ON SOUND RADIATION FROM AN OPEN-ENDED NON-UNIFORMLY LINED CYLINDRICAL NOZZLE

L. M. B. C. Campos , J. M. G. S. Oliveira
Secção de Mecânica Aeroespacial, Instituto Superior Técnico,
1049-001 Lisboa Codex, Portugal

Keywords: *duct, impedance, noise attenuation, sound radiation*

Abstract

Acoustic liners are widely used in jet engine inlet and exhaust ducts, as a passive means of noise reduction. One possible way of liner optimization is the use of a non-uniform acoustic impedance distribution. It has been shown that such a liner can lead to an attenuation in the acoustic amplitude of some of the modes present in the duct. However, the sound attenuation perceived by an observer in the far-field is arguably the most important effect to be achieved.

These issues may be illustrated by considering the radiation of sound from a cylindrical duct with impedance varying circumferentially, axially or in both directions and the acoustic pressure at the far-field in each situation.

The radiation of sound from an open pipe is represented by a pressure distribution on a disk, viz. *the exit plane. The radiation of sound in free space,* i.e. *without obstacles, is specified by a Kirchhof integral. The source distribution on the duct exit plane, which allows the evaluation of radiation integrals, is specified by the radial, axial, and circumferential modes, in the cylindrical nozzle.*

The evaluation of the radiation integrals show that (i) the total acoustic field consists of a spherical wave multiplying a sum of radial modes $n = 1,\dots,\infty$ *and azimuthal modes of odd order only* $m = 1,3,5,\dots$*; (ii) each mode consists of a monopole term and a dipole term and depend on the frequency and the radial wavenumbers, determined by the boundary condition at the duct wall. When the impedance distribution varies circumferentially or axially, the evaluation of the wavenumbers involves the determination of the roots of an infinite determinant, while when the impedance aries both axially and circumferentially, the roots of a doubly infinite determinant have to be calculated.*

In the case of external noise of an aircraft, the observer is on the ground, at a distance much greater than the duct diameter, and the radiation integrals for an observer in the far-field can be simplified, since the dipole term is weak. The evaluation of the acoustic pressure for an observer in the far-field, shows that it depends on the radial wavenumbers in the nozzle, which are specified by the wall boundary conditions, and thus depend on the acoustic impedance distribution. This allows comparison of hard-walled nozzles, with liners with constant impedance and non-uniform liners, the latter with impedance distribution varying circumferentially, axially or in both directions. The far-field acoustic pressure is specified by a directivity factor that is calculated for several values of the parameters for each of the cases mentioned above.

1 Introduction

In a cylindrical nozzle noise can be absorbed in its interior by vortical flow [1–10] and at the walls by acoustic liners, which may have uniform [11–16] or non-uniform [17–24] impedances. The sound field received by an observer in the far-field is determined by radiation out of the open end of the nozzle [25–32] or inlet of a fan [33,34]. In the case of nozzle there is a refraction effect which can be represented either by a vortex sheet [35] or an irregular shear layer [1,2] issuing from the lip.

The aim of the present paper is to relate the acoustic field radiated through the nozzle exit to the far-field to the acoustic modes in a nozzle with non-uniform wall impedance. The radiation from the disk in the nozzle exit plane, to an observer in the far-field, consists, to leading order, of monopole and dipole terms. The acoustic pressure distribution in the nozzle exit plane, is expressed in terms of duct modes, allowing the evaluation of radiation integrals. The latter involve the acoustic eigenfunction, corresponding to the eigenvalues for propagating or cut-on modes, in the case of rigid walls or walls with uniform impedance, and also walls with impedance varying circumferentially, axially or in both directions. A numerical application is made, including the calculation of radial eigenvalues and plotting od acoustic eigenfunctions for uniform liner duct in comparison with weakly and strongly circumferentially non-uniform liners; the benefits of the non-uniform lining are assessed, not including the effects of transmission across the irregular and turbulent shear layer issuing from the jet nozzle lip.

2 Sound radiation from an open pipe

The radiation of sound from an open pipe (Fig. 1) is represented by a pressure distribution on a disk (2.1), *viz.* the exit plane; the radiation integrals are simplified (2.3) for an observer in the far-field (2.2).

2.1 Source distribution on a circular disk

The radiation of sound in free space, *i.e.* without obstacles, is specified by the Kirchhof integral:

$$p(\mathbf{x}) = \frac{1}{4\pi}\int_{\mathcal{D}} \frac{1}{|\mathbf{x}-\mathbf{y}|} \mathrm{e}^{-\mathrm{i}\omega(t-|\mathbf{x}-\mathbf{y}|/c)} q(\mathbf{y})\, \mathrm{d}^3\mathbf{y}, \tag{1}$$

for a spatial source distribution of strength q at position $\mathbf{y}$ in the domain $\mathcal{D}$, with frequency ω,

