ROBUST CONTROL AND CLOSED-LOOP IDENTIFICATION BY NORMALIZED COPRIME FACTORIZATION

Takeharu Kameyama*
***National Defense Academy**

Keywords: *failure tolerant control, normalized coprime factorization, coprime factorization, robust stability margin, ν-gap metric*

Abstract

This paper deals with design of a failure tolerant control system where failures are identified using the normalized coprime factorization method. The identification method employed is a closed-loop one, which is also based on coprime factorization; therefore, the method is suitable to the robust control systems. The impact of failure on the closed-loop stability is evaluated by ν-gap metric obtained from the estimated plant. To illustrate the effectiveness of the control and identification method, a simple design example and simulation results are shown.

1 Introduction

This Future high performance aircraft require robust flight control that can maintain stability, controllability and mission attainability in the presence of failures, damages and so on, and the self-repairing function to reconfigure control systems by identifying damages. In the research on Self-Repairing Flight Control System (SRFCS) [1]-[6], we have designed a robust control law by using the normalized coprime factorization (NCF) method [7]-[9], and a reconfigurable law using damage information which is assumed to be known. In the first research we evaluated the laws by mathematical simulation [1], [2], and next we applied the design method to a dynamic wind tunnel test model (DWTTM), where real-time hardware-in-the-loop simulation called dynamic wind tunnel test (DWTT) was performed under real aerodynamic environments in the wind tunnel, and the laws were evaluated [3]-[6].

In this paper, I deal with a problem of parameter identification for reconfigurable flight control systems which is a remaining research theme. Specifically I employed the coprime factorization (CF) method [10], which is a parameter identification method for closed-loop systems. This method is suitable for the NCF method used for the design of the robust control law, and can estimate the plant parameters by using transfer functions of the nominal plant and the controller, and the input and output signals.

In the next section, I explain how to incorporate a function of parameter identification into the SRFCS, based on the results of DWTT. In Section 3, I describe an outline of the NCF method [7]-[9] which is a design method of the robust control law, and I reinforce the explanation in Section 2 using the relationships between the robust stability margin [11]-[13] derived from the NCF method and the ν-gap metric [11]-[13] which is the index of the norm of the model error viewed from the controller. In Section 4, I describe an outline of the CF method [10] that is a parameter identification method for closed-loop systems and a procedure to apply the CF method to the closed-loop system which includes the controller designed by the NCF method. In Section 5, I simulate and evaluate the effectiveness of the control and identification methods applied to a simple second-order system. Finally I will give conclusions in Section 6.

2 Failure Tolerant Control Strategy

From DWTT, the following results have been obtained [6].

· The robust control law designed for expected model errors is sufficiently robustly stable against simulated damages.

· The robust control law degrades the performance for significant damages. But when the reconfigurable law is applied, the performance can be recovered.

· The damages for which the robust control law cannot guarantee the stability destabilize the system, even if the reconfigurable law is used.

Therefore, I assume a limited role of parameter identification for SRFCS, as follows.

· Not an adaptive (reconfigurable) control law but a robust control law is used for less significant damages/failures.

· The ν-gap metric, which is the difference (model error) between the estimated plant obtained using the estimated parameters and the nominal plant used for the design of the controller, is calculated on line.

· If the ν-gap metric nears the robust stability margin, which will be defined in Section 3, the control law is changed to the reconfigurable law to recover the control performance.

This procedure is shown in Fig.1.

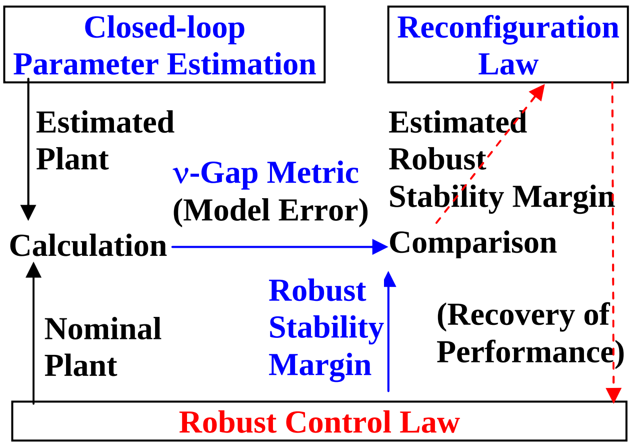

Fig.1. Failure Tolerant Control System

3 Normalized Coprime Factorization Method and Indexes of Model Error

I describe NCF method [7]-[9], on which control system design and parameter estimation are based, and then I define robust stability margin [11]-[13] and ν-gap metric [11]-[13], which are the indexes of failure influence on stabilization.

Let RH_∞ be the set of stable proper transfer function matrices and let GH_∞ be the set of stable proper transfer function matrices whose inverse matrices also belong to RH_∞.

3.1 Normalized Coprime Factorization Method

NCF is a method to design a controller that guarantees the stability for model errors by making the H_∞ norm of the closed-loop transfer function matrix as small as possible.

Using NCF representation [12], [14], the nominal plant $P_0(s)$ and the perturbed plant $P(s)$ can be expressed respectively as

$$P_0(s) = M(s)^{-1} N(s) \tag{1}$$

$$P(s) = \left(M(s) + \Delta_M\right)^{-1} \left(N(s) + \Delta_N\right) \tag{2}$$

where let Δ_N and Δ_M belong to the set defined as the following equation

$$D_\delta = \left\{ \Delta = \left[\Delta_N \quad \Delta_M\right] \mid \Delta \in RH_\infty, \|\Delta\|_\infty < \delta \right\} \tag{3}$$

for a positive constant δ.

