos\phi
\end{aligned} \tag{18}$$

$$\begin{aligned}
\dot{\phi} &= \left(p_m-\Delta p-\omega_p\right)+\left(q_m-\Delta q-\omega_q\right)sen\phi\tan\theta \\
&+(r_m-\Delta r-\omega_r)\cos\phi\tan\theta \\
\dot{\theta} &= \left(q_m-\Delta q-\omega_q\right)\cos\phi-(r_m-\Delta r-\omega_r)sen\phi \\
\dot{\psi} &= \left(q_m-\Delta q-\omega_q\right)sen\phi\sec\theta+(r_m-\Delta r-\omega_r)\cos\phi\sec\theta
\end{aligned} \tag{19}$$

$$\begin{bmatrix} \dot{x}_E \\ \dot{y}_E \\ \dot{z}_E \end{bmatrix} = L_{EB}\begin{bmatrix} u \\ v \\ w \end{bmatrix} - \begin{bmatrix} W_{XE} \\ W_{YE} \\ W_{ZE} \end{bmatrix} \tag{20}$$

control signals:

$$\begin{aligned}
a_{xm} &= a_x+\Delta a_x \\
a_{ym} &= a_x+\Delta a_x \\
a_{zm} &= a_x+\Delta a_x \\
p_m &= p+\Delta p \\
q_m &= q+\Delta q \\
r_m &= r+\Delta r
\end{aligned} \tag{21}$$

output signals:

$$\begin{aligned}
\alpha_m &= \arctan\left(\tfrac{w-x_\alpha q+y_\alpha p}{u}\right)+K_{\alpha\alpha}\alpha+K_{\beta\alpha}\beta \\
&+\Delta\alpha+\nu_\alpha \\
\beta_m &= \arctan\left(\frac{v+x_\beta r-z_\beta p}{u}\right)+K_\beta\beta+\Delta\beta+\nu_\beta \\
V_{Tm} &= K_v V_T+\Delta V_T+\nu_V \\
x_{Em} &= x_E+\Delta x_E+\nu_{x_E} \\
y_{Em} &= y_E+\Delta y_E+\nu_{y_E} \\
z_{Em} &= z_E+\Delta z_E+\nu_{z_E}
\end{aligned} \tag{22}$$

where the last terms in (22) stand for sensor noise.

In the dynamic model given by (20)-(22), the parameter vector Θ to be estimated is formed by 28 components, namely

$$\begin{aligned}
\Theta = [&\Delta a_x\ \Delta a_y\ \Delta a_z\ \Delta p\ \Delta q\ \Delta r\ \Delta\alpha\ \Delta\beta\ K_\alpha\ K_\alpha^\beta\ K_\beta \\
&\Delta V_T\ K_{V_T}\ W_{x_E}\ W_{y_E}\ W_{z_E}\ \Delta_{x_E}\ \Delta_{y_E}\ \Delta_{z_E}\ u(0)\ v(0) \\
&w(0)\ \phi(0)\ \theta(0)\ \psi(0)\ x_E(0)\ y_E(0)\ z_E(0)]^T
\end{aligned} \tag{23}$$

where the last 9 terms in (23) denote initial conditions of the state vector in (11).

Therefore, from equations (20)-(23) we conclude that the FPR constitutes a parametric identification problem applied to a dynamic system of the form,

$$\begin{aligned}
\dot{x}(t) &= f(x,u,\Theta),\ x(0)=g(\Theta) \\
y(t) &= h(x,u,\Theta)+\upsilon(t)
\end{aligned} \tag{24}$$

where $x \in \Re^9$, $u \in \Re^6$, $y \in \Re^6$ e $\Theta \in \Re^{28}$.