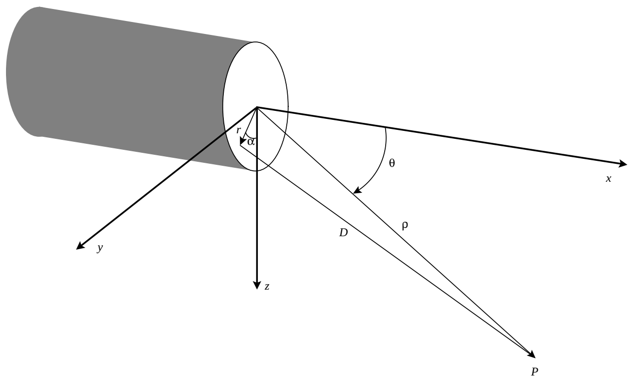

Fig. 1 Sound radiation from the disk on the exit plane of a cylindrical pipe to an observer in the far-field

and radiation to the observer at $\mathbf{x}$, in a homogeneous medium at rest, for which the sound speed is c. In the case of a disk of radius a on the XOY-plane with centre at the origin, the position of the source is written in polar coordinates (R,α):

$$0 \le \alpha \le 2\pi, \quad 0 \le R \le a; \qquad \mathbf{y} = R(\mathbf{e}_x \cos\alpha + \mathbf{e}_y \sin\alpha), \tag{2}$$

and the position of the observer in spherical coordinates (r,θ,φ):

$$\mathbf{x} = r(\mathbf{e}_x \cos\theta + \mathbf{e}_z \sin\theta), \tag{3}$$

where $\varphi = 0$ because the XOZ-plane can be taken through the observer. The radiation integral (1) becomes in this case

$$p(r,\theta,t) = \frac{\mathrm{e}^{-\mathrm{i}\omega t}}{4\pi} \times \int_0^{2\pi} \mathrm{d}\alpha \int_0^a \mathrm{d}R \frac{R}{D(R,\alpha)} \mathrm{e}^{\mathrm{i}(\omega/c)D(R,\alpha)} q(R,\alpha) \tag{4}$$

where $q(R,\alpha)$ is the source distribution, and

$$D(R,\alpha) = |\mathbf{x}-\mathbf{y}| = |R^2 + r^2 - 2Rr\sin\theta\cos\alpha|^{1/2}, \tag{5}$$

is the distance between observer and source.

2.2 Reception by an observer in the far-field

In the case of external noise of an aircraft, the observer is on the ground, at a distance much greater than the nozzle diameter,

$$R^2 \leq a^2 << r^2 :$$
$$D(R,\alpha) = r - R\sin\theta\cos\alpha + O(R^2/r^2), \quad (6a)$$

and the distance may be simplified by (6a), and the inverse distance by:

$$1/D(R,\alpha) = 1/r + R/r^2 \sin\theta\cos\alpha + O(R^2/r^2). \quad (6b)$$

Substitution of (6a, 6b) in the radiation integral (4) specifies

$$p(r,\theta,t) = \frac{e^{-i\omega t}}{4\pi}\int_0^{2\pi} d\alpha \int_0^a dR\, q(R,\alpha) \times \frac{R}{r}\left[1 + \frac{R}{r}\sin\theta\cos\alpha\right] e^{i(\omega/c)(r - R\sin\theta\cos\alpha)} \quad (7)$$

the acoustic field received by the observer in the far-field.

2.3 Decomposition into monopole and dipole terms

The acoustic pressure received in the far-field (7) may be written

$$p(r,\theta,t) = \frac{e^{i\omega(r/c-t)}}{4\pi r}(I_1 + I_2), \quad (8)$$

as a spherical wave radiated from the origin (or disk or nozzle centre) to the observer, multiplied by monopole (9a) and dipole (9b) terms

$$I_1 = \int_0^{2\pi} d\alpha \int_0^a dR\, q(R,\alpha)\, R e^{-i(\omega R/c)\sin\theta\cos\alpha)} \quad (9a)$$

$$I_2 = \int_0^{2\pi} d\alpha \int_0^a dR\, q(R,\alpha) \times R e^{-i(\omega R/c)\sin\theta\cos\alpha)}(R/r)\sin\theta\cos\alpha \quad (9b)$$

where the latter is related to the former by

$$I_2 = -\frac{1}{r}\frac{d}{d(i\omega/c)} I_1, \quad (10)$$

since differentiation with regard to $i\omega/c$ is equivalent to multiplication by $R\sin\theta\cos\alpha$. Substitution of (9a) and (10) in (8) yields:

$$p(r,\theta,t) = \frac{e^{i\omega(r/c-t)}}{4\pi r}\left\{1 + i\frac{d}{d(\omega r/c)}\right\} \int_0^{2\pi} d\alpha \int_0^a dR\, q(R,\alpha)\, R e^{-i(\omega R/c)\sin\theta\cos\alpha}, \quad (11)$$

as the acoustic pressure received by an observer in the far-field, from a source distribution $q(R,\alpha)$ on a disk. The latter is specified next in terms of the acoustic modes of a nozzle.

3 Modal structure in a cylindrical nozzle

The source distribution on the nozzle exit plane (3.2), which allows the evaluation of radiation integrals (3.3), is specified by the radial, axial, and circumferential modes, in the cylindrical nozzle.

