DEVELOPMENT OF A SIMPLE AND FAST COMPUTATIONAL ROUTINE TO SOLVE THE FULL POTENTIAL EQUATION OF THE TRANSONIC AXI-SYMMETRIC FLOW

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ABSTRACT

The present work is a further development of the authors' effort to carry out numerical computational study of transonic flow around two-dimensional and slender axi-symmetric bodies, with a view to develop simple and fast computational routines (using PC's) for drag minimization purposes. Therefore a computational method to solve the full potential equation for axi-symmetric flow will be developed. The results of the computation was validated by comparison with other existing methods, as well as, in the degenerate case of slender body, with the small perturbation method developed earlier by the authors. Experimental results for the latter case are also available for comparison. The AF2 algorithm for solving transonic full potential equation for two-dimensional cases is here adopted and modified accordingly for the axi-symmetric flow, and has given excellent results.

1. INTRODUCTION

Transonic flow around axi-symmetric bodies has been the subject of interest for many investigations due to its relevance to the flight of aircrafts and projectiles, for which relatively large drag compared to the sub- and supersonic regimes is encountered and should be minimized. Many commercial computational fluid dynamic (CFD) codes have been developed using high level complexity and accuracy based on the complete numerical solution of the associated Euler or Navier-Stokes equation using finite difference approach (such as FLUENT). The computation may require considerable computation time, both in mainframe as well as personal computers. For preliminary design analysis, it is desirable to have access to relatively simple and fast computational analysis using personal computers This has motivated the authors to study and develop a relatively simple and fast method, in particular to analyze axi-symmetric bodies.

The effort was initiated with the utilization of transonic small perturbation approach as reported earlier [1][2][3]. As a continuation of the effort, in this paper attention is focused on the development of a computational routine to solve the full potential equation of the transonic axi-symmetric flow. Therefore, it is our objective, particularly for efficient use in the preliminary design and optimization, to develop an in house efficient computational scheme.

The development of the transonic full potential equation code for axi-symmetric flow capitalizes on the success of similar method that was applied to 2D problem. This study is limited to the axi-symmetric flow about an axi-symmetric body.

Axi-symmetric bodies can be described by two coordinates that are analogous to two dimensional problems, allowing the relatively simple formulation of the computational scheme somewhat similar to the two dimensional case. The method developed also capitalizes on the successful pioneering transonic small perturbation (TSP) scheme that was introduced by Murman and Cole[4], first for two dimensional problem, in particular the upwinding scheme. A transonic small perturbation computational code has been developed for computing slender axi-symmetric body using Murman Cole algorithm. This code has been validated for slender body. The formulation of the conservative form of the full potential equation amenable to numerical computation follows the rotated difference scheme introduced by Jameson[5][6]. Following closely the philosophy and procedures introduced in these schemes, the transonic full potential equation (TFPE) has been rederived for axi-symmetric flow, leading to the development of a TFPE code which has higher order of accuracy compared to the TSP code. There is no small perturbation assumption for TFPE and the body involved is not necessarily slender; hence the TFPE code will have a wider range of applications.

To obtain a convenient form of the TFPE for efficient numerical computation, the second variant of Approximate Factorization (AF2) method is considered by the authors to be most appropriate for solving transonic full potential equation. The algorithm has been developed for two-dimensional problem by Holst and Ballhaus[7][8][9] to arrive at accurate and more efficient results compared to classical algorithm like successive line over relaxation (SLOR). This algorithm will be applied and adapted for the present axi-symmetrical flow case.

The computational code is then validated by comparing the results with simliar ones obtained by Green and South [10] for the full potential solution of transonic flow over a sphere. For the degenerate cases of slender axi-symmetric bodies, comparison of the results to those obtained using TSP approach (including those obtained using the author's earlier work), as well as to the experimental data obtained by MBB[9], have given confidence to the accuracy and effectiveness of the TFPE computational code developed.

