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Heat Transfer with Ablation in Cylindrical Bodies

1 ABSTRACT

The phenomenon of ablation is a process of thermal protection with several applications, mainly, in mechanical and aerospace engineering. Ablative thermal protection is applied using special materials (named ablative materials) externally on the surface of a structure in order to isolate it against thermal effects. The ablative phenomenon is a complex process involving phase changes with partial or total loss of the material. So the position of the boundary it is unknown a priori. The governing equations of the process is a non-linear system of coupled partial differential equations. The uni-dimensional analysis of ablative process in a cylindrical body is performed by using the generalized integral transform technique – GITT for solution of the system of governing equations. By application of this technique of solution the system of partial differential equations is transformed in a system of infinite ordinary differential equations that can be solved by numerical techniques available in computational codes after the truncation of that system. As boundary condition is considered a transient heat flux like ones that occur, for example, in re-entrance of aerospace vehicles in the atmosphere. The results of interest are the thickness and the rate of loss of the ablative material. The obtained results are compared with available results of other techniques of solution in the literature.

2 INTRODUCTION

The phenomenon of ablation is a process of thermal protection with several applications, mainly, in mechanical and aerospace engineering. Ablative thermal protection is applied using special materials (named ablative materials) externally on the surface of a structure in order to isolate it against thermal effects. The ablative phenomenon is a complex process involving phase changes with partial or total loss of the material. So the position of the boundary it is unknown a priori.

The diagrams shown in Figures 1 and 2 illustrate the ablation phenomenon. In Figure 1 the following proccesses are presented:

1. Heat transfer by convection in the boundary layer, that represents the main thermal load;
2. Heat transfer by radiation;
3. Heat transfer by conduction in the virgin material that should satisfy the temperature approach limits in the substructure or in the thermal shield on the structure;
4. Resin decomposition;
5. Fibers decomposition;
6. Passage of the gas produced through of the residuals;
7. Retreat of the surface;
8. Radiation in the wall;
9. Shock in the boundary layer;
10. Combustion in the boundary layer.

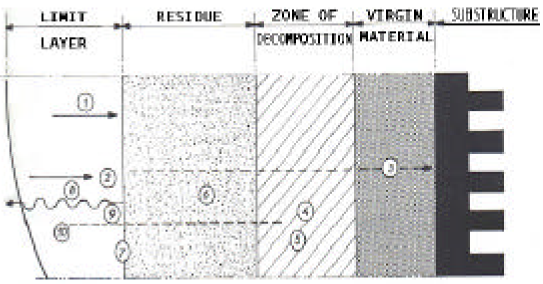

Figure 1: Illustration of ablation phenomenon.

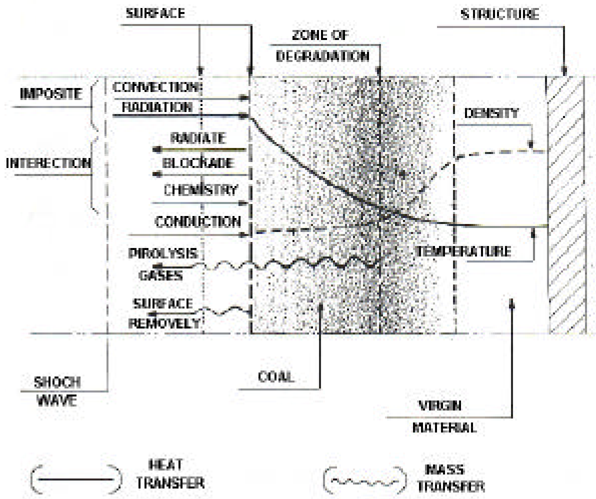

Figure 2: Physical representation of the ablation process involving the melting of the ablative material.