3.1 Radial, axial and circumferential modes

The modes in a cylindrical nozzle are specified by the solution of the classical wave equation in cylindrical coordinates:

$$\left\{\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R} + \frac{1}{R^2}\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right\} Q = 0, \quad (12)$$

in the absence of mean flow. Since the coefficients of the wave equation depend only on radius R, it is convenient to use a Fourier decomposition in (z,α,t), *viz.*:

$$Q(R,\alpha,z,t) = e^{-i\omega t}\sum_{m=-\infty}^{+\infty} e^{im\alpha}\int_{-\infty}^{+\infty} dk\, e^{ikz} P_m(R,k), \quad (13)$$

a wave of frequency ω, with longitudinal wavenumber k and circumferential wavenumber m. Substitution of (13) in (12) shows that:

$$\left[R^2\frac{d^2}{dR^2} + R\frac{d}{dR} + \kappa^2R^2 - m^2\right] P_m(R,k) = 0, \quad (14)$$

the radial dependence is specified, for a cylindrical nozzle, by a Bessel function of order m:

$$P_m(R,k) = J_m(\kappa R), \quad (15a)$$

with radial wavenumber

$$\kappa^2 = \sqrt{(\omega/c)^2 - k^2}, \tag{15b}$$

specified by a boundary condition at the nozzle wall $R = a$, which specifies the radial modes κ_{nm} with $n = 1, \ldots, \infty$, which may be distinct for each circumferential order m.

3.2 Amplitudes of sound generation in a nozzle

Thus the acoustic field in the nozzle consists of a superposition (13, 15a) of:

$$Q(R,\alpha,z,t) = e^{-i\omega t} \sum_{m=-\infty}^{+\infty} e^{im\alpha} \times \sum_{n=1}^{\infty} J_m(\kappa_{mn}R) e^{ik_{mn}z} A_{mn}, \tag{16}$$

where m is the azimuthal and n the radial order, the radial wavenumbers κ_{mn} are specified by the boundary condition at the nozzle wall $R = a$, the axial wavenumbers are related by (15b), *viz.*:

$$k_{mn} = \sqrt{(\omega/c)^2 - (\kappa_{mn})^2}, \tag{17}$$

and the amplitudes A_{mn} of each mode are specified by the source distribution in the nozzle. The pressure distribution $q(R,\alpha)e^{-i\omega t}$ in the nozzle exit plane $z = 0$ is thus specified by:

$$q(R,\alpha) = Q(R,\alpha,0,0) = \sum_{m=-\infty}^{+\infty} e^{im\alpha} \sum_{n=1}^{\infty} J_m(\kappa_{mn}R) A_{mn}, \tag{18}$$

which may be substituted in the radiation integral (11) to specify the sound field received by the observer in the far-field. Substitution of (17) into (11) shows that the sound field received by the observer in the far-field

$$p(r,\theta,t) = \frac{e^{i\omega(r/c-t)}}{4\pi r} \sum_{m=-\infty}^{+\infty} \sum_{n=1}^{\infty} p_{mn}(r,\theta) A_{mn}, \tag{19a}$$

is specified by a superposition of radiation integrals for each mode:

$$p_{mn}(r,\theta) = \left\{1 - i\frac{d}{d(\omega r/c)}\right\} \int_0^{2\pi} d\alpha \int_0^a dR\, R e^{-i(\omega R/c)\sin\theta\cos\alpha} e^{im\alpha} J_m(\kappa_{mn}R), \tag{19b}$$

which are evaluated next.

3.3 Evaluation of radiation integrals for each mode

The acoustic pressure in the far-field for the n-th radial and m-th azimuthal mode is given by an integration in R:

$$p_{mn}(r,\theta) = \left\{1 + i\frac{d}{d(\omega r/c)}\right\} \int_0^a dR\, R\, J_m(\kappa_{mn}R)\, I_m\left((\omega R/c)\sin\theta\right), \tag{20a}$$

where the integration in α appears in

$$\zeta = -(\omega R/c)\sin\theta: \quad I_m(\zeta) = \int_0^{2\pi} e^{im\alpha + i\zeta\cos\alpha} d\alpha. \tag{20b}$$

The integral may be re-written:

$$I_m(\zeta) = \int_0^{\pi} e^{im\alpha} \left(e^{i\zeta\cos\alpha} + (-1)^m e^{-i\zeta\cos\alpha}\right) d\alpha = \int_0^{\pi} \cos(m\alpha) \left[e^{i\zeta\cos\alpha} + (-1)^m e^{-i\zeta\cos\alpha}\right] d\alpha,$$

which is expressible in terms of Bessel functions [36]:

$$J_m(\zeta) = \frac{i^{-m}}{\pi} \int_0^{\pi} e^{iz\cos\alpha} \cos(m\alpha) d\alpha, \tag{21}$$

by:

$$I_m(\zeta) = \pi i^m \left[J_m(\zeta) + (-1)^m J_m(-\zeta)\right]. \tag{22}$$

The series expansion for the Bessel functions

$$J_m(\zeta) = (\zeta/2)^m \sum_{s=0}^{\infty} \left\{(-\zeta^2/4)^s / (s!(s+m)!)\right\}, \tag{23}$$

implies that

$$J_m(-\zeta) = (-)^m J_m(\zeta), \tag{24}$$

so that

$$I_m(\zeta) = 2\pi \mathrm{i}^m J_m(\zeta). \tag{25}$$

4 Acoustic effects of non-uniform impedance

The evaluation of the acoustic pressure for an observer in the far-field (4.1), shows that it depends on the radial wavenumbers in the nozzle, which are specified by the wall boundary conditions (4.2). This allows comparison of hard-walled nozzles, with liners with constant impedance and non-uniform liners, the latter with impedance distribution varying circumferentially, axially or in both directions.