2.2 Dynamic model of lateral-directional movement of aircraft

The aircraft dynamic system is described by a stochastic nonlinear hybrid model in the form of eq. (24). In this section the inverse problem formulation is applied to the lateral-directional movement of the aircraft, for which the linear state and output equations can be written as [8],

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{p}(t) \\ \dot{r}(t) \\ \dot{\phi}(t) \end{bmatrix} = \begin{bmatrix} Y_\beta & \sin(\alpha_e) & -\cos(\alpha_e) & a_{14} \\ L'_\beta & L'_p & L'_r & 0 \\ N'_\beta & N'_p & N'_r & 0 \\ 0 & 1 & a_{43} & 0 \end{bmatrix}\begin{bmatrix} \beta(t) \\ p(t) \\ r(t) \\ \phi(t) \end{bmatrix} + \begin{bmatrix} Y_{\delta_a} & Y_{\delta_r} \\ L'_{\delta_a} & L'_{\delta_r} \\ N'_{\delta_a} & N'_{\delta_r} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_a(t) \\ \delta_r(t) \end{bmatrix}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \\ y_4(t) \\ y_5(t) \end{bmatrix} = \begin{bmatrix} x_1(t)+\beta_{bias} \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ c_{51} \end{bmatrix}\begin{bmatrix} \beta(t) \\ p(t) \\ r(t) \\ \phi(t) \\ a_y(t) \end{bmatrix} + \upsilon(t) \tag{25}$$

where $a_{14}=\cos(\phi_e)\cos(\theta_e).(g/V_0)$, $a_{43}=\cos(\theta_e)tg(\theta_e)$ and $c_{51}=(V_0/g).(Y_\beta x_1(t)+Y_{\delta_a}\delta_a(t)+Y_{\delta_r}\delta_r(t))$; $\upsilon(t)$ represents the measurement noise.

Equation (25) has 14 unknown parameters

that need to be estimated, giving $\Theta \in R^{14}$, i.e.,

$$\Theta = [Y_\beta\; L'_\beta\; L'_p\; L'_r\; N'_\beta\; N'_p\; N'_r \\ Y_{\delta_a}\; Y_{\delta_r}\; L'_{\delta_a}\; L'_{\delta_r}\; N'_{\delta_a}\; N'_{\delta_r}\; \beta_{bias}]^T \tag{26}$$

As usually formulated in the aeronautical literature [5, 6], the components of the vector Θ, are the dimensional aerodynamic derivatives, e.g. $Y_\beta = \frac{\rho_e S V_e^2}{2m} C_{Y_\beta}$, which in turn can be written in term of nondimensional coefficients, e.g. C_{Y_β}, by proper choice of flight parameters, such as ρ_e, V_e, S, m, all assumed known a priori. More details about this conversion can be found in [1].

3 Parametric estimation method

In this section, the parametric identification, in particular the parameter estimation applied to a linear causal model of an aircraft, in space state formulation according to eq. (24). The output-error method is one of the most used estimation methods in aircraft identification and aerodynamic parameter estimation [6], [7], [8]. It has several desirable statistical properties, including its application to nonlinear dynamical systems and the proper accounting of measurements noise [8].

The structure of the model is considered to be known, and the identification process consists in determining the parameter vector Θ, which gives the best prediction of the output signal $y(t)$, using some sort of optimization criteria. The attainment of an estimate through optimization of a cost function based on the prediction error of the plant requires, usually, the minimization of a nonlinear function. Thus, the Lenberg-Marquardt method is used here to estimate the parameters in model (25). Therefore, the cost function to be minimized involves the prediction error,

$$e(k) = \hat{y}(k) - y(k) \tag{27}$$

where $\hat{y}(k)$ is the output prediction based on the actual estimate $\hat{\Theta}$ of the parameter vector Θ.