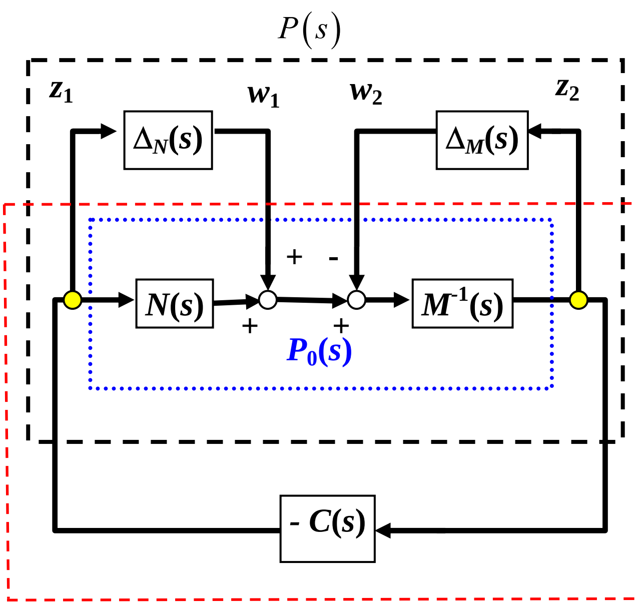

Fig.2. NCF Control System for a Perturbed Plant

In the control system of Fig.2, the maximum value $\varepsilon_{\max}$ of model error with which $P(s)$ can be stabilized by a controller $C(s)$ for all $\Delta \in D_\delta$ is expressed as

$$\varepsilon_{\max} = \left(\inf_C \left\| T_{zw} \right\|_\infty \right)^{-1} \tag{4}$$

where

$$T_{zw}(s) = \begin{bmatrix} C(s) \\ I \end{bmatrix} \left(I + P_0(s) C(s) \right)^{-1} \begin{bmatrix} P_0(s) & I \end{bmatrix} \tag{5}$$

is the closed-loop transfer function matrix from the plant input/output signals $[w_1^T \ w_2^T]^T$ to the controller input/output signals $[z_1^T \ z_2^T]^T$. We can design a controller by the NCF method after assigning admissible model error $\varepsilon \, (\leq \varepsilon_{\max})$.

3.2 Indexes of Model Error

The model error ε admissible for the controller $C(s)$ which is designed for the nominal plant $P_0(s)$ is defined as

$$\varepsilon = \left(\left\| T_{zw} \right\|_\infty \right)^{-1} \tag{6}$$

where ε is called robust stability margin. Let $\varepsilon_{\max}$ be the maximum value of ε, which is the one determined from only the plant $P_0(s)$. If $\varepsilon_{\max}$ is less than the ν-gap metric (defined below), the controller that makes the system with considerable model errors stable may not exist. So $\varepsilon_{\max}$ can be an index which indicates the maximum value of admissible (stabilizable) model error.

An index of the model errors viewed from the controller which stabilizes the nominal plant $P_0(s)$ is the one that indicates how easily the controller stabilizes the perturbed plant $P(s)$. We can use the ν-gap metric as such an index. The ν-gap metric ν, which is a function of $P_0(s)$ and $P(s)$, is defined as

$$\nu = \delta_\nu(P(s), P_0(s)) = \left\| \Psi\left(P(s), P_0(s)\right) \right\|_\infty \tag{7}$$

$$\Psi(X,Y) \equiv \left(I + YY^{\sim} \right)^{-\frac{1}{2}} (X - Y) \left(I + X^{\sim} X \right)^{-\frac{1}{2}} \tag{8}$$

If $\varepsilon > \nu$, the controller is robustly stable for the perturbed plant [11]-[13]. Hence if we know ν and ε, we can estimate how critical the model error of the perturbed plant $P(s)$ is for $C(s)$ by comparing ν with ε. The relationship between the robust stability margin and ν-gap metric allows one to use ν as an effective index which expresses the degree of failure.

If $\varepsilon_{\max}$ is less than the ν-gap metric of an envisioned model error, we consider the augmented system $W(s)P_0(s)V(s)$ a new design plant where the compensators $V(s)$ and $W(s)$ loop-shape the open-loop transfer function matrix as shown in Fig.3. This manipulation can make $\varepsilon_{\max}$ large. We design a controller $C(s)$ for the augmented plant and make $V(s)C(s)W(s)$ a controller (augmented controller) for the original plant $P_0(s)$.

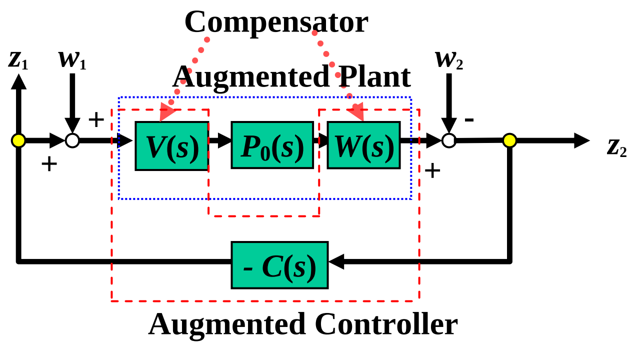

Fig.3. NCF Control System for an Augmented Plant

4 Parameter Identification by Coprime Factorization

I describe an outline of the CF method [10] that is a parameter identification method for closed-loop systems. Inputs must generally be independent of outputs to identify parameters of a dynamic system. However, it is difficult to identify parameters of closed-loop systems, since inputs correlate with outputs through a feedback controller and inputs may not be independent of outputs [15]. Particularly, this problem is significant for a failure tolerant control system which needs to know dynamic characteristics of the (augmented) plant on line. Parameter identification method based on the CF method is one of the ways of closed-loop identification.