4.1 Selection of cut-off and cut-on modes

When substituting equations (25) in (19b):

$$p_{mn}(r,\theta) = \left\{1 + \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}(\omega r/c)}\right\} \int_0^a J_m(\kappa_{mn}R) J_m((\omega R/c)\sin\theta) R\,\mathrm{d}R, \tag{26}$$

the property [37] of Bessel functions:

$$J_{-m}(\zeta) = (-)^m J_m(\zeta), \tag{27}$$

implies that

$$p_{mn}(r,\theta) = p_{-m,n}(r,\theta), \tag{28}$$

so that the total pressure field (19a) simplifies to:

$$p(r,\theta,t) = \frac{\mathrm{e}^{\mathrm{i}\omega(r/c-t)}}{2\pi r} \sum_{n=1}^{\infty} \sum_{m=0}^{+\infty} p_{mn}(r,\theta) A_{mn}. \tag{29}$$

The second term of (26) involves:

$$\frac{\mathrm{d}}{\mathrm{d}(\omega r/c)} J_m((\omega R/c)\sin\theta) = \frac{R}{r}\frac{\mathrm{d}}{\mathrm{d}(\omega R/c)} J_m((\omega R/c)\sin\theta) = \frac{R}{r}\sin\theta J_m'((\omega R/c)\sin\theta), \tag{30}$$

where prime denotes derivative of the Bessel function with regard to its argument:

$$J_m'(\zeta) = J_{m-1}(\zeta) - \frac{m}{\zeta} J_m(\zeta). \tag{31}$$

Substitution of (31) in (30) yields

$$\frac{\mathrm{d}}{\mathrm{d}(\omega r/c)} J_m((\omega R/c)\sin\theta) = \frac{R}{r}\sin\theta\, J_{m-1}\left(\frac{\omega R}{c}\sin\theta\right) - \frac{mc}{\omega r} J_m\left(\frac{\omega R}{c}\sin\theta\right), \tag{32}$$

which specifies the dipole term in the acoustic pressure received in the far-field:

$$p_{mn}(r,\theta) = a^2 \int_0^1 J_m(\kappa_{mn} a s) \left\{ J_m(s\Omega\sin\theta) - \mathrm{i}\frac{a}{r}\left[\frac{m}{\Omega} J_m(s\Omega\sin\theta) - \sin\theta J_{m-1}(s\Omega\sin\theta)\right]\right\} s\mathrm{d}s, \tag{33}$$

where a dimensionless radial distance (34a) and a dimensionless frequency (34b) were introduced:

$$s = R/a \tag{34a}$$

$$\Omega = \omega a/c. \tag{34b}$$

Thus: (i) the total acoustic field (29) consists of a spherical wave multiplying a sum of radial modes $n = 1, \ldots, \infty$ and azimuthal modes of order $m = 0, 1, \ldots, \infty$; (ii) each mode (33) consists of a monopole term (first in the curly brackets) and a dipole term (in square brackets); (iii) the parameters are the dimensionless frequency (34b) calculated from the nozzle radius a, the ratio of the latter to the distance of the observer a/r in the dipole term (which is weak because $a^2 << r^2$ for observers in the far-field), and $\kappa_{mn} a$ which are the radial wavenumbers, determined by the boundary condition at the duct wall.

4.2 Rigid, impedance and non-uniform walls

The simplest case (I) is a nozzle with rigid walls, for which the normal velocity at the wall is zero, implying from the momentum equation that the normal derivative of the pressure is zero:

$$0 = i\omega v_n(R = a) = \rho^{-1}\left.\frac{\partial p}{\partial r}\right|_{r=a}, \quad (35)$$

where ρ is the mass density. Thus:

$$0 = J'_m(\kappa_{mn}a), \quad (36)$$

so that the radial wavenumbers κ_{mn} are determined by the zeros j_{mn} of the derivative of the Bessel function J_m:

$$J'_m(j_{mn}) = 0: \quad \kappa_{mn} = j_{mn}/a. \quad (37)$$

Since these zeros are real, the radial wavenumbers κ_{mn} are real, and the corresponding (17) axial wavenumbers:

$$k_{mn}a = \sqrt{\Omega^2 - (j_{mn})^2}, \quad (38)$$

are: (i) either real, for propagating or cut-on modes, if $|j_{mn}| \le \Omega$; (ii) or imaginary, for evanescent or cut-off modes, if $|j_{mn}| > \Omega$. Since the zeros of the derivative of the Bessel function J'_m form an unbounded sequence $j_{m1}, j_{m2}, \ldots$, there is a finite number of cut-on modes, larger for larger dimensionless frequency. The cut-off modes make a negligible contribution to radiation to the far-field, so the sum in the total acoustic pressure (29) is restricted to cut-on modes. The amplitude of the latter cut-on modes has been found to be weakly dependent on mode order for turbomachinery noise, and thus A_{mn} may be taken as a constant factor, and omitted together with the spherical wave term $e^{i\omega(r/c-t)}/2\pi r$ and a^2, which are common factors, regardless of the wall condition, and thus do not affect the comparison between rigid lined walls. For a rigid wall, the far-field acoustic pressure is thus specified by a directivity factor:

$$P(\theta) = \sum_{m=0}^{\infty}\sum_{n=1}^{j_{mn}<\Omega}\int_0^1 J_m(j_{mn}s)\,J_m(s\Omega\sin\theta)\,s\,ds, \quad (39)$$

where only the monopole term was considered, since it dominates the dipole term.