3.1 Maximum Likelihood Estimation criteria

Consider a dynamic system, identifiable, with model structure $M(\Theta)$ defined and output y. Suppose that $p(y|\Theta)$ is the conditional probability gaussian distribution of the random variable y with dimension m, mean $f(\Theta)$ and covariance R, with dimension $m \times m$. $p(y|\Theta)$ is known as the likelihood functional, and in [2] the authors attribute its name due to the fact that it is a measure of the probability of occurrence of the observation y for a given parameter Θ. The Maximum Likelihood Estimate is defined as the value of Θ which maximizes this functional, in such a way that the best estimate of Θ, according to the MLE criteria is

$$\hat{\Theta} = Arg\,Max\, p(y|\Theta) \tag{28}$$

Thus, the likelihood functional is

$$p(y|\Theta) = \frac{1}{(2\pi)^{m/2}|R|^{n/2}} \cdot \\ exp\left\{-\frac{1}{2}\sum_{k=1}^{n}[e(k,\Theta)]^T[R]^{-1}[e(k,\Theta)]\right\} \tag{29}$$

whose maximization is equivalent to the minimization of

$$J(\Theta) = \sum_{k=1}^{n}\frac{1}{2}\{[e(k,\Theta)]^T[R]^{-1}[e(k,\Theta)] + ln|R|\} \tag{30}$$

since, in the optimization process, $J(\Theta)$ is equivalent to $-ln\,p(y|\Theta)$, except for a constant term.

3.2 Minimization of the cost function by Levenberg-Marquardt

The identification algorithms based on the Gauss-Newton method is of second order. This method, although complex, is suitable for a quadratic cost function, and is expected to converge quickly. First, we approximate $J(\Theta)$ by a parabolic function $J_L(\Theta)$ under the condition Θ_L (retaining only the 3 first Taylor series terms),

$$J_L(\Theta) \cong J(\Theta_L) + (\Theta - \Theta_L)^T \nabla_\Theta^T J(\Theta_L) \\ + \frac{1}{2}(\Theta - \Theta_L)^T[\nabla_\Theta^2 J(\Theta_L)](\Theta - \Theta_L) \tag{31}$$

The optimization condition is obtained when,

$$\nabla_\Theta J(\Theta^*) = 0 \qquad (32)$$

Applying (32) to equation (31), results, for Θ close to the local minima Θ^*,

$$\nabla_\Theta J_L(\Theta) \cong \nabla_\Theta J(\Theta_L) + (\Theta - \Theta_L)^T [\nabla^2_\Theta J(\Theta_L)] = 0 \qquad (33)$$

which can be used to find the minima of the original cost function through the recursion,

$$\Theta_{i+1} = \Theta_i - [\nabla^2_\Theta J(\Theta_i)]^{-1} \nabla^T_\Theta J(\Theta_i) \qquad (34)$$

The complexity in the calculation of the Hessian matrix, $\nabla^2_\Theta J(\Theta_L)$ in (34), is avoided through the Gauss-Newton method, which uses the approximation,

$$\nabla^2_\Theta J(\Theta) \approx \sum_{k=1}^{n} [\nabla_\Theta \hat{y}_k(\Theta)]^T \left[\hat{R}\right]^{-1} [\nabla_\Theta \hat{y}_k(\Theta)] \qquad (35)$$

where the terms involving the second derivative are discarded. The gradient of the estimated output, $\nabla_\Theta \hat{y}_k(\Theta)$, is called *Sensibility Function*.

The Levenberg-Marquardt algorithm is an extension of the Gauss-Newton [10]. The idea is to modify (35) to

$$\nabla^2_\Theta J(\Theta) \approx \sum_{k=1}^{n} [\nabla_\Theta \hat{y}_k(\Theta)]^T \left[\hat{R}\right]^{-1} [\nabla_\Theta \hat{y}_k(\Theta)] + \lambda I \qquad (36)$$

and the inversion in (34) is not performed in an explicit manner, i.e., tipically the original equation

$$\left[\nabla^2_\Theta J(\Theta) + \lambda I\right] \Delta\hat{\Theta} = \nabla^T_\Theta J(\Theta_i) \qquad (37)$$

is solved via SVD.