4.1 Coprime Factorization Method

When the nominal plant $P_0(s)$ is stabilized by the controller $C(s)$, the following equations hold.

$$P_0(s) = D_{0l}(s)^{-1} N_{0l}(s) \quad (9)$$

$$C(s) = X_r(s) Y_r(s)^{-1} = Y_l(s)^{-1} X_l(s) \quad (10)$$

$$N_{0l}(s) X_r(s) + D_{0l}(s) Y_r(s) = U(s) \quad (11)$$

where $N_{0l}(s)$, $D_{0l}(s)$, $X_r(s)$, $Y_r(s)$, $X_l(s)$, $Y_l(s) \in RH_\infty$ and $U(s) \in GH_\infty$ are stable proper transfer function matrices. Eqs.(9) and (10) are called CF representation of $P_0(s)$ and $C(s)$ respectively, and Eq.(11) is called the Bezout identity. Then all the perturbed plants $P(s)$ that $C(s)$ can stabilize are assumed to be expressed also as follows:

$$\begin{aligned} P(s) &= \left(D_{0l}(s) - R(s) X_l(s)\right)^{-1} \\ &\times \left(N_{0l}(s) + R(s) Y_l(s)\right) \end{aligned} \quad (12)$$

where $R(s)$ is an arbitrary stable proper transfer function matrix.

Equation(12) is expressed by the block diagram in the dashed-line box in Fig.4. By solving Eq.(12) for $R(s)$,

$$\begin{aligned} R(s) &= \left(D_{0l}(s) P(s) - N_{0l}(s)\right) \\ &\times \left(X_l(s) P(s) + Y_l(s)\right)^{-1} \\ &= D_{0l}(s)\left(P(s) - P_0(s)\right) \\ &\times \left(C(s) P(s) + I\right)^{-1} Y_l(s)^{-1} \end{aligned} \quad (13)$$

is derived. From Eq.(13), we can see that the poles of $R(s)$ include the ones of the closed-loop system and that the order of $R(s)$ is generally more than the one of the closed-loop system.

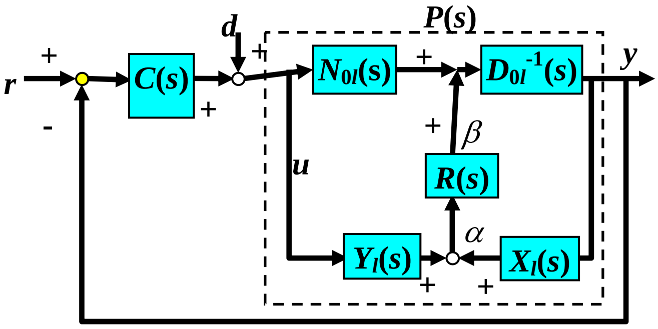

Fig.4. Control System for the Plant expressed by Eq.(12)

Let α and β be the inputs to $R(s)$ and the outputs from $R(s)$, respectively. From Fig.4, we have

$$\begin{aligned} \alpha &= X_l(s) r + Y_l(s) d \\ \beta &= D_{0l}(s) y - N_{0l}(s) u \end{aligned} \quad (14)$$

where r represents reference signals and d external input signals. We can estimate the parameters of $R(s)$ as those of the open-loop system $\beta = R(s)\alpha$, since α is independent of β. Here the least squares method with upper and lower limited trace gain for discrete-time systems is used as a parameter estimation algorithm [16].

We define $R(s)$ whose parameters are replaced with the estimated values as $\hat{R}(s)$. If $\hat{R}(s)$ is obtained, from Eq.(12) we can estimate the transfer function matrix of the unknown perturbed plant $\hat{P}(s)$ as follows:

$$\begin{aligned} \hat{P}(s) &= \left(D_{0l}(s) - \hat{R}(s) X_l(s)\right)^{-1} \\ &\times \left(N_{0l}(s) + \hat{R}(s) Y_l(s)\right) \end{aligned} \quad (15)$$

From Eq.(15), we can see that generally $\hat{P}(s)$ has the order more than the one of the original plant $P(s)$, since $\hat{R}(s)$ has the order more than the one of the closed-loop system. But if the ν-gap metric between $\hat{P}(s)$ and $P(s)$ nearly equals zero, there is no problem for the robust control law. That is, we can decide that we do not have to reconfigure a control law for a failure tolerant control system in this case. This means that we can obtain significant information for a failure tolerant control system since we can estimate a degree of model error (that is, ν-gap metric) even if we cannot estimate accurate parameters of the plant using the parameter identification method based on the CF method. In practice, however, the estimated plant obtained by Eq.(15) can usually be reduced and we can obtain the true parameters of real plant if we can reduce the estimated plant to the system that has the same order as the real plant. We can reconfigure a control law using the estimated parameters if necessary in that case.

4.2 Application of CF Method to an NCF Augmented Controller

I describe a procedure to apply the CF parameter identification method to a closed-loop system which includes an augmented controller designed by the NCF method described in Section 3.

In Fig.3 suppose that a closed-loop system which includes a controller $C(s)$ designed by the NCF method for an augmented plant $P_a(s)=W(s)P_0(s)V(s)$ is stabilized. Using NCF representations [12], [14] of a nominal plant $P_0(s)$ and an augmented controller $C_a(s)=V(s)C(s)W(s)$,

$$P_0(s)=M_n(s)^{-1}N_n(s) \tag{16}$$

$$C_a(s)=X_{ar}(s)Y_{ar}(s)^{-1}=Y_{al}(s)^{-1}X_{al}(s) \tag{17}$$

respectively, the following equation holds

$$M_n(s)Y_{ar}(s)+N_n(s)X_{ar}(s)=U_a(s) \tag{18}$$

where

$$\begin{gathered} U_a(s)\in GH_\infty \\ W(s)Y_{ar}(s),\ V(s)^{-1}X_{ar}(s)\in RH_\infty \\ N_n(s)V(s),\ M_n(s)W(s)^{-1}\in RH_\infty \end{gathered} \tag{19}$$

If we choose

$$\begin{gathered} N_{0l}(s)=N_n(s),\ D_{0l}(s)=M_n(s) \\ X_r(s)=X_{ar}(s),\ Y_r(s)=Y_{ar}(s) \\ X_l(s)=Y_{al}(s),\ Y_l(s)=X_{al}(s) \\ U(s)=U_a(s) \end{gathered} \tag{20}$$

in Eqs.(9) and (10), the Eq.(20) satisfies Eq.(11) and we can estimate a perturbed plant by the method of Section 4.1.