In the case (II) of a wall with uniform impedance $\bar{Z}_0$, the radial wavenumbers are specified by the roots of:

$$iZ_0 J'_m(\kappa_{mn}a) = J_m(\kappa_{mn}a), \quad (40)$$

where Z_0 is the specific impedance, *i. e.* the impedance divided by that of a plane wave:

$$Z_0 = \bar{Z}_0/\rho c. \quad (41)$$

In this case the radial wavenumbers κ_{mn} are generally complex, and the axial wavenumbers (17) also:

$$k_{mn}a = \sqrt{\Omega^2 - (\kappa_{mn}a)^2}. \quad (42)$$

Although the distinction between cut-on and cut-off modes is not so clear in the case of an impedance wall, the sum for the total acoustic field is taken for the cut-on modes of a rigid wall, as in (39), but for the complex radial wavenumbers, so that (39) remains valid, and can be evaluated (cfr. [36]). The acoustic pressure received in the far-field is:

$$\begin{aligned} p_{mn}(\theta) = -2ik_{mn}a\left[(\kappa_{mn}a)^2 - (\Omega\sin\theta)^2\right]^{-1} \\ \left[\Omega\sin\theta J_{mn}(\kappa_{mn}a)J'_m(\Omega\sin\theta) - \right. \\ \left. \kappa_{mn}aJ_m(\Omega\sin\theta)J'(\kappa_{mn}a)\right] \end{aligned} \quad (43)$$

We proceed to consider the calculation of radial eigenvalues k_{mn} in the case of liners of circumferentially or axially non-uniform impedance or both.

4.3 Axially and /or circumferentially non-uniform liners

This expression (43) can also be used in the case of non-uniform wall impedance $Z(\theta, z)$, except that radial wavenumbers are no longer determined as the roots of (40). In the case of a circumferentially non-uniform distribution, which is represented by the first two terms of a Fourier series:

$$Z_A(\theta) = Z_0(1 + 2\varepsilon\cos\theta), \quad (44)$$

the radial wavenumbers are specified exactly by the roots of an infinite determinant,

$$\det\left[i\frac{\Omega}{\kappa a}J_m(\kappa a)\delta_{mm'} - Z_{m'-m}J'_m(\kappa a)\right] = 0, \quad (45)$$

where all the impedance Fourier coefficients $Z_m = 0$ except Z_0 and $Z_{\pm 1} = \varepsilon Z_0$.

In the case of an axially non-uniform wall impedance over a length L of duct, represented by the first two terms of a Fourier series:

$$Z_B(z) = Z_0\left[1 + 2\delta\cos(2\pi z/L)\right], \qquad (46)$$

the radial wavenumbers are specified exactly by the roots of an infinite determinant,

$$\det\left[i\sqrt{1+\frac{2\pi l}{\kappa a}}J_m(\kappa a)\delta_{ll'} - Z_{l-l'}J'_m(\kappa a)\right] = 0, \qquad (47)$$

where all the impedance Fourier coefficients $Z_l = 0$ except Z_0 and $Z_{\pm 1} = \delta Z_0$.

In the case of impedance distribution varying both axially and radially, represented by the first two terms of the double Fourier series:

$$Z_C(\theta, z) = Z_0(1 + \varepsilon\cos\theta)\left[1 + \delta\cos(2\pi z/L)\right], \qquad (48)$$

the radial wavenumbers are specified exactly by the roots of a double infinite determinant, *i.e.* an infinite determinant whose terms are infinite determinants in a combination of (45) and (47). The directivity of the acoustic pressure in the far-field is specified by (43) also in the case of non-uniform impedance, except that the radial wavenumbers are given by the roots of (45) for circumferentially non-uniform impedance (44), the roots of (47) for axially non-uniform impedance (46), and roots of a combination of (45) and (47) for impedance varying both axially and circumferentially (48).

5 Noise reduction for far-field observer

The eigenvalues and eigenfunctions are calculated for circumferentially non-uniform liners, to assess the noise reduction benefit relative to uniform liners.

5.1 Eigenvalues for the dimensionless radial wavenumbers

Arguably the best measure of the effectiveness of an acoustic liner is the effect on the reduction of far-field noise. As a numerical example consider a nozzle of radius $a = 1$m, and a wave frequency $f = 1$kHz or $\omega = 2\pi f = 6.28 \times 10^3 \mathrm{s}^{-1}$, corresponding, for a sound speed $c = 340\mathrm{ms}^-1$, to a dimensionless frequency or Helmholtz number $\Omega = \omega R/c = 18.5$.

The condition $j_{mn} < \Omega$ for the roots j_{mn} of the derivatives of the Bessel functions $J'(j_{mn}) = 0$, specifies the propagating or cut-on modes.

The radial wavenumbers can be determined using (40) for a uniform specific impedance impedance:

$$Z_0 = 2.5 - i0.4, \qquad (49)$$

and (45), for circumferentially non-uniform impedances (44), with relative amplitude of the harmonic

$$\varepsilon = 0.1 + 0.1i, \qquad (50a)$$

$$\varepsilon = 0.2 - 0.3i. \qquad (50b)$$

The former is designated comparatively the ‘weakly’ (50a) and the latter the ‘strongly’ (50b) non uniform liner, although in both cases the impedance is the same (49) on the mean, and the variation from the mean is small in absolute terms in both cases.