The analytical and numerical, as well as analytical-numerical solutions have been done. In a two-dimensional, geometry the method of Approximate Integral Balance, was presented by Hsiao et all (1984). The physical and mathematical models of the ablation process have been presented by Lacase (1967) and Zaparoli (1989). The solution of the diffusion problem with variable coefficients was studied by Özisik and Cotta (1987). A generalized study of the ablative phenomenon was done by Adams and Sutton (1982 and 1959). The using of the Generalized Integral Transform Technique (GITT), was presented by Cotta (1993), Diniz et all (1990, 1993). They solved the uni-dimensional problems of heat diffusion for several geometries. Vollerani (1974) applied the Integral Method for problems of simple classes of ablation. The Classical Integral Transform Technique for linear problems was presented by Mikhailov and Özisik (1984). The GITT is a generalization of that technique for non-linear problems

In the present work the uni-dimensional analysis of the ablative process in a cylindrical body has been done using the Generalized Integral Transform Technique as an analytical tool for solution of the differential partial equation, considering the isolated internal side and the external side subject to a transient heat flow. In the solution of the problem the values of the depth S(t) and the ablative speed represented by V(t), are obtained and compared with results from the literature.

ANALYSIS

Here it is considered the conduction of heat in a hollow cylinder of finite thickness (l) with constant physical properties. Initially, the cylinder is subjected to a temperature T0 and its external surface is submitted to transient heat flux and the internal surface is thermally isolated. The problem is solved in two steps: the heating of the material to the phase change temperature (Pre-ablation Period) and the phase change with removal of the protection material (Ablation Period). Figure 3 shows the cylindrical geometry

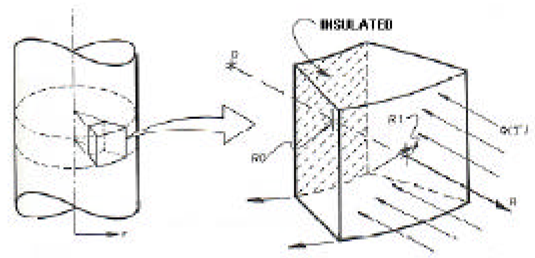

Figure 3: Cylindrical geometry.

- **Preablation Period:**

The Figure 4 illustrate the phenomenon for the Pre-ablation Period:

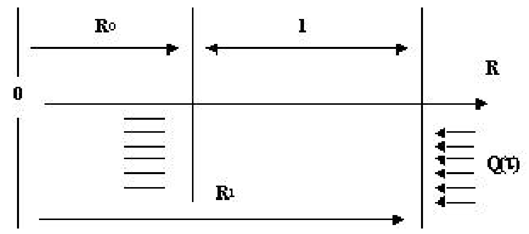

Figure 4: Cylindrical geometry in the radial direction.

The uni-dimensional heat conduction equation for the cylindrical geometry is the form

$$\frac{\partial q(R,t)}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[R\frac{\partial q(R,t)}{\partial R}\right]; \qquad (1)$$

The following initial and boundary conditions, are considered

$$\theta\,(R,0)=0\;;\;\tau=0 \qquad (2)$$

$$\left.\frac{\partial q(R,t)}{\partial R}\right|_{R=R_0}=0 \;\;\text{e}\;\; \left.\frac{\partial q(R,t)}{\partial R}\right|_{R=R_1}=Q(t) \qquad (3)$$

In order to homogenize the equations (1) and (3) a new variable is defined:

$$q^*(R,t)=q(R,t)-\left(RR_0-\frac{R^2}{2}\right)Q(t) \qquad (4)$$

The substitution the equation (4) in the equation (1) and in the boundary and initial conditions results in the following equations:

$$R\frac{\partial q^*}{\partial R}=\frac{\partial}{\partial R}\left[R\frac{\partial q^*(R,t)}{\partial R}\right]+P(R,t) \qquad (5)$$

where,

$$P(R,t)=(2R-R_0)Q(t)-\left(\frac{R^3}{2}-R^2R_0\right)Q(t)$$

The initial and the boundary conditions become

$$q^*(R,t)=\left(RR_0-\frac{R^2}{2}\right)Q(t)\,,\quad \tau=0\,; \qquad (6)$$

$$\left.\frac{\partial q^*(R,t)}{\partial R}\right|_{R=R_0}=0 \quad \text{e} \quad \left.\frac{\partial q^*(R,t)}{\partial R}\right|_{R=R_1}=0 \qquad (7)$$

where, Q(t) is the heat flux imposed at the external surface of the cylinder.