5 Simulation

5.1 Case of SISO model

For a simple SISO model which is a second-order oscillation system with an unstable longitudinal short period mode,

$$P_0(s)=\frac{3s+2}{s^2-s+2} \tag{21}$$

we loop-shape $P_0(s)$ using a first order compensator

$$W(s)=\frac{2(s+1)}{s} \tag{22}$$

to add a servo function to the system and to enlarge ε_{max} of the system. And we design a controller by the NCF method for the augmented plant $W(s)P_0(s)$. As a result, the second order controller

$$C(s)=\frac{1.316s^2+0.9012s+0.4925}{s^2+1.617s+0.6483} \tag{23}$$

stabilizes the system with a robust stability margin ε of about 0.60. We obtain the augmented controller $C(s)W(s)$ as follows.

$$C(s)W(s)=\frac{2(s+1)(1.316s^2+0.9012s+0.4925)}{s(s^2+1.617s+0.6483)} \tag{24}$$

Using the normalized coprime factorization, we have NCF representations of the nominal plant $P_0(s)$ and the augmented controller $C(s)W(s)$ as follows

$$N_{0l}(s)=\frac{3s+2}{s^2+3.414s+2.828} \tag{25}$$

$$D_{0l}(s)=\frac{s^2-s+2}{s^2+3.414s+2.828} \tag{26}$$

$$X_l(s)=X_r(s)=\frac{2(s+1)(1.316s^2+0.9012s+0.4925)}{s^3+1.708s^2+1.061s+0.3498} \tag{27}$$

$$Y_l(s)=Y_r(s)=\frac{s(s^2+1.617s+0.6483)}{s^3+1.708s^2+1.061s+0.3498} \tag{28}$$

Those transfer functions satisfy Eqs.(9) and (10). In fact, we have

$$N_{0l}(s)X_r(s)+D_{0l}(s)Y_r(s)=\frac{s^5+8.515s^4+19.60s^3+19.82s^2+9.827s+1.970}{s^5+5.123s^4+9.722s^3+8.805s^2+4.196s+0.9894}\in GH_\infty \tag{29}$$

$$W(s)Y_r(s)=\frac{2(s+1)(s^2+1.617s+0.6483)}{s^3+1.708s^2+1.061s+0.3498}\in RH_\infty$$

$$D_{0l}(s)W(s)^{-1}=\frac{s(s^2-s+2)}{2(s+1)(s^2+3.414s+2.828)}\in RH_\infty \tag{30}$$

Equations (29) and (30) indicate that Eqs.(11), (18) and (19) hold. We can use the transfer functions Eqs. (25), (26), (27), and (28) in Eq.(14), which generate α and β used to identify $R(s)$.

I will estimate an unknown perturbed plant $P(s)$ and ν-gap metrics between $\hat{P}(s)$ and $P(s)$ or $P_0(s)$. For example, the real plant is assumed to be changed by influence of failure as follows.

$$P(s) = \frac{2s+1}{s^2 - 2s + 3} \tag{31}$$

By applying Eqs.(25), (26), (27), (28), and (31) to Eq.(13), the seventh order system

$$R(s) = \frac{-s^6 + 1.292s^5 + 2.064s^4 - 4.583s^3 - 7.906s^2 - 4.944s - 1.399}{s^7 + 8.297s^6 + 31.41s^5 + 68.06s^4 + 86.72s^3 + 62.24s^2 + 22.32s + 2.786} \tag{32}$$

is obtained. I will estimate $R(s)$ as a seventh order system assuming that the perturbed plant is unknown but its order is the same as the one of the nominal plant, that is, the second order.

Setting the reference signal $r(t) = 1$ and the external input signal $d(t) = 0$, the parameters of $\hat{R}(s)$ estimated using the new input α and output β (so-called generalized input and output) are shown in Fig.5. Although the estimated parameters are different from the ones of real $R(s)$ in Eq.(32), the parameters obtained by reducing the estimated plant $\hat{P}(s)$ (the twelfth order system) to a second order system are very close to the true values as shown in Fig.6. We can see that although the parameters of $R(s)$ cannot be estimated, the model of $P(s)$ can be estimated.

The ν-gap metrics ν between the estimated plant $\hat{P}(s)$ and the real plant $P(s)$ are shown in Fig.7. From the estimated parameters, ν is calculated to be about 0.005. The plant model has successfully been estimated, since ν nearly equals 0.

The ν-gap metrics ν_0 between the augmented estimated plant $W(s)\hat{P}(s)$ and the augmented nominal plant $W(s)P_0(s)$ are shown in Fig.8. For the estimated parameters, ν_0 is calculated to be about 0.29 and nearly equals the true value (0.29). The estimation results allow us to expect that the closed-loop system is sufficiently robustly stable, since ε is much larger than ν_0.