The inclusion of λI in (37) solves the problem of an ill conditioned approximated Hessian. The Levenberg-Marquardt algorithm can be interpreted in the following way: for small values of λ it behaves like the Gauss-Newton algorithm, while for high values of λ it behaves like the gradient method. More details about the Levenberg-Marquardt method can be found in [11].

4 Experimental results

4.1 Flight path reconstruction

A flight test was performed and data was gathered with sampling time T=0,09s. The input signals relative to this maneuver are shown in figures 1 and 2, containing the accelerations and angular velocities, respectivaly. These signals are referred to the control signal u in eq. (24). The vertical scales are omitted.

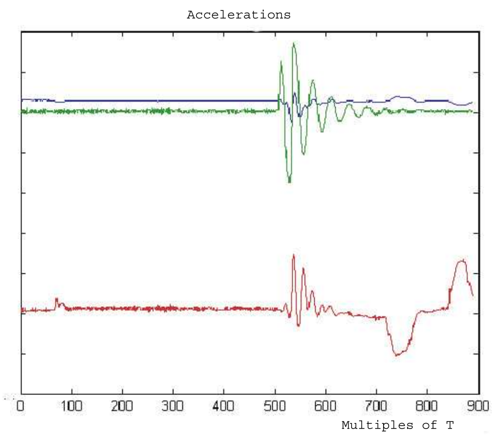

Fig. 1 Acceleration measurements of the maneuver employed. The horizontal scale is given in multiples of sampling time T.

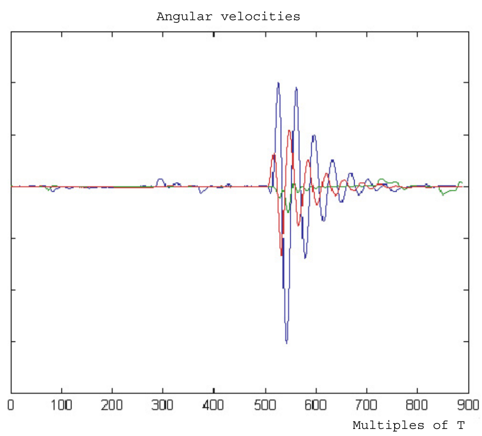

Fig. 2 Angular velocities measurements of the maneuver employed. The horizontal scale is given in multiples of sampling time T.

Based on the input signals indicated in figures 1 and 2 and in the measured variables according to the output vector (22), the identification algorithm was used to determine the parameter vector containing the 28 parameters indicated in eq. (23). The identification algorithm was executed many times, aiming to investigate the influence of the design parameters. The influence of the integration method used to solve the state equation (20) was also investigated, concluding that the Euler method is not suitable, but the 4th order Runge-Kutta method produces adequate results.

After executing the identification, we must evaluate its performance. The first quality measure is the mean square prediction error. The plots of these errors in the present case indicate small values, and these plots are omitted, except for the air speed, which is shown in figure 3. The relative vertical scale indicates a total range of 6 m/s. Hence, the difference between the measured and the predicted values of air speed is small.

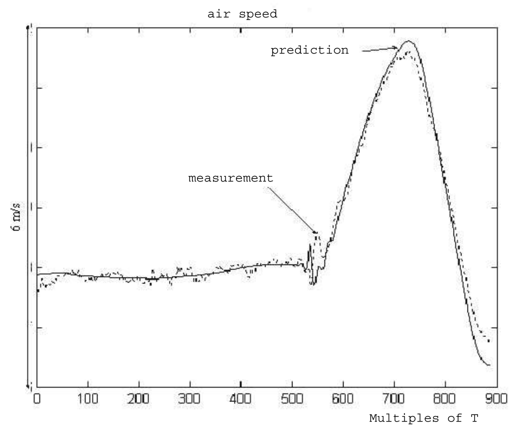

Fig. 3 True air speed: measurements and prediction.

Next, it is considered the main performance measure here, the comparison of the estimated values with those obtained by EMBRAER via traditional procedures. Two of these variables are considered here: the angle of attack and the side slip angle, respectively the variables α and β in eq. (14).