For the solution of the problem by the Generalized Integral Transform Technique, the following auxiliary problem of eigenvalue, with two homogeneous boundary conditions is defined

$$\frac{d}{dR}\left(R\frac{dy_i(R)}{dR}\right)+m_i^2 R y_i(R)=0 \qquad (8)$$

$$\left.\frac{dy_i(R)}{dR}\right|_{R=R_0}=0 \quad \text{e} \quad \left.\frac{dy_i(R)}{dR}\right|_{R=R_1}=0 \qquad (9)$$

The eigenfunctions, eigenvalues and the norm are of the form

$$y_i(m_i,R)=Y_1(m_i,R_1)J_0(m_i,R)-J_1(m_i,R_1)Y_0(m_i,R); \qquad (10)$$

$$J_1(m_i,R_0)Y_1(m_i,R_1)-Y_1(m_i,R_0)J_1(m_i,R_1)=0 \quad \text{e} \qquad (11)$$

$$N_i=\frac{2}{(m_i p)^2}\left\{1-\frac{J_1^2(m_i R_1)}{J_1^2(m_i R_0)}\right\} \quad ; \quad i=1,2,3,.... \qquad (12)$$

The using of the Generalized Integral Transform Technique, after a sequence of mathematical manipulations, permits to define the following Integral Transform and its inverse pair:

$$\tilde{q}_i^*(t)=\int_{R_0}^{R_1} R y_i(m_i,R)q^*(R,t)dR \qquad (13)$$

$$q^*(R,t)=\sum_{i=1}^{\infty}\frac{1}{N}y_i(m_i,R)\tilde{q}_i^*(t) \qquad (14)$$

Applying the transformations in the equation (5) it is obtained:

$$\frac{d\tilde{q}_i^*(t)}{dt}+m_i^2\tilde{q}_i^*(t)=g_i(t) \qquad (15)$$

where,

$$g_i(t)=g_i^*Q(t)+f_i^*\dot{Q}(t); \qquad (16)$$

$$f_i^*=-\frac{R_i}{m_i^2}Z_0(m_i,R_1)-\frac{R_0}{m_i^2}\int_{R_0}^{R_1}Z_0(m_i,R)dR; \qquad (17)$$

$$g_i^*=-R_0\int_{R_0}^{R_1}Z_0(m_i,R)dR \qquad (18)$$

with

$$Z_0(m_i,R)=Y_1(m_i,R_0)J_0(m_i,R)-J_1(m_i,R_0)Y_0(m_i,R) \qquad (19)$$

The solution of the equation (15) is,

$$\tilde{q}_i^*(t)=f_i^*Q(0)e^{-m^2t}+\int_0^t\left[g_i^*Q(t'')-f_i^*\dot{Q}(t')\right]e^{-m^2t'}dt' \qquad (20)$$

The temperature field of the pre-ablative phase is obtained substituting the equation (20) in the equation (14), thus:

$$q(R,t)=q_{av}(t)+\sum_{i=1}^{\infty}\frac{y_i(m_i,R)}{N_i}q_i^*(t)+\left(\frac{R^2}{2}-R_0R\right)Q(t) \qquad (21)$$

with,

$$q_{AV}=\frac{1}{1+2R_0}\left\{R_1\cdot t\left(A+\frac{Bt}{2}+\frac{Ct^2}{3}\right)+\left[\frac{5}{24}\left(R_0^2R_1+R_0R_1^2+R_0^3\right)-\frac{R_1^3}{8}\right]Q(t)\right\} \qquad (22)$$

To obtain the t_f (time at the beginning of the ablation period), the equation is used (21) is solved for $\theta(1,\tau_f)\equiv 1$, thus:

$$1\equiv q_{av}(t_f)-\left[R_0-\frac{1}{2}\right]Q(t)+\sum_{i=1}^{\infty}\frac{y_i(m,R)\tilde{q}_i^*(t)}{N_i} \qquad (23)$$

The temperature of the period Pre-ablation Period is the initial condition for the Ablation Period, which involves the transient motion of the external surface in R coordinate.

- **Ablation Period:**

The Figure 5 shows the Ablation Period.