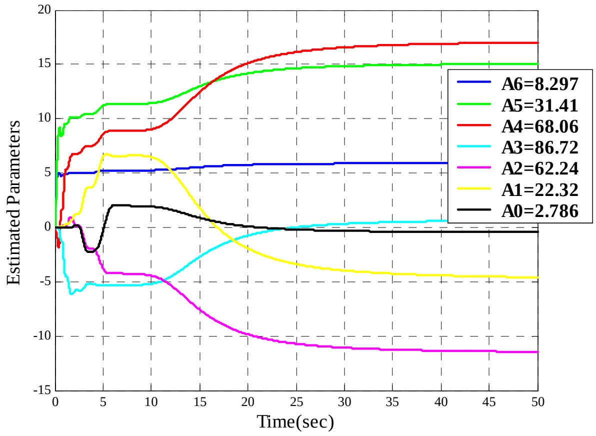

Fig.5a. Estimated Parameters (Denominator)

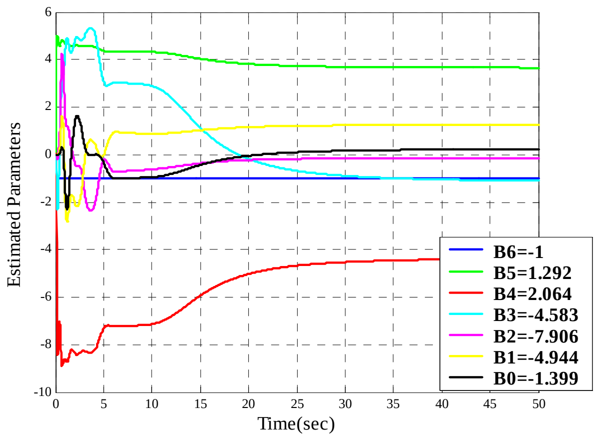

Fig.5b. Estimated Parameters (Numerator)

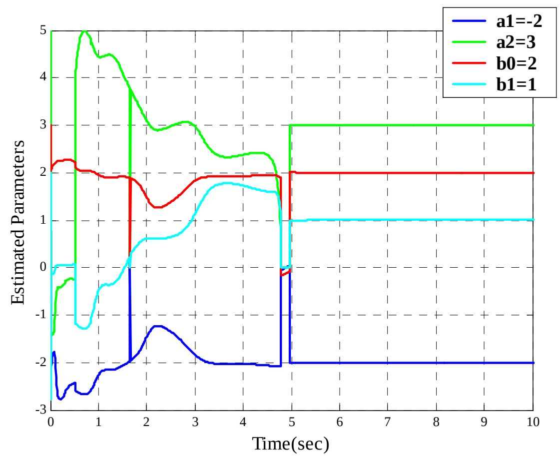

Fig.6. Calculated Plant Parameters

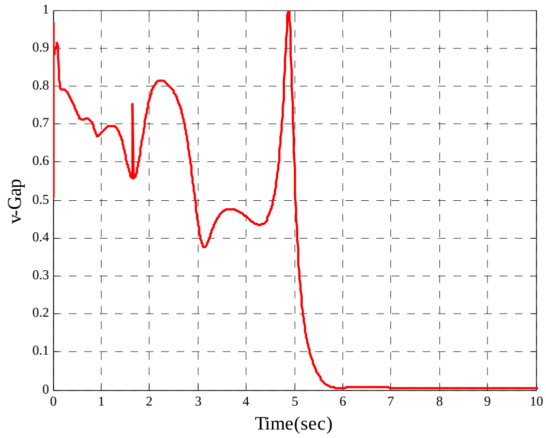

Fig.7. ν-Gap Metric (versus Real Plant)

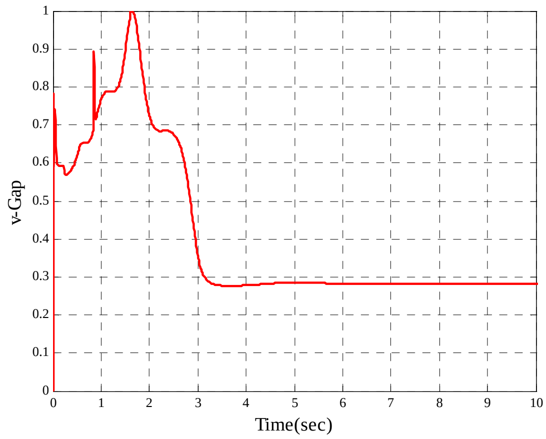

Fig.8. ν-Gap Metrics (versus Original Plant with Weight)

Next I will show that ν-gap metric can be a measure of norm of failure that gives a great influence for stability of closed-loop system.

Assume that a real plant is changed by influence of failure as follows:

$$P(a,s) = \frac{3s+2}{s^2 - as + 2} \tag{33}$$

where 'a' is a parameter and when a = 1, the real plant equals the nominal plant. The relation of the ν-gap metrics between $W(s)P_0(s)$ and $W(s)P(a, s)$, and the robust stability margin ε is shown in Fig.9. In Fig.9 the blue line with * indicates the ν-gap metrics of each perturbed plant when an integer from 0 to 10 is substituted for the perturbed parameter 'a'. From Fig.9 we can see that ν-gap metric is effective as an index of model error, since the ν-gap metric gets larger as 'a' is changed apart from 1, that is, the change of the perturbed plant from the nominal plant gets larger. In Fig.9 the red line indicates the robust stability margin ε about the nominal plant and the stability of closed-loop system can be guaranteed if the ν-gap metric of a perturbed plant is smaller than ε. In Fig.9 the green line indicates a desirable margin from ε and the performance of closed-loop system can be guaranteed if the ν-gap metric is below the line. The ν-gap metric ν^* of this line is decided as the robust stability margin about the perturbed plant calculated from the estimated plant is larger than a threshold value (setting 0.3), that is, the following equation holds [16].

$$\sin\left(\sin^{-1}\varepsilon - \sin^{-1}\nu^*\right) \geq 0.3 \tag{34}$$

From Fig.9 it is shown that the closed-loop system is stable when 'a' is within 0 to 7 and that the closed-loop system satisfies the performance within 0 to 4. This is proved from the step response of each closed-loop system (Fig.10). We can prevent the vital degradation of the performance if we execute reconfiguration of the control law when the ν-gap metric calculated from the estimated plant exceeds ν^*.

