REDUCED-ORDER MODELING OF UNSTEADY FLOWS WITHOUT STATIC CORRECTION REQUIREMENT

M. Behbahani-Nejad*, H. Haddadpour, V. Esfahanian*****
***Chamran University of Ahwaz, Ahwaz, Iran**
****Sharif University of Technology, Tehran, Iran**
*****University of Tehran, Tehran, Iran**

Keywords: *Reduced Order Modeling, Unsteady Flows, Fluid Eigenmodes*

Abstract

In this article, a new reduced-order modeling approach is presented. This approach is based on fluid eigenmodes and without using the static correction. Vortex lattice method is used to analyze unsteady flows over two-dimensional airfoils and three-dimensional wings. Eigenanalysis and reduced-order modeling are performed using conventional method with and without the static correction technique. In addition to the conventional method, Eigenanalysis and reduced-order modeling are also performed using the new proposed method, i. e., without static correction requirement. Numerical examples are presented to demonstrate the accuracy and computational efficiency of the proposed method. Based on the obtained results, it has been shown that the accurate reduced order models of unsteady flows can be constructed without using the static correction technique.

1 Introduction

Reduced-order modeling (ROM) is a conceptually novel and computationally efficient technique that is recently used in analysis of unsteady flows. Unsteady flow eigenmodes are used to construct reduced-order unsteady flow models similar to the normal mode analysis in structural dynamics. Although the modal analysis of structures is quite routine, the modal analysis of unsteady flows is still in the developing stage. The advantage to a modal approach is that one may construct a reduced-order model by retaining only a few of the original modes. Eigenanalysis of unsteady potential flows about flat airfoils, cascades and wings has been applied by Hall [1]. He constructed reduced-order models based on unsteady incompressible vortex lattice method and found that in order to obtain satisfactory results, the static correction technique must be used. Romanowski and Dowell [2] applied ROM to the subsonic unsteady flows based on the Euler equations about a NACA 0012 airfoil. ROM of unsteady viscous flow in a compressor cascade based on the coupled potential flow and boundary-layer approximation has been applied by Florea et al. [3], and the status of ROM of unsteady aerodynamic systems has been reviewed by Dowell et al. [4].

Behbahani-nejad [5] and Esfahanian and Behbahani-nejad [6] applied ROM to the subsonic unsteady flows about complex configurations using boundary element method. They indicated that the number of zero eigenvalues of unsteady model is equal to the number of elements that lie on the body. Hence, some of the eigenmodes which are equal to the body's elements behave exactly in quasistatic fashion and ROM without the static correction can not generate satisfactory results even with the large number of eigenmodes. On the other hand, ROM based on body and its wake eigenmodes (conventional ROM) can give satisfactory results if and only if the static correction technique is applied. However, when the static correction technique is applied, the quasisteady part of the solution must be computed for each time step which alteres the efficiency of ROM. By constructing reduced-order model based only on the wake eigenmodes, the body qusistatic eigenmodes are removed and satisfactory results will be obtained without using the static correction technique.

In this context a new formulation based on vortex lattice method is presented by which the eigenvalue problem is defined based only on the unknown wake vortices. The eigenvalues of the new eigensystem are nonzero and therefore the new system has no quasistatic eigenmodes. Eigenanalysis results show that the eigenvalues of the proposed method are equal to the corresponding nonzero eigenvalues of the conventional method. To demonstrate the present approach, reduced-order models are constructed for unsteady flows over a two dimensional airfoil and a three dimensional wing. The results show that the present ROM can accurately and more efficiently analyze unsteady flows in comparison with the conventional reduced-order models.