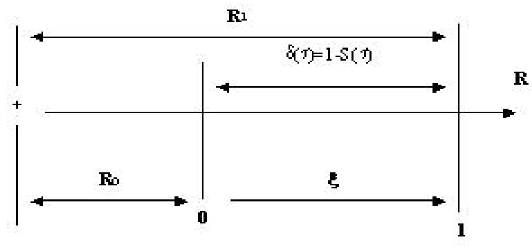

Figure 5: Cylindrical geometry in function of the new variable (ξ).

The equation that governs the ablation period becomes:

$$R\frac{\partial \hat{q}(R,t)}{\partial t}=\frac{\partial}{\partial R}\left[R\frac{\partial \hat{q}(R,t)}{\partial R}\right]$$, $R_0<R<R(\tau)$ e $\tau>\tau_m$

(24)

with the following conditions initial and of boundary;

$$\hat{q}(R,t)=q_{inicial}(R,t)-1, \quad \tau=\tau_m$$
(25)

$$\left.\frac{\partial \hat{q}(R,t)}{\partial R}\right|_{R=R_0}=0 \quad \text{e} \quad \left.\frac{\partial \hat{q}(R,t)}{\partial R}\right|_{R=R(t)}=0 \qquad (26)$$

The values of the ablation speed are imposed by the couplement equation.

$$\frac{\partial \hat{q}(R,t)}{\partial R}-n\frac{dR}{dt}=Q(t), \quad R=R(\tau) \qquad (27)$$

The equation (27) with the equations (24) and (25), are solved using the Generalized Integral Transform Technique. The ablative depth is $l_1(t)=f(R(t)=1-S(t))$: and the eigenfunctions and the eigenvalues are of the form

$$f_i[l_i(t),R]=Y_1[l_i(t),R_0]J_0[l_i(t),R]-J_1[l_i(t),R_0]Y_0[l_i(t),R]$$
(28)

$$J_0[l_i(t)R(t)]Y_1(l_i(t)R_0)-Y_0[l_i(t)R(t)]J_1(l(t)R_0)$$
29)

Defined a normalized eigenfunction Ki(R,t), in the following way:

$$K_i(R,t)=f_i[l_i(t),R]\cdot\left[\int_{R_0}^{1}Rf_i^2[l_i(t),R]dR\right]^{\frac{-1}{2}} \qquad (30)$$

ones can obtain

$$\int_0^{R(t)}K_i(R,t)K_j(R,t)dR=d_{ij} \qquad (31)$$

where the Kronecker's delta is defined as

$$d_{ij}=\begin{Bmatrix}1, i=j\\ 0, i\neq j\end{Bmatrix}$$

The values of the integral transform and its inverse are obtained after the solution of the coupled system of ordinary differential equations

$$\frac{\partial \tilde{\hat{q}}_i(t)}{\partial t}+l_i^2(t)\tilde{\hat{q}}_i(t)+\sum_{j=1}^{\infty}A_{ij}^*(t)\hat{q}_j(t)=0 \qquad (34)$$

$$\sum_{i=1}^{\infty}\tilde{\hat{q}}_i(t)\frac{\partial K_i(R,t)}{\partial R}=n\frac{\partial R}{\partial t}+Q(t), \quad R=R(\tau) \qquad (35)$$

where,

$$A_{ij}(t)=\int_{R_0}^{R(t)}RK_i(R,t)\frac{\partial K_j(R,t)}{\partial t}dR,$$

for, i=1,2,3...

with the initial condition, for $\tau=\tau_M$,

$$\tilde{\hat{q}}_i(t)=\int_{R_0}^{R(t)}K_i(R,t)\left[\sum_{j=1}^{\infty}\frac{y_j(m_j,R)}{N_j}\hat{q}_j^*(t)+\left(\frac{R^2}{2}-R_0R\right)Q(t)\right]dR$$
$$+(q_{AV}-1)\int_{R_0}^{R(t)}K_i(R,t)dR$$

(36)

The equations (34) to (36) form a infinite system of differential ordinary equations. For the calculation of the speed and ablative depth it is necessary to truncate the system for finite of order N.