For example, when 'a' equals 5, we execute reconfiguration of the control law, since the ν-gap metric calculated from the estimated plant $\hat{P}(s)$ exceeds ν^* as shown in Fig.9. We calculate the estimated plant $\hat{P}(s)$ and redesign the modified controller

$$C_{new}(s) = \frac{2.173s^2 + 0.6212s + 0.3301}{s^2 + 1.754s + 0.7175} \tag{35}$$

by applying NCF method to $W(s)\hat{P}(s)$, when the closed-loop system is still stable. We change the nominal controller $C(s)W(s)$ to the modified augmented controller $C_{new}(s)W(s)$ after the transient response caused by the parameter perturbation converges.

We show a chain of profile of the above events in Fig.11. The response during t = 0 ~ 15 (sec) shows the step response for the nominal plant $P_0(s)$ with the nominal controller $C(s)W(s)$. It indicates that both the stability and the performance are desirable. When the plant is changed to $P(5, s)$ by influence of failure at t = 15 (sec) under the step input, the great transient response occurs, but the response converges during about 10 secs for the robust stability possessed by $C(s)W(s)$ as shown from the response during t = 15 ~ 35 (sec). We estimate the perturbed plant by CF method the while. We put the input to zero again at t = 35 (sec). The response during t = 35 ~ 60 (sec) shows the step response for the perturbed plant $P(5, s)$ with $C(s)W(s)$. It indicates that the damping is worse than the one in $P_0(s)$, that is, the performance is degraded. It is also proved from the fact that the ν-gap metric (≅0.45) calculated from the estimated plant exceeds ν^* (=0.43). We execute redesign logic and obtain the redesign controller $C_{new}(s)W(s)$. We change $C(s)W(s)$ to $C_{new}(s)W(s)$ at t = 60 (sec) and no transient response occurs as shown from the response during t = 60 ~ 80 (sec). The response during t = 80 ~ 100 (sec) shows the step response for the perturbed plant $P(5, s)$ with the redesign controller $C_{new}W(s)$. It indicates that the damping is better than the one during t = 35 ~ 60 (sec), that is, the performance is recovered.

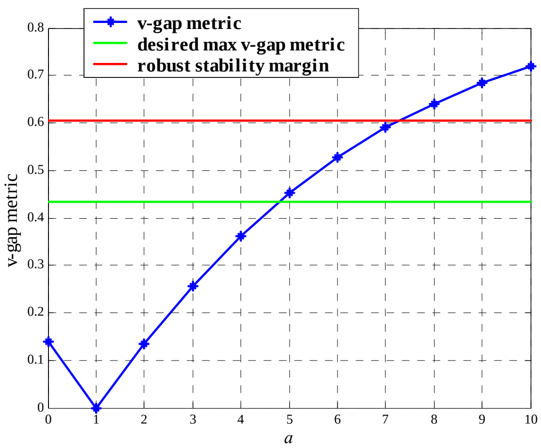

Fig.9. ν-Gap Metric and Robust Stability Margin

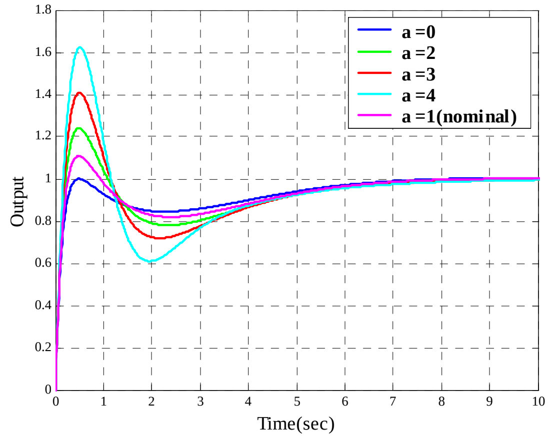

Fig.10a. Step Responses (Perturbed Plant: a = 0 ~ 4)

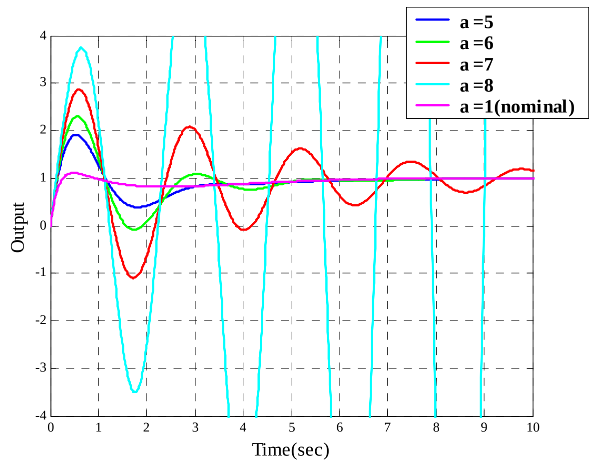

Fig.10b. Step Responses (Perturbed Plant: a = 5 ~ 8)

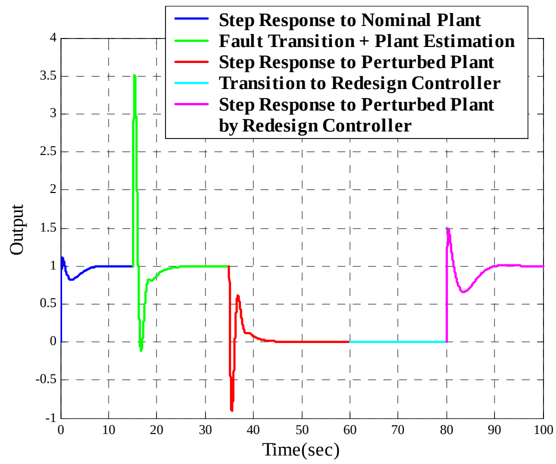

Fig.11. Time Responses
for a sequence of failure and reconfiguration steps

5.2 Case of SITO model

For another simple unstable model of SITO,

$$P_0(s)=\begin{bmatrix}\dfrac{3s+2}{s^2-s+2}\\ \dfrac{2}{s+1}\end{bmatrix} \tag{36}$$

we design a controller which stabilizes the closed-loop system. As a result, the below controller is obtained.

$$C(s)=\begin{bmatrix}\dfrac{1.4s^2+2.5s+0.37}{s^2+1.7s+1.0} & \dfrac{0.41s^2+0.50s+0.054}{s^2+1.7s+1.0}\end{bmatrix} \tag{37}$$

which stabilizes the system with a robust stability margin ε of about 0.56.