2 Eigenanalysis and ROM

In vortex lattice method (VLM) for unsteady flow computations, the following matrix quation can be obtained [1]

$$A\Gamma^{n+1} + B\Gamma^{n} = w^{n+1} \qquad (1)$$

where Γ is the vector of vortex strengths, w is the known downwash, and A and B are known sparse matrices. When Γ is computed from Eq.(1), unsteady lift can be calculated as [7]

$$L = \int_{-b}^{b} \rho \left[U\gamma(x) + \frac{d}{dt}\int_{-b}^{x} \gamma(x_1)dx_1 \right] dx \qquad (2)$$

where $\gamma(x) = \frac{d\Gamma}{dx}$ and b is semichord of the airfoil. For zero downwash, One can set $\Gamma = x_i e^{\lambda_i t}$, and $z_i = e^{\lambda_i \Delta t}$ to obtain the following generalized eigenvalue problem

$$z_i A x_i + B x_i = 0 \qquad (3)$$

where λ_i and z_i are i^{th} eigenvalues in λ-plane and z-plane respectively and x_i is the corresponding eigenvector. More generally Eq.(3) can be written as

$$AXZ + BX = 0 \qquad (4)$$

where Z is a diagonal matrix containing the eigenvalues and X is a matrix that its columns are the right eigenvectors. On the other hand, the left eigenvectors satisfy the following relation

$$A^T YZ + B^T Y = 0 \qquad (5)$$

where Y is a matrix that its rows are the left eigenvectors. If the eigenvectors are normalized suitable, they satisfy the orthogonality conditions

$$Y^T AX = I \qquad Y^T BX = -Z \qquad (6)$$

The dynamic behavior of the fluid can be represented as the sum of the individual eigenmodes, i.e.,

$$\Gamma = Xc \qquad (7)$$

where c is the vector of normal mode coordinates. Substitution of Eq. (7) into Eq. (1), premultiplying by Y^T, and making use of the orthogonality condition gives a set of N uncoupled equations for the modal coordinates c, i.e.

$$c^{n+1} - Zc^{n} = Y^T w^{n+1} \qquad (8)$$

Now one may construct a reduced-order model by retaining only a few of the original modes. However, the above reduced-order model does not produce satisfactory results unless the static correction technique is applied. Therefore, it is a normal procedure to decompose the unsteady solution into two parts. One part is equivalent to the response of the system if the disturbance is quasisteady, and the other part is the dynamic part. Therefore, the unsteady solution can be defined as the following equation

$$\begin{aligned}\Gamma^n &= \Gamma_s^n + \Gamma_d^n \\ &= \Gamma_s^n + X\bar{c}^n\end{aligned} \quad (9)$$

The quasistatic portion Γ_s is given by

$$[A+B]\Gamma_s^n = w^n \quad (10)$$

Thus, Eq. (7) is replaced by

$$c_d^{n+1} - Zc_d^n = Y^T w^{n+1} - Y^T\left(A\Gamma_s^{n+1} + B\Gamma_s^n\right) \quad (11)$$

3 ROM without Static Correction Requirement

Esfahanian and Behbahani-Nejad studies show that the existence of zero eigenvalues in the eigensystem is the main reason for applying static correction technique [6]. Here, another approach is proposed. The proposed method removes zero eigenvalues by defining a new eigenvalue problem that its eigenvalues are the same as the nonzero eigenvalues of the previous eigensystem. If Γ_b and Γ_w are defined as the vector of body's and wake vortex strengths respectively, one can write

$$\Gamma = \begin{Bmatrix}\Gamma_b \\ \Gamma_w\end{Bmatrix} \quad (12)$$

Now, Eq. (1) can be rewritten as

$$\begin{bmatrix}A_{11} & A_{12} \\ A_{21} & A_{22}\end{bmatrix}\begin{Bmatrix}\Gamma_b \\ \Gamma_w\end{Bmatrix}^{n+1} + \begin{bmatrix}B_{11} & B_{12} \\ B_{21} & B_{22}\end{bmatrix}\begin{Bmatrix}\Gamma_b \\ \Gamma_w\end{Bmatrix}^{n} = \begin{Bmatrix}w_b \\ 0\end{Bmatrix}^{n+1} \quad (13)$$

and therefore

$$A_{11}\Gamma_b^{n+1} + A_{12}\Gamma_w^{n+1} + B_{11}\Gamma_b^n + B_{12}\Gamma_w^n = w_b^{n+1} \quad (14)$$

$$A_{21}\Gamma_b^{n+1} + A_{22}\Gamma_w^{n+1} + B_{21}\Gamma_b^n + B_{22}\Gamma_w^n = 0 \quad (15)$$