In figures 4 are presented the relative values of angle of attack. Three plots compose figure 4: the values measured by the aircraft sensors, the values calibrated by Embraer through traditional procedures, and the values calibrated by the method proposed in this paper. The vertical scale omits the absolute values of the angles, but presents the total variation.

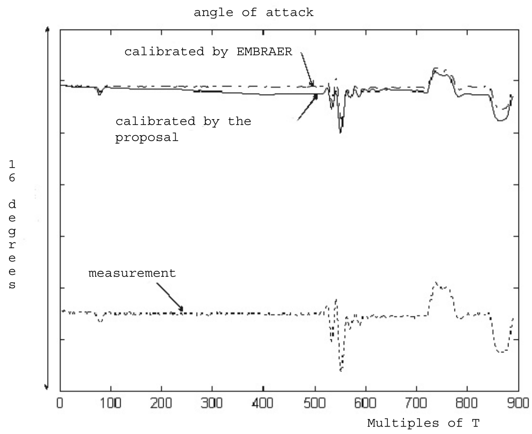

Fig. 4 Measured and calibrated values for the angle of attack.

Based on figure 4, we conclude that the procedure proposed here for the flight path reconstruction presents results compatible with that obtained via the techniques employed by EMBRAER.

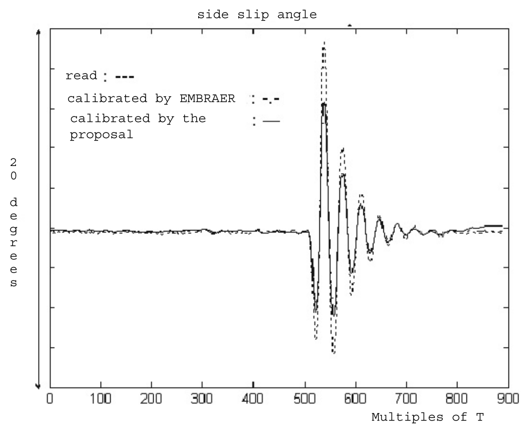

Fig. 5 Measured and calibrated values for the side slip angle.

Figure 5 presents results for the side slip angle. This figure indicates that the method proposed for FPR presents calibrated values of the side slip angle which are similar to those obtained by EMBRAER traditional techniques.

4.2 Matching of flight test data for lateral-directional movement

The aerodynamic derivatives associated with the lateral-directional model, as shown in eq. (25), were estimated by matching the real flight test data with the model predicted simulation. A dutch-roll maneuver of a regional transport aircraft was used to investigate the effectiveness of the discussed output-error method (the Levenberg-Marquardt), applied to estimate the aerodynamic parameter vector defined in eq. (26).

The aircraft input signals are the aileron $\delta a(t)$ and rudder deflections $\delta r(t)$, and the output signals are five attitude parameters: sideslip angle $\beta(t)$, roll rate $p(t)$, yaw rate $r(t)$, bank angle $\phi(t)$, and lateral acceleration $a_y(t)$. The experimental input signals are shown in fig. 6 and the output signals are shown in figs. 7 to 11, represented by the red lines.

The time history of the aircraft input-output relationship was measured with a sampling time of 0.0312 s, and the 914 measured points gives an observation time window of approximately 28 s.

Table 1 shows the final values of the non-dimensional aerodynamic derivatives obtained by the Levenberg-Marquardt algorithm. We use the values achieved by the Nelder-Mead method to initialize the Levenberg-Marquardt algorithm, since this method is more computationally demanding and a good initial estimate can speed up its convergence. A maximum likelihood cost was used, in which case the weighting factor was the estimated covariance matrix associated to the prediction errors.

Since the flight data employed to generate Table 1 was obtained experimentally and no wind tunnel tests are available, an indirect measure of performance is used, based on the prediction error. So, the main focus of the present inverse aerodynamic modeling is to check that this local

Table 1 Estimation of the non-dimensional stability and control derivatives.