5.2 Sound attenuation due to non-uniform liner

In order to assess the sound attenuation due to the non-uniform liner, the eigenfunctions for the acoustic pressure (39) with non-uniform $p_{mn}(\theta; Z_0, \varepsilon)$ and uniform $p_{mn}(\theta; Z_0, 0)$ liner are compared in a logarithmic scale:

$$\mathcal{A}_{mn}(\theta; Z_0, \varepsilon) \equiv \log_{10}\left|\frac{p_{mn}(\theta, Z_0, \varepsilon)}{p_{mn}(\theta, Z_0, 0)}\right|, \qquad (51)$$

in decibels; the effect on energy would be the double of that given by (51), and is plotted as a function of azimuthal angle θ in Figs. 2 and 3. In Fig. 2 the axisymmetric mode $m = 0$ is considered for increasing radial orders $n = 1, \ldots, 6$. The fundamental radial mode $n = 1$ (top left) is not much affected by the non-uniform impedance, but the effect becomes visible for the second

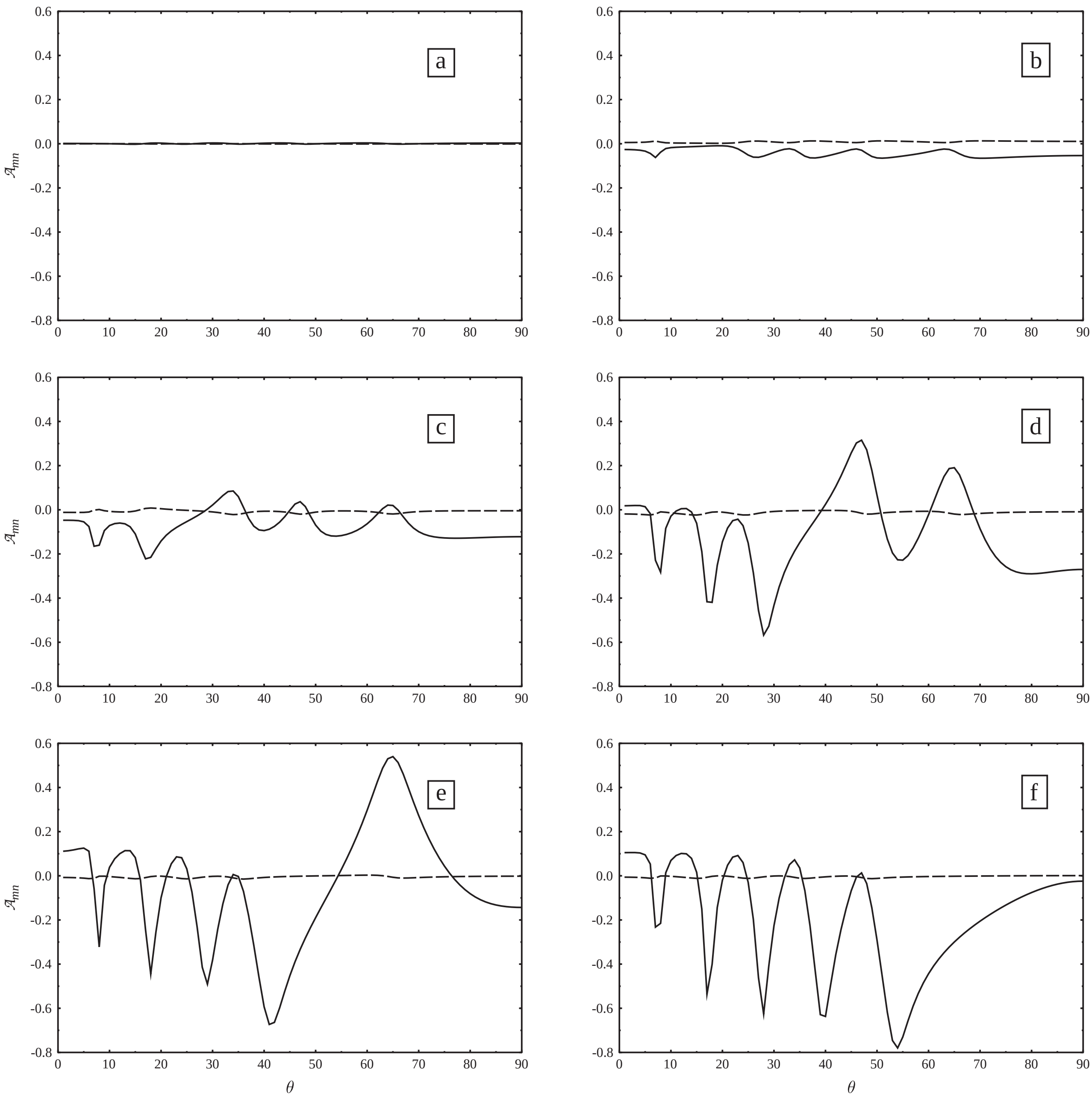

Fig. 2 Comparison on a decimal logarithmic scale (51) of the ratio of amplitudes of the acoustic pressure (43), for a non-uniform (44) and a uniform (49) liner with the same mean impedance, in the cases of 'weak' (- -) and 'strong' (—) non-uniformity, as a function of azimuthal angle θ, for axisymmetric modes $m = 0$ and (a) the fundamental radial mode $n = 1$ and the harmonics (b) $n = 2$, (c) $n = 3$, (d) $n = 4$, (e) $n = 5$, and (f) $n = 6$.