It can be shown that matrices B_{11} and B_{12} are zero and therefore Eq. (14) results in

$$w_b^{n+1} = \begin{bmatrix}A_{11} & A_{12}\end{bmatrix}\begin{Bmatrix}\Gamma_b \\ \Gamma_w\end{Bmatrix}^{n+1} \quad (16)$$

The above equation gives

$$\Gamma_b^{n+1} = A_{11}^{-1}w_b^{n+1} - A_{11}^{-1}A_{12}\Gamma_w^{n+1} \quad (17)$$

Substitution of Eq. (17) into Eq. (15) results in

$$\begin{aligned}&\left[A_{22} - A_{21}A_{11}^{-1}A_{12}\right]\Gamma_w^{n+1} + \left[B_{22} - B_{21}A_{11}^{-1}A_{12}\right]\Gamma_w^n = \\ &-A_{21}A_{11}^{-1}w_b^{n+1} - B_{21}A_{11}^{-1}w_b^n\end{aligned} \quad (18)$$

or

$$A_{new}\Gamma_w^{n+1} + B_{new}\Gamma_w^n = w_{new}^{n+1} \quad (19)$$

where

$$A_{new} = A_{22} - A_{21}A_{11}^{-1}A_{12} \quad (20)$$

$$B_{new} = B_{22} - B_{21}A_{11}^{-1}A_{12} \quad (21)$$

$$w_{new} = -A_{21}A_{11}^{-1}w_b^{n+1} - B_{21}A_{11}^{-1}w_b^n \quad (22)$$

Because Eq. (19) is versus Γ_w, the corresponding eigensystem has no zero eigenvalue. Therefore one may construct accurate reduced-order models without using the static correction technique.

4 Results and Discussion

4.1 Present Vortex Lattice Model

In this section the results are presented to validate the proposed unsteady vortex lattice model. The unsteady lift due to plunging motion of an isolated airfoil is computed for a range of reduced frequencies and compared with the Theodorsen exact solution as shown in Fig. 1. The airfoil is modeled using 20 vortex elements. The wake length is taken to be 10 chord lengths and it is discretized using 200 vortex elements. The apparent mass effects have been added to Theodorsen circulatory lift function to obtain circulatory and noncirculatory lift [7]. The results are quite satisfactory for the range of reduced frequencies considered here.

The lift acting on the airfoil due to a step change in airfoil downwash (Wagner problem) is considered to validate the present vortex lattice computations in time domain. As is shown in Fig. 2, the present vortex lattice model results are in perfect agreement with the Wagner function. Moreover, Fig. 2 shows the indicial response of a rectangular wing due to rigid-body plunging motion in comparison with the corresponding results presented in Ref. [1]. As in Ref. [1], the wing aspect ratio is 5.0 and it is modeled with eight vortex elements in the streamwise direction, and 10 in the spanwise direction. The wake is taken to be five chords long and is modeled using 40 vortex elements in the streamwise direction and 10 in spanwise direction. As is shown in the figure, the results of the present 3-D vortex lattice model are in good agreement with those of Ref. [1].

4.2 Eigenanalysis

The results of conventional and present eigenanalysis are presented in this section. Eigenvalues of the proposed method are shown in Fig. 3 in comparison with those of the conventional method for 2D airfoil. Moreover, in Fig. 4 the eigenvalues of the proposed method and conventional method are plotted with respect to the eigenvalues numbers. The results show that the eigenvalues of the proposed method are the same as the nonzero eigenvalues of the conventional method. In the proposed eigenanalysis, the eigensystem is interpreted only by the wake elements. Therefore, there are not any zero eigenvalues related to the body's elements. On the other hand, in the conventional method, the eigensystem is constructed using the airfoil elements as well as the wake elements. Therefore, there are 20 zero eigenvalues related to the body's elements, and 200 nonzero eigenvalues related to the wake elements.