Coefficient	Initial values	LM method
C_{Y_β}	-0.0068	-0.0058
$C_{L'_\beta}$	-0.1861	-0.1514
$C_{L'_p}$	-0.3562	-0.4718
$C_{L'_r}$	-1.1700	1.4942
$C_{N'_\beta}$	0.0678	0.0415
$C_{N'_p}$	0.0616	-0.0275
$C_{N'_r}$	-2.7110	-0.6765
$C_{Y_{\delta_a}}$	0.0068	0.0052
$C_{Y_{\delta_r}}$	-0.0068	-0.0073
$C_{L'_{\delta_a}}$	-0.0001	-0.0029
$C_{L'_{\delta_r}}$	-0.0198	0.0704
$C_{N'_{\delta_a}}$	0.0016	-0.0405
$C_{N'_{\delta_r}}$	-0.2037	-0.1029

minimization procedure can provide good matching to the experimental flight data and stable input-output modeling for the aircraft. This prediction capability, as obtained by the output-error method, can be accessed from the model validation results shown in figs. 7 to 11, where the estimation error are small for most of the output variables.

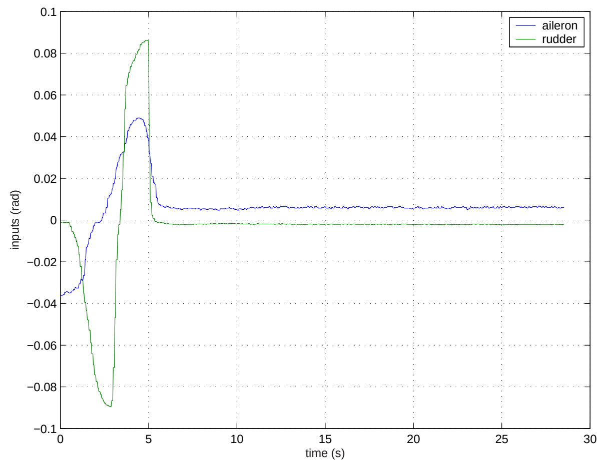

Fig. 6 Aileron δa and rudder δr inputs for the dutch-roll maneuver.

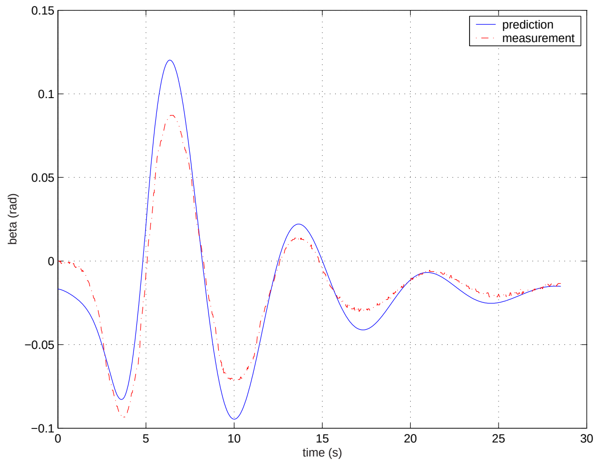

Fig. 7 Measured side slip angle β and estimated values.

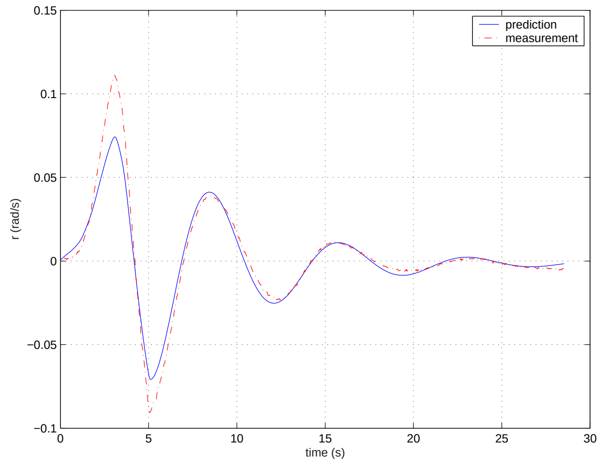

Fig. 9 Measured yaw velocity r and estimated values.