$n = 2$ (top right) and third $n = 3$ (middle left) radial modes. Significant amplitude variation occur for the fourth $n = 4$ (middle right), fifth $n = 5$ (bottom left), and sixth $n = 6$ (bottom right) radial modes; these amplitude variations are mostly reductions, although some increases occur.

Broadly similar conclusions can be drawn from Fig. 3, which considers the fundamental radial mode $n = 1$ for several azimuthal orders $m = 0, 2, \ldots, 14$. The axisymmetric mode $m = 0$

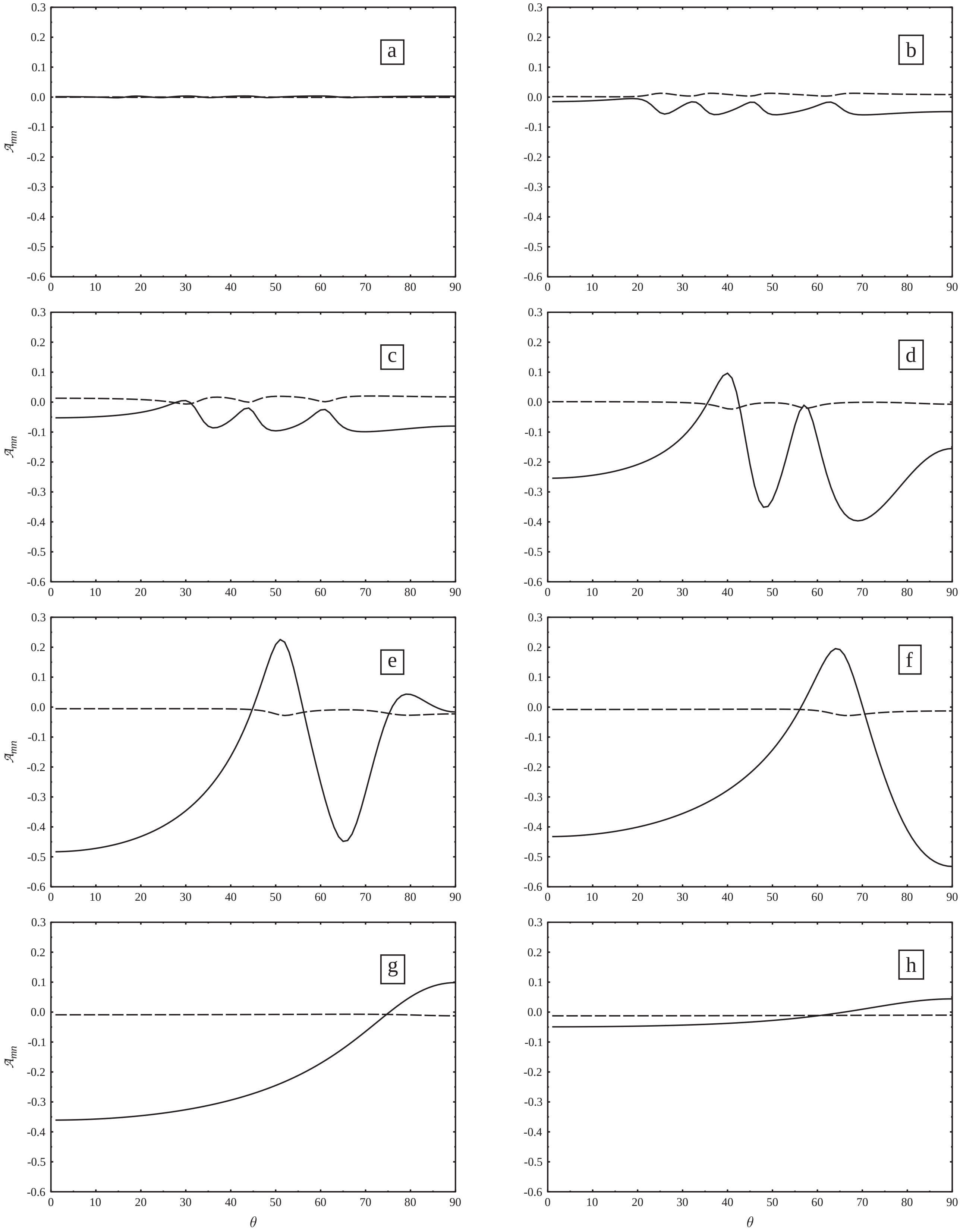

Fig. 3 As Fig. 2, for the fundamental axial mode $n = 1$ and (a) axisymmetric $m = 0$, and non-axisymmetric modes of increasing even azimuthal order (b) $m = 2$, (c) $m = 4$, (d) $m = 6$, (e) $m = 8$, (f) $m = 10$, (g) $m = 12$, and (h) $m = 14$.

(top left) is not much affected by the non-uniform liner. The azimuthal modes $m = 2$ (top right) and $m = 4$ (second row, left) show visible reductions. The higher order azimuthal modes $m = 6$ (third row, left), $m = 10$ (third row, right), $m = 12$ (bottom left), and $m = 14$ (bottom right) show a clear amplitude reduction due to the 'strongly' non-uniform liner over most directions, although an increase is seen for some directions. It is clear that a non-uniform liner, with relative impedance variations in the range $20-30\%$, can provide an overall sound reduction.