Eigenvalues of vortex lattice model of unsteady flow about the three dimensional wing are plotted in Fig. 5. Similarly, as is shown in the figure the nonzero eigenvalues of conventional eigenanalysis are the same as the eigenvalues of the proposed method. Some of the nonzero eigenvalues in the figure which do not coincide with the eigenvalues of the proposed method, are corresponded to the wing elements and indeed they must be zero, but the numerical errors in computational problem made them nonzero eigenvalues.

4.3 Reduced Order Models

Next, we use the eigenvalues computed in the previous section to construct reduced order aerodynamic models. At first we review the conventional reduced order models (CROM). Figures 6 and 7 illustrate the unsteady lift predicted using the reduced order modeling technique without the static correction for 2D airfoil and 3D wing, respectively. As expected and shown in the figures, CROM without the static correction can not produce satisfactory results even if a large number of modes are used. It is due to the fact that the effects of the eigenmodes corresponding to the zero eigenvalues may be only considered using static correction.

The accuracy of the proposed method is shown in Fig. 8 where the unsteady lift of the airfoil predicted using the CROM with static correction and the present reduced order models (PROM) are compared. As is shown in the figure, PROM without static correction, can produce satisfactory results as accurate as CROM with static correction technique. That is due to the absence of zero eigenvalues in the new eigensystem. Figure 9 illustrates the effect of number of modes for the present method. As is shown, the exact solution can be produced when large number of modes is used. The unsteady lift of the 3D wing predicted using CROM with static correction, and PROM are compared in Fig. 10. The figure reveals that the same accuracy will be obtained for the 3D wing.

The efficiency of the proposed method will be more clear if the method is used in the time domain analysis. CROM and PROM are used to predict the unsteady lift in time domain due to plunging motion with reduced frequency k=0.5. Computational results of the lift variation during some heaving oscillation cycles of the airfoil are presented in Fig. 11. The results of the PROM and CROM with static correction are in perfect agreement with those of the direct method. However, the results of CROM without the static correction show considerable error, which is expected. Figure 12 presents the same comparison for the 3D wing. Similarly, the results of PROM show the same accuracy as CROM with the static correction.

4.4 Efficiency Analysis

Finally, the efficiency of PROM is presented for both frequency and time domains. To clarify the efficiency analysis, CPU times for PROM and CROM with static correction are compared. In addition, CPU times for eigenvalue computations and ROM are presented separately. Table 1 and 2 indicate CPU times in seconds for the 2D airfoil and the 3D wing. For each case, ROM is performed in the time domain (TD) and in the frequency domain (FD). The results are based on numerical computations using a P3-1000 MHz with 1 GB RAM. Results presented in tables 1 and 2 reveal that the proposed method can analyze either eigensystem or ROM more efficiently than CROM with the static correction. Efficiency of the proposed method from eigenanalysis point of view is due to the fact that the resulting eigensystem has a smaller rank than the conventional method, since it is represented based only on the wake elements. Therefore, the proposed method will be more efficient when the ratio of number of body's elements to the number of wake elements be increased. The application of the proposed method for ROM is more efficient than CROM because there is no need to compute the quasisteady solution in each time step. Hence, the present method will be more efficient as the time increases in the time domain analysis.

5 Conclusion and Remarks

Conventional reduced order modeling can generate satisfactory results when the static correction is used. This effect is due to the existence of zero eigenvalues in z-plane. It is shown that the existence of zero eigenvalues depend on the number of computational elements on the body. For implementation of static correction one needs to find quasisteady solution in each time step. In the present work a reduced order model for unsteady flows is developed without need to the static correction technique. In this method the numerical eigensystem is constructed using the wake variables only and the rank of the eigensystem is lower than the corresponding eigensystem in the conventional method. Therefore, the eigensystem in z-plane does not contain any zero eigenvalue and there is no need for the static correction.

The results indicate that the proposed method is computationally more efficient than the conventional method which requires the static correction. Moreover, the obtained results indicate that the proposed method can produce satisfactory results as accurate as the conventional method.

5 Acknowledgment

Chamran university of Ahwaz, sharif university of technology and university of Tehran are acknowledged for providing technical, administrative, and financial assistance.

References

[1] Hall, K. C. Eigenanalysis of unsteady flows about airfoils, cascades, and wings. *AIAA Journal*, Vol. 32, pp 2426-2432, 1994.