Using the normalized coprime factorization, we have NCF representations of the nominal plant $P_0(s)$ and the controller $C(s)$ as follows

$$N_{0l}(s)=\begin{bmatrix}\dfrac{3.0s^2+5.1s+3.2}{s^3+5.0s^2+7.2s+4.9}\\ \dfrac{2.0s^2+2.6s+3.1}{s^3+5.0s^2+7.2s+4.9}\end{bmatrix} \tag{38}$$

$$D_{0l}(s)=\begin{bmatrix}\dfrac{s^3+0.86s^2+0.14s+3.7}{s^3+5.0s^2+7.2s+4.9} & \dfrac{-1.2s^2-1.5s-0.27}{s^3+5.0s^2+7.2s+4.9}\\ \dfrac{-1.2s^2+1.2s-2.5}{s^3+5.0s^2+7.2s+4.9} & \dfrac{s^3+4.2s^2+6.0s+2.8}{s^3+5.0s^2+7.2s+4.9}\end{bmatrix} \tag{39}$$

$$X_l(s)=\begin{bmatrix}\dfrac{1.4s^2+2.5s+0.37}{s^2+1.8s+0.62} & \dfrac{0.41s^2+0.50s+0.054}{s^2+1.8s+0.62}\end{bmatrix} \tag{40}$$

$$Y_l(s)=\frac{s^2+1.7s+1.0}{s^2+1.8s+0.62} \tag{41}$$

$$X_r(s)=\begin{bmatrix}\dfrac{1.4s^2+2.4s+0.36}{s^2+1.8s+0.62} & \dfrac{0.41s^2+0.58s+0.070}{s^2+1.8s+0.62}\end{bmatrix} \tag{42}$$

$$Y_r(s)=\begin{bmatrix}\dfrac{1.1s^2+1.8s+1.0}{s^2+1.8s+0.62} & \dfrac{-0.21s^2-0.23s-0.038}{s^2+1.8s+0.62}\\ \dfrac{-0.21s^2-0.41s-0.059}{s^2+1.8s+0.62} & \dfrac{1.7s^2+3.1s+1.1}{s^2+1.8s+0.62}\end{bmatrix} \tag{43}$$

Those transfer functions satisfy Eqs.(9) and (10). In fact, we have

$$\begin{aligned}&N_{0l}(s)X_r(s)+D_{0l}(s)Y_r(s)\\&=\begin{bmatrix}\dfrac{1.1s^2+1.9s+1.0}{s^2+1.8s+0.62} & \dfrac{-0.21s^2-0.25s-0.014}{s^2+1.8s+0.62}\\ \dfrac{-0.21s^2+1.3s-0.33}{s^2+1.8s+0.62} & \dfrac{1.7s^2+2.7s+0.66}{s^2+1.8s+0.62}\end{bmatrix}\\&\in GH_\infty\end{aligned} \tag{44}$$

Equation (44) indicates that Eq.(11) holds. We can use the transfer functions Eqs. (38), (39), (40), and (41) in Eq.(14), which generate α and β used to identify $R(s)$.

I will estimate an unknown perturbed plant $P(s)$ and ν-gap metrics between $\hat{P}(s)$ and $P(s)$ or $P_0(s)$. For example, the real plant is assumed to be changed by influence of failure as follows.

$$P(s)=\begin{bmatrix}\dfrac{2s+1}{s^2-2s+3}\\ \dfrac{4}{s+5}\end{bmatrix} \tag{45}$$

By applying Eqs.(38), (39), (40), (41), and (45) to Eq.(13), the eighth order system

$$R(s)=\begin{bmatrix}\dfrac{-s^7-8.1s^6+9.4s^5+24s^4-32s^3-88s^2-75s-20}{s^8+14s^7+72s^6+206s^5+429s^4+613s^3+586s^2+327s+89}\\ \dfrac{2s^7+1.1s^6-8.6s^5-17s^4-1.7s^3-9.6s^2-39s-16}{s^8+14s^7+72s^6+206s^5+429s^4+613s^3+586s^2+327s+89}\end{bmatrix} \tag{46}$$

is obtained. I will estimate $R(s)$ as an eighth order system assuming that the perturbed plant is unknown but its order is the same as the one of the nominal plant.

Setting the reference signal $r(t)$ = 1 and the external input signal $d(t)$ = 0, the parameters of $\hat{R}(s)$ are estimated using the new input α and output β. The parameters obtained by reducing the estimated plant $\hat{P}(s)$ (the twentieth order system) to a second/first order system are very close to the true values as shown in Fig.12. We can see that the model of $P(s)$ can be estimated.

The ν-gap metric ν between the estimated plant $\hat{P}(s)$ and the real plant $P(s)$ converges about 0.002. The plant model has successfully been estimated, since ν nearly equals 0.