2. FULL POTENTIAL EQUATION

3.1. The Governing Equation of Transonic Full Potential for Axi-Symmetric Flow

The Divergence Form of Transonic Full Potential Equation for two dimensional flow is well known and given by[6][11]-[16] :

$$(\bar{\rho}\varphi_x)_x + (\hat{\rho}\varphi_r)_r = 0 \qquad (1.a)$$

where $\bar{\rho} = \rho - \mu\Delta x \rho_x$ (1.b)

$$\hat{\rho} = \rho - \mu\Delta y \rho_y \qquad (1.c)$$

This equation is based on Jameson's rotated difference scheme that is applied for transonic full potential equation . The scheme introduces $\bar{\rho}$ and $\hat{\rho}$ as biased density to accommodate the characteristic of supersonic flow , where disturbance of a point in physical domain propagates only in the area that is limited by the Mach cone. This scheme is similar with Murman-Cole scheme for transonic small perturbation equation (TSP). The approach introduced Jameson for two-dimensional problem is now adapted for the transonic axi-symmetric flow problem.

3. GRID GENERATION

A grid system is generated by the algebraic method, transfinite interpolation and then smoothed to obtain orthogonality and smooth grid distribution by Partial Differential Equation [17]:

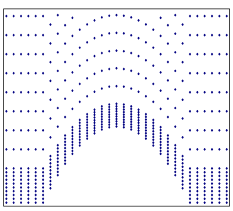
$$\alpha\, x_{\xi\xi} + 2\beta\, x_{\xi\eta} + \gamma\, x_{\eta\eta} = 0 \qquad (2.a)$$

$$\alpha\, r_{\xi\xi} + 2\beta\, r_{\xi\eta} + \gamma\, r_{\eta\eta} = 0 \qquad (2.b)$$

$$\begin{aligned} \alpha &= x_\xi^2 + r_\xi^2 \\ \beta &= -x_\xi x_\eta - r_\xi r_\eta \\ \gamma &= x_\eta^2 + r_\eta^2 \end{aligned} \qquad (2.c)$$

The equation is discretized by finite difference approach and solved by relaxation method.

Because of singularity, it is convenient to make Δr very small therefore the tendency of solution near singularity can be captured. For this purpose, we define 'inner grid', it is inspired by 'inner solution' of Slender Body Theory (see for example Ashley and Landahl[18]).



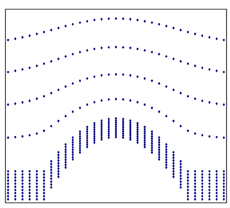


Figure 1: Grid before and after being smoothed by PDE

4. TRANSFORMED VARIABLES AND EQUATION

To carry out computation in the transformed space, the governing equation (1) is transformed into

$$\frac{\partial(r\bar{\rho}U)}{\partial\xi}+\frac{\partial(r\hat{\rho}V)}{\partial\eta}=0 \qquad (3.a)$$

The contravariant velocities are expressed as :

$$U=u\xi_x+v\xi_y \qquad (3.b)$$

$$V=u\eta_x+v\eta_y \qquad (3.c)$$

Parameters of the grids are derived from the relationships :

$$\xi_x=Jr_\eta \qquad (4.a)$$

$$\eta_x=-Jr_\xi \qquad (4.b)$$

$$\xi_r=-Jr_\eta \qquad (4.c)$$

$$\eta_r=Jx_\xi \qquad (4.d)$$

$$J\approx\frac{1}{x_\xi r_\eta-x_\eta r_\xi} \qquad (4.e)$$

The other properties can be determined using the thermodynamic relations

$$u=\varphi_x \quad v=\varphi_r \qquad (5.a)$$

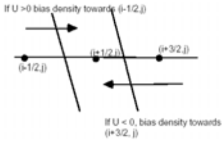
$$a^2=a_\infty^2+\frac{\gamma-1}{2}\left[u_\infty^2+v_\infty^2-u^2+v^2\right] \qquad (5.b)$$