3 DISCUSSION AND CONCLUSION

The result s of interest are the thickness S(t) of the melted material and the speed represented by the ablation rate V(t), due to the loss of the thickness of the ablation material.

The results shown in the Figures (6 to 8) were obtained by numerical solution of equations (34)-(36) using the IMSL Library (1979

Initially the problem was developed considering the conditions that simulated the case of a plane plate and using a constant flux, Figure (6), The results is in good agreement to results by the exact solution obtained by B. T. F. Chung and J. S. Hsiao (1983) and Diniz (1990). For large values of the radio of curvature the solution approaches of the exact and when the radio decreases, it is noticed the curvature influence in the calculated parameters, according to values indicated by τ_M in the Figures.

In Figures (7) and (8) are presented results for linear and parabolic heat fluxes.

The Generalized Integral Transform Technique has been applied with success in the present work, for the solution of the ablation problem in a cylindrical geometry. More results for diffusion problems of heat and mass are presented by Diniz (1990-1993), Cotta (1989-1993) and Kurokawa (2002-2003).

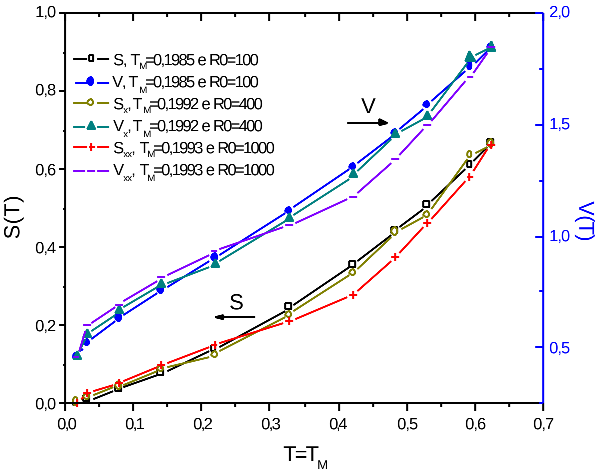

Figure 6: Comparison with the solution of the plane plate, for $Q(\tau) = 2$.

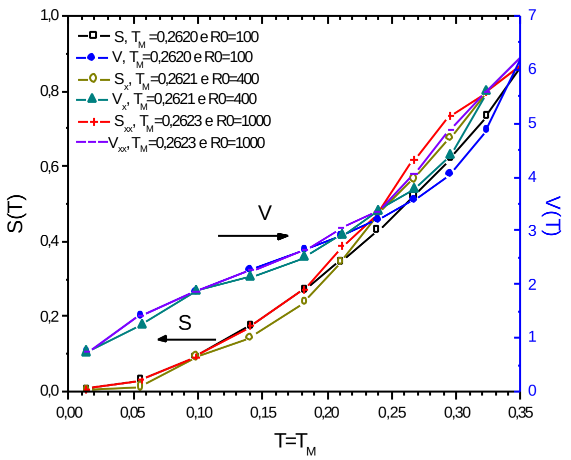

Figure 7: Influence of the ray $Q(\tau) = 10\tau$, with R=1000.

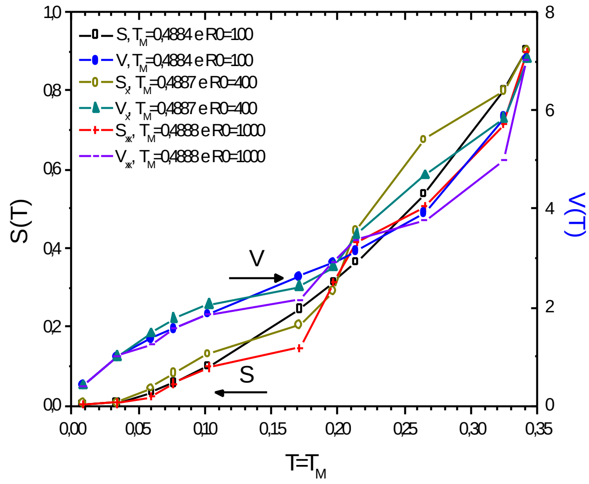

Figure 8: Comparison with the result of the plane plate, for R=1000 and $Q(t) = 10\tau^2$.

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