[2] Romanowski, M. C. and Dowell, E. H. Reduced order Euler equations for unsteady aerodynamic flows: Numerical techniques. AIAA paper 96-0528, 1996.

[3] Florea, R., Hall, K. C. and Cizmas, P. G. Reduced order modeling of unsteady viscous flow in a compressor cascade. *AIAA Journal*, Vol. 36, pp 1039-1048, 1998.

[4] Dowell, E. H., Hall, K. C. and Romanowski, M. C. Eigenmode analysis in unsteady aerodynamics: Reduced order models. *Appl. Mech. Rev.*, Vol. 50, No. 6, pp 371-385, 1997.

[5] Behbahani-Nejad, M. *Reduced order modeling of 3-D unsteady flows using fluid eigenmodes and boundary element method.* PhD Thesis, University of Tehran, Tehran, Iran, 2002.

[6] Esfahanian V. and Behbahani-Nejad, M. Reduced order modeling of unsteady flows about complex configurations using the boundary element method. *ASME Journal of Fluids Engineering*, Vol. 124, No. 4, pp. 988-993, 2002.

[7] Katz, J. and Plotkin, A. *Low-speed aerodynamics from wing theory to panel methods.* 2nd edition, Cambridge, New York, 2001.

Table 1: CPU Time for Eigenanalysis Computations

Case		Eigenanalysis	
		Conventional	Present
2D	TD	10	8
	FD	10	8
3D	TD	119	79
	FD	119	79

Table 2: CPU Time for ROM Computations

Case		ROM	
		Conventional	Present
2D	TD	7.9	5.7
	FD	0.7	0.2
3D	TD	12.5	4.9
	FD	3.8	2.2