$$a_\infty=V_\infty/M_\infty \quad \text{with} \quad V_\infty=1 \qquad (5.c)$$

$$\frac{\rho}{\rho_\infty}=\left(\frac{a^2}{a_\infty^2}\right)^{\frac{1}{\gamma-1}} \qquad (5.d)$$

$$M^2=\frac{u^2+v^2}{a^2} \qquad (5.e)$$

The biased density is computed by following manner :



if $U>0$

$$\rho_{i+1/2,j}=\rho_{i+1/2,j}-\text{Max}\left[0,1-\frac{2}{M_{i+1/2,j}^2+M_{i-1/2,j}^2}\right]\left(\rho_{i+1/2,j}-\rho_{i-1/2,j}\right)$$

otherwise (6)

$$\rho_{i+1/2,j}=\rho_{i+1/2,j}-\text{Max}\left[0,1-\frac{2}{M_{i+3/2,j}^2+M_{i+1/2,j}^2}\right]\left(\rho_{i+3/2,j}-\rho_{i+1/2,j}\right)$$

The computation of $\bar{\rho}$ and $\hat{\rho}$ are carried out in similar manner.

For:

- Solid Surface and Symmetrical Line

$V=0$ at $j=0$, it imply

$$\Phi_\eta=\frac{-\left[\Phi_\xi\left(\xi_x\eta_x+\xi_y\eta_y\right)++\eta_x\right]}{\eta_x^{\,2}+\eta_y^{\,2}} \qquad (7)$$

- Far Field : $\Phi = x$, or all the properties are not disturbed.

All variables described above are evaluated using central difference scheme at the half nodal points, to reduce the number of grid points utilized for the finite difference equation.

5. APPROXIMATE FACTORIZATION

After writing the governing equation (3) in finite difference form applied to the grid, the resulting equation is numerically solved using Approximate Factorization 2 (AF2) Algorithm . This Algorithm was introduced by Ballhaus, devoleped to obtain fast and accurate result for solving transonic full potential flow of 2D. The Algorihm was developed form original version of Approximate Factorization for transonic potential equation to counter instability because of moving shock during iteration process. The algoritm is adopted because of its effectiveness.

The resulting algebraic equation obtained from the finite difference formulation of equation (3) , which will be solved using the approximate factorization scheme, is computed through an iterative process. For iterative purposes, the equation can be cast into the folloeing form:

$$N\Delta\varphi^n + \sigma R^n_{i,j} = 0 \qquad (8)$$

N is an operator that is performed to $\Delta\varphi^n$. $R^n_{i,j}$ is discretized part of equation Eq(8) and is called the residual term. It can be written as follows :

for interior point :

$$R_{i,j} = \left(\frac{r\bar{\rho}U}{J}\right)_{i+1/2,j} - \left(\frac{r\bar{\rho}U}{J}\right)_{i-1/2,j} + \left(\frac{r\hat{\rho}V}{J}\right)_{i,j+1/2} - \left(\frac{r\hat{\rho}V}{J}\right)_{i,j-1/2} \qquad (9.a)$$

for j = 0

$$R_{i,0} = \left(\frac{r\bar{\rho}U}{J}\right)_{i+i/2,j} - \left(\frac{r\bar{\rho}U}{J}\right)_{i-i/2,j} + 2\left(\frac{r\hat{\rho}V}{J}\right)_{i,j+1} \qquad (9.b)$$

N can be defined as product of approximate factroization :

$$\alpha N\Delta\Phi_{i,j} = -\left[\alpha - \vec{\delta}_\eta\left(\frac{r\hat{\rho}A_3}{J}\right)_{i,j-1/2}\right]\left[\alpha\overleftarrow{\delta}_\eta - \overleftarrow{\delta}_\xi\bar{\rho}_i\left(\frac{rA_1}{J}\right)_{i+1/2,j}\delta_\xi\right]\Delta\Phi_{i,j}$$