The ν-gap metric ν_0 between the estimated plant $\hat{P}(s)$ and the nominal plant $P_0(s)$ converges about 0.55 and nearly equals the true value (0.55). The estimation results allow us to expect that the closed-loop system is robustly stable but degrades the performance, since ε is only a little larger than ν_0. It is proved from the step responses of the closed-loop system in case of both the nominal plant $P_0(s)$ and the perturbed plant $P(s)$ in Fig.13.

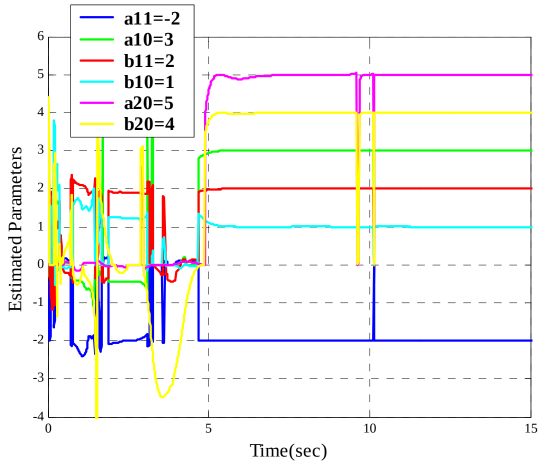

Fig.12. Calculated Plant Parameters

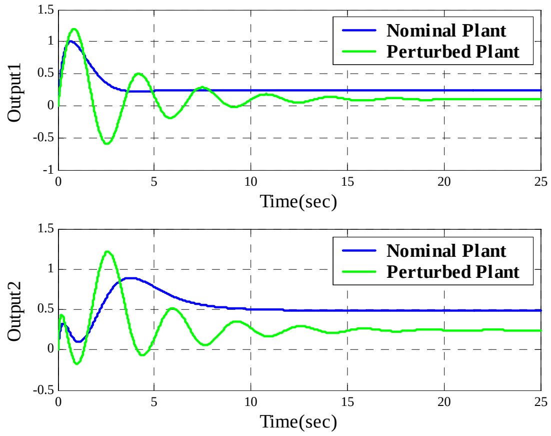

Fig.13. Step Responses

6 Conclusions

The identification method based on the coprime factorization proves to be effective for closed-loop identification of control systems designed by the normalized coprime factorization method. The method will work for failure detection and identification of a reconfigurable (self-repairing) FCS.

References

[1] Kameyama T, Senuma T, Matsumoto Y, Ide M, Shimizu E, Hisano T, Yasui H and Goto K. The Study on Self Repairing Flight Control System - Overview of Self Repairing Flight Control Law, *The 36th Aircraft Symposium*, Yokosuka, pp 457-460, 1998.

[2] Kameyama T, Senuma T, Matsumoto Y, Ide M, Shimizu E, Tokuda E, Masuko Y and Seto J. The Study on Self Repairing Flight Control System - Inner Loop Robust Control Law Design, *The 36th Aircraft Symposium*, Yokosuka, pp 461-464, 1998.

[3] Kameyama T, Matsumoto Y, Ohsawa H, Horie K, Kataoka K, Hisano T, Sataka M, Goto K and Yasui H. The Study on Self Repairing Flight Control System - Overview of Evaluation to Flight Control System Using Dynamic Wind Tunnel, *The 39th Aircraft Symposium*, Gifu, CD-ROM, pp 347-350, 2001.

[4] Masuko Y, Goto K, Hatakenaka H, Hisano T, Tokuda E, Yasui H, Kameyama T, Matsumoto Y, Ohsawa H, Horie K and Kataoka K. The Study on Self Repairing Flight Control System - The Design of Self Repairing Flight Control System, *The 39th Aircraft Symposium*, Gifu, CD-ROM, pp 351-357, 2001.

[5] Ishiguro M, Yamaguchi Y, Arakawa M, Nonaka R, Sataka M, Takagi M, Kameyama T, Matsumoto Y, Ohsawa H, Horie K and Kataoka K. The Study on Self Repairing Flight Control System - The Design of The Dynamic Free-Flight Wind Tunnel Test System, *The 39th Aircraft Symposium*, Gifu, CD-ROM, pp 358-363, 2001.

[6] Matsumoto Y, Kameyama T, Ohsawa H, Horie K and Kataoka K. The Study on Self Repairing Flight Control System - The Results of the Dynamic Free-Flight Wind-Tunnel Testings, *The 39th Aircraft Symposium*, Gifu, CD-ROM, pp 364-373, 2001.

[7] D. C. McFarlane, K. Glover: *Robust Controller Design Using Normalized Coprime Factor Plant Descriptions*, Lecture Notes in Control and Information Sciences, No. 138, Springer-Verlag, 1990.

[8] S. Skogestad, I. Postlethwaite: *Multivariable Feedback Control*, John Wiley & Sons, 1996.

[9] M. Saeki, et al.: *Design of Control System - H_∞ Control and its Application*, Asakura-Shoten, 1994.

[10]F. Hansen, G. Franklin: On a Fractional Representation Approach to Closed-Loop Experiment Design, *Proc. of Am. Control Conf.*, Atlanta, pp 1319-1320, 1988.

[11]G. Vinnicombe: Frequency domain uncertainty and the graph topology, *IEEE Trans. on Automatic Control*, Vol. 38, No. 9, pp 1371-1383, 1993.

[12]G. Vinnicombe: *Uncertainty and Feedback - H_∞ loop-shaping and the ν-gap metric*, Imperial College Press, 2001.

[13]K. Zhou, J. C. Doyle: *Essentials of Robust Control*, Prentice Hall, 1998.

[14]A. Fujimori: *Robust Control*, Corona Publishing, 2001.

[15]S. Adachi: *System Identification Theory for Users*, Society of Instrument and Control Engineers, 1993.

[16]S. Shinnaka: *Adaptive Algorithm*, Sangyo-Tosho, 1990.