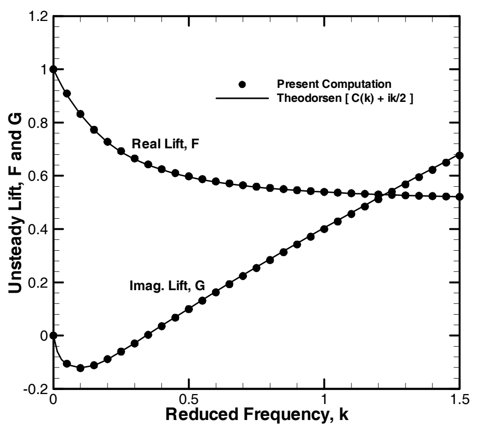

Fig. 1. Unsteady Lift Due to Plunging Motion of an Isolated Airfoil

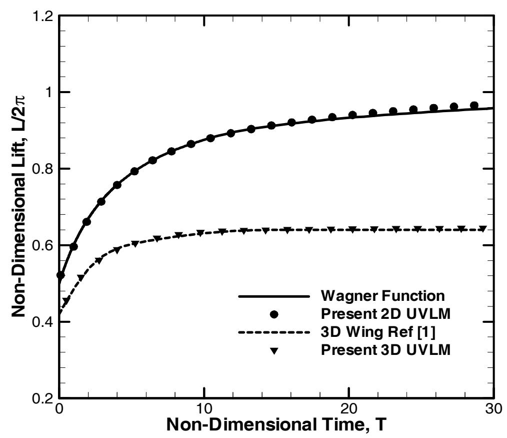

Fig. 2. Lift Acting on the Airfoil Due to a Step Change in Airfoil Downwash

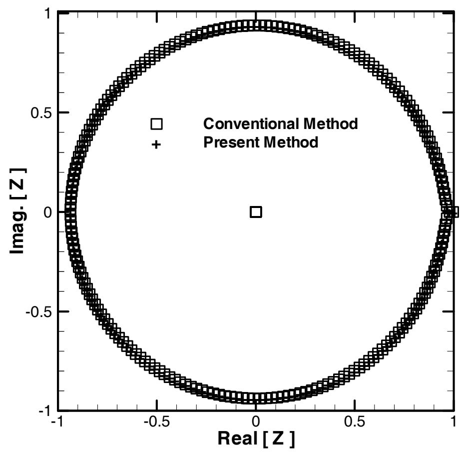

Fig. 3. Eigenvalues for 2D Airfoil

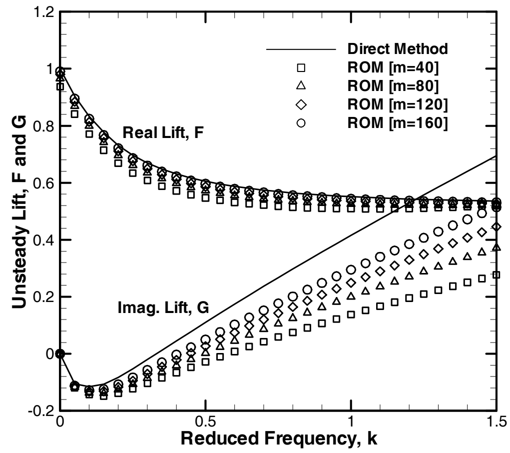

Fig. 6. Unsteady Lift for 2D Airfoil Predicted Using PROM Without the Static Correction

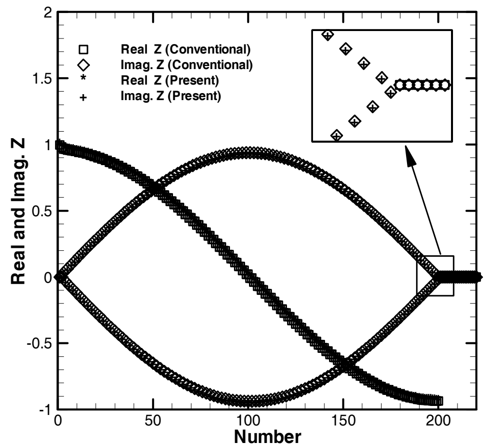

Fig. 4. Eigenvalues Versus Eigenvalues Numbers for 2D Airfoil

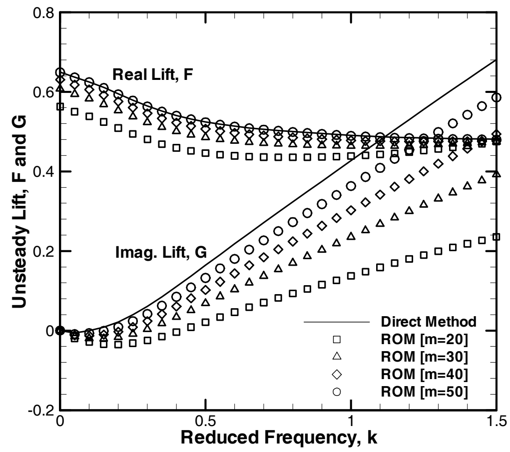

Fig. 7. Unsteady Lift for 3D Wing Predicted Using PROM Without the Static Correction

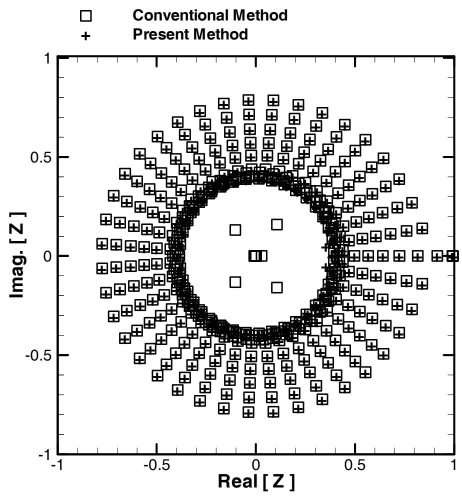

Fig. 5. Eigenvalues for 3D Wing

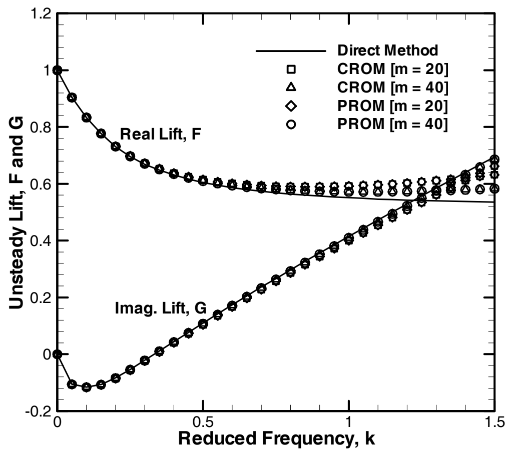

Fig. 8. Unsteady Lift for 2D Airfoil Predicted Using CROM With Static Correction and PROM