(10.a)

$$A_1 = \frac{\xi_x{}^2 + \xi_r{}^2}{J} \qquad A_2 = \frac{\eta_x{}^2 + \eta_r{}^2}{J}$$

(10. b-c)

( ) = ξ- wise backward difference operator = ( )i+1,j − ()I

( ) = ξ-wise forward difference operator = ( )i,j+1 − ( )I

( ) = η- wise forward difference operator = ( )i,j − ()i,j-1

The computation is carried out into two-step as follows

Step 1 :

$$\left[\alpha - \vec{\delta}_\eta\left(\frac{r\rho A_3}{J}\right)_{i,j-1/2}\right] f_{i,j} = \alpha\,\omega\, L\Phi^n_{i,j} \qquad (11)$$

Step 2

$$\left[\alpha\overleftarrow{\delta}_\eta - \overleftarrow{\delta}_\xi\bar{\rho}_i\left(\frac{rA_1}{J}\right)_{i+1/2,j}\vec{\delta}_\xi\right]\Delta\Phi^N_{i,j} = f_{i,j} \qquad (12)$$

The first step is performed to all point in the i-direction and the second step is perform to all points in the j-direction.The result of each iteration is ΔΦ at all points.

ω is a relaxation factor; it is selected to obtain efficient computation, and its value can be chosen between 1 and 2.

Definition of α

α is not determined in an exact manner. α can be considered as a parameter that describes the pseudo-time for each step while the iteration is running. Jameson[18] suggests the value of α as follows:

α is the geometric sequence :1, 2, 3..M. M is the maximum number of sequence that is chosen so that the scheme run effectively.

$$\alpha_k = \alpha_h\left(\frac{\alpha_l}{\alpha_h}\right)^{(k-1)/(M-1)}$$

K=1,2,3,...M (13)

- α_l = α for low frequency of error
- α_h = α for high frequency of error

Based on the full potential code for 2-dimension, Sankar [6] uses:

- α_l = 0.5
- α_h = 0.1
- M = 5

6. RESULTS

First the computational results for a sphere at M=0.7 is compared to those obtained by Green and South[10], who uses a different computational approach. The comparison is shown in Fig. 2, which shows that excellent agreement is obtained.

Next, for the degenerate case of slender axi-symmetric flow, the TFPE computational code developed is applied to a slender body with a parabolic profile at Mach numbers 0.975 and 0.99. The computational results are compared with those obtained using the TSP code developed earlier by the authors as well as computational and experimental results reported by Railey[19], as shown in Fig.3. The pressure distribution curve obtained by the present code agree closely with those other results, although relatively qualitative agreement is indicated in the region of the shock.

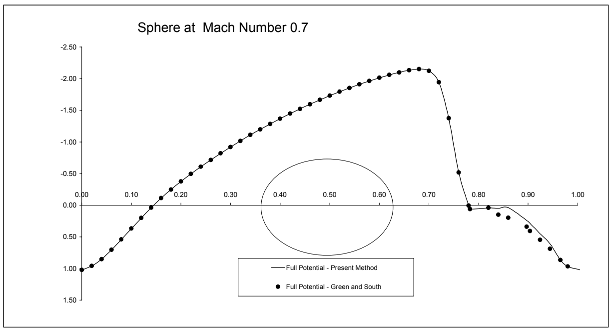
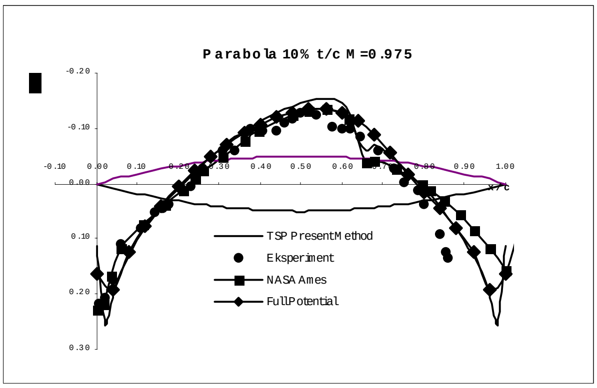


Fig.2 Comparison between Green and South's results and those of present method.