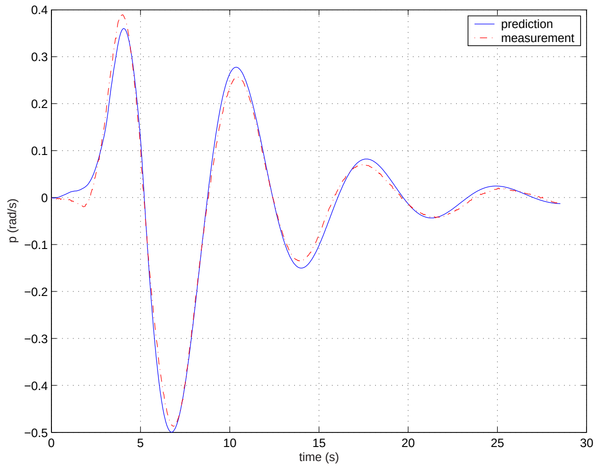

Fig. 8 Measured roll velocity p and estimated values.

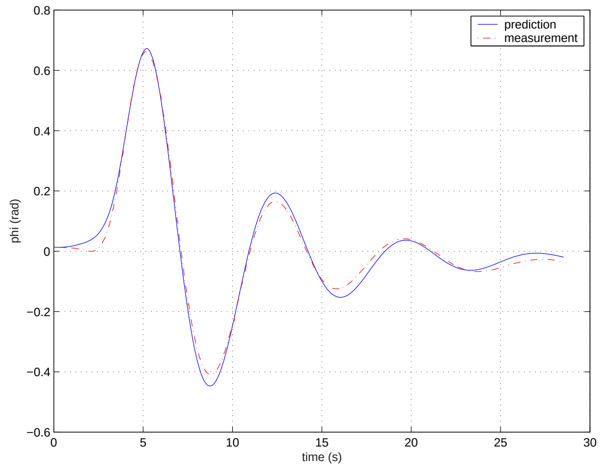

Fig. 10 Measured bank angle ϕ and estimated values.

5 Conclusions

Based on the small prediction error (as seen in figure 3) and good agreement between the calibrated values (see figures 4 and 5), we can conclude that the proposed procedure for FPR, based on parametric identification via optimization using the Levenberg-Marquardt method, exhibited satisfactory performance. Thus, the proposed procedure can be added to the repertory of FPR techniques employed by EMBRAER, and constitutes a relevant alternative for practical applications.

This work also presented the estimation of an aircraft linear aerodynamic derivatives, which presented good convergence properties and good matching to the experimental flight data. The results obtained with the Levenberg-Marquardt algorithm demonstrate the feasibility of the method.

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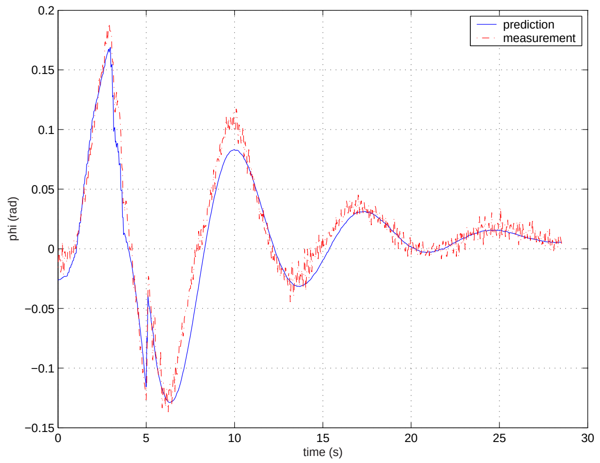

Fig. 11 Measured lateral acceleration a_y and estimated values.

as part of the PITE Program.

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