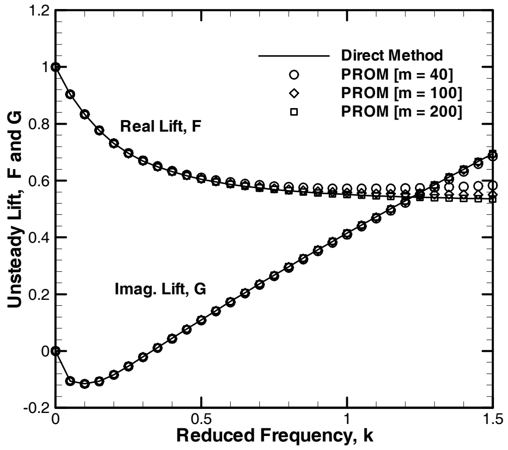

Fig. 9. The Number of Modes Effects on the Unsteady Lift for 2D Airfoil

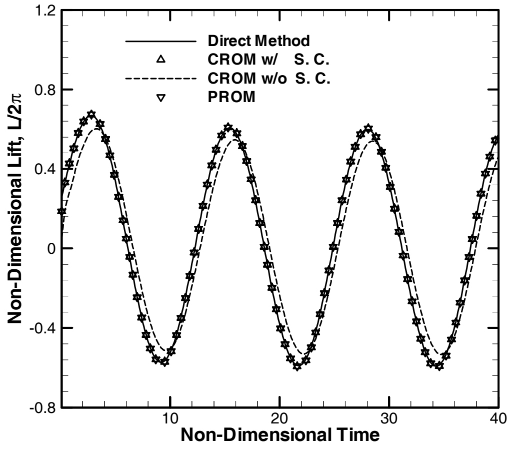

Fig. 11. Lift Variation During Some Heaving Oscillation Cycles of the 2D Airfoil

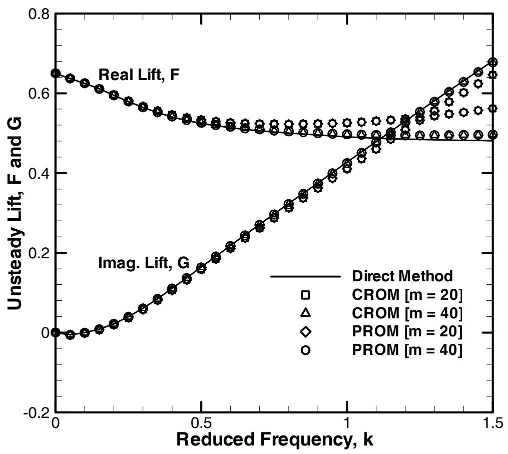

Fig. 10. Unsteady Lift for 3D Wing Predicted Using CROM With Static Correction and PROM

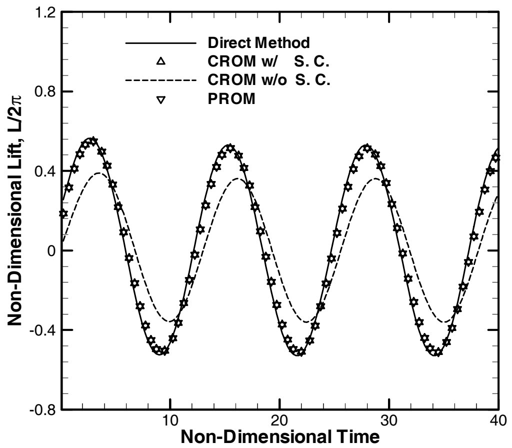

Fig. 12. Lift Variation During Some Heaving Oscillation Cycles of the 3D Wing