Figure 4 shows the comparison of the pressure distribution curve obtained using the present method with the results using TSP code developed by Sekar[20] and experimental results obtained by Lorenz-Meyer, W. and Aulehla at MBB[21]. Again excellent agreement is indicated. Note that the present full potential code developed can be used for slender body as well as bulky ones.

The computation using the TFPE code was carried out using 222 x 223 grid points. The body length is 0.25 of horizontal boundary's length of the grid system. The computation was performed up to 5000 iteration and took about 5 minutes using Generic PC with Pentium 450 Hz CPU.



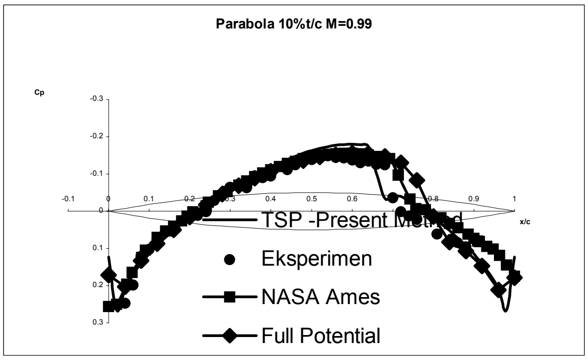
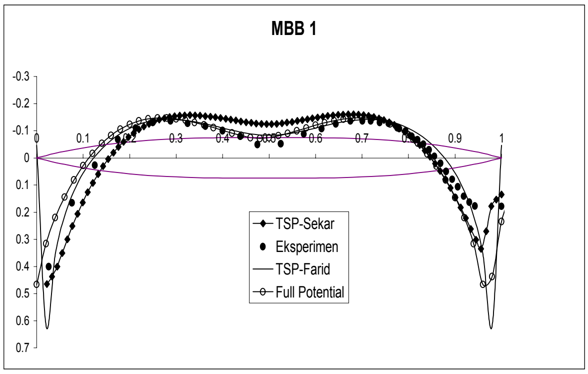
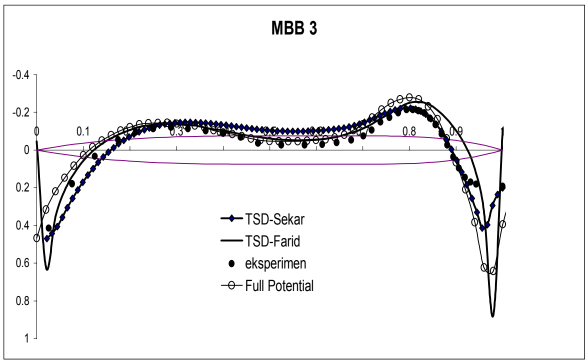


Figure 3 : Presentation of results, parabolic arc in mach number 0.9,0.975, 0.99





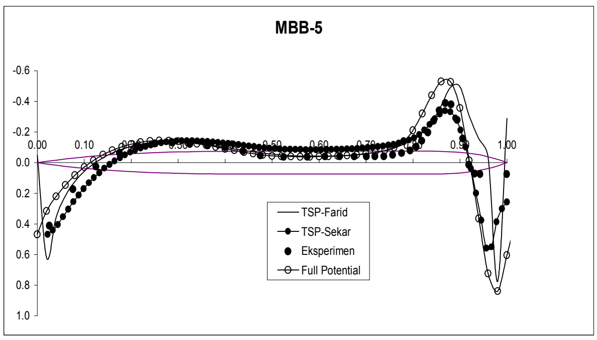


Figure 4 : Comparison of results for MBB Bodies (MBB 1, MBB3, MBB5) at Mach number 0.8.

7. CONCLUSIONS

The Transonic Full Potential Equation computational code has been developed with good accuracy and efficiency. The code has also been validated by comparison of results for spherical and slender axi-symmetric geometries.

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