6 Discussion

The effect of non-uniform impedance on sound radiation to the far-field has been assessed on the basis of the acoustic pressure on the disk corresponding to the nozzle exit plane, without accounting for edge diffraction effects. The latter can be quite important [35], in particular in the presence of a mean flow, when a turbulent and irregular shear layer [1,2] is issued from the nozzle lip. The latter causes spectral and directional broadening of sound, which further reduces the noise levels. Although the latter effects have not been modelled here, it is clear that an attenuation of the 'input' sound field incident on the shear layer from the interior of the jet will result in a further attenuated sound field transmitted to an observer in the far-field outside the shear layer. The attenuations shown for the basic in-duct sound field are clear even though: (i) the non-uniform liner (44) has the same average impedance as the uniform liner; (ii) only one liner impedance 'harmonic' was used, involving a single parameter ε, which was not optimized. By considering multi-harmonic impedance distribution

$$Z(\theta) = Z_0 \left[1 + \sum_{l=1}^{L} \varepsilon_l \cos(l\theta) \right], \qquad (52)$$

and optimizing the parameter ε_l, greater noise reduction could be obtained. In the present example a relative impedance variation $|\varepsilon|$ of $20-30\%$ was shown to be sufficient to affect significantly the sound fields.

References

[1] L. M. B. C. Campos. The spectral broadening of sound in turbulent shear layers. Part 1: The transmission of sound through turbulent shear layers. *Journal of Fluid Mechanics*, 89:723–749, 1978.

[2] L. M. B. C. Campos. The spectral broadening of sound in turbulent shear layers. Part 2: The spectral broadening of sound and aircraft noise. *Journal of Fluid Mechanics*, 89:751–783, 1978.

[3] H. E. Plumblee and P. E. Doak. Duct noise radiation through a jet flow. *Journal of Sound and Vibration*, 65(4):453–491, 1979.

[4] P. A. Nelson and C. L. Morfey. Aerodynamic sound production in low speed flow ducts. *Journal of Sound and Vibration*, 79(2):263–289, 1981.

[5] M. S. Howe. The damping of sound by turbulent wall shear layers. *Journal of the Acoustical Society of America*, 98:1723–1730, 1995.

[6] C. K. W. Tam and L. Auriault. The wave modes in ducted swirling flows. *Journal of Fluid Mechanics*, 371:1–20, 1998.

[7] L. M. B. C. Campos and P. G. T. A. Serrão. On the acoustics of an exponential boundary layer. *Philosophical Transactions of the Royal Society of London, Series A*, 356:2335–2378, 1998.

[8] L. M. B. C. Campos, J. M. G. S. Oliveira, and M. H. Kobayashi. On sound propagation in a linear flow. *Journal of Sound and Vibration*, 219(5):739–770, 1999.

[9] L. M. B. C. Campos and M. H. Kobayashi. On the reflection and transmmission of sound in a thick shear layer. *Journal of Fluid Mechanics*, 420:1–24, 2000.

[10] M. Willatzen. Sound propagation in a moving fluid confined by cilindrical walls — exact series solutions for radially dependent flow profiles. *Acustica*, 87:552–559, 2001.

[11] A. D. Rawlins. Radiation of sound from an unflanged rigid cylindrical duct with an acoustically absorbing internal surface. *Proceedings of the Royal Society of London, Series A*, 361:65–91, 1978.

[12] K. Ogimoto and G. W. Johnston. Modal radiation impedances for semi-infinite unflanged circular ducts including flow effects. *Journal of*

Sound and Vibration, 62(4):598–605, 1979.

[13] W. Koch and W. Möhring. Eigensolutions for liners in uniform mean flow ducts. *AIAA Journal*, 21(2):200–213, 1983.

[14] W. Koch. Radiation of sound from a two-dimensional acoustically lined duct. *Journal of Sound and Vibration*, 55(2):255–274, 1977.

[15] S. W. Rienstra. Contributions to the theory of sound propagation in ducts with bulk-reacting lining. *Journal of the Acoustical Society of America*, 77(5):1681–1685, May 1985.

[16] D. A. Bies, C. H. Hansen, and G E Bridges. Sound propagation in rectangular and circular cross-section ducts with flow and bulk-reacting liner. *Journal of Sound and Vibration*, 146(1):47–80, 1991.

[17] M. S. Howe. The attenuation of sound in a randomly lined duct. *Journal of Sound and Vibration*, 87(1):83–103, 1983.

[18] W. R. Watson. An acoustic evaluation of circumferentially segmented duct liners. *AIAA Journal*, 22(9):1229–1233, September 1984.

[19] C. R. Fuller. Propagation and radiation of sound from flanged circular ducts with circumferentially varying wall admittances, I: semi-infinite ducts. *Journal of Sound and Vibration*, 93(3):321–340, 1984.

[20] C. R. Fuller. Propagation and radiation of sound from flanged circular ducts with circumferentially varying wall admittances, II: finite ducts with sources. *Journal of Sound and Vibration*, 93(3):341–351, 1984.

[21] P. G. Vaidya. The propagation of sound in ducts lined with circumferentially non-uniform admittance of the form $\eta_0 + \eta_q \exp(iq\theta)$. *Journal of Sound and Vibration*, 100(4):463–475, 1985.

[22] B. Regan and J. Eaton. Modelling the influence of acoustic liner non-uniformities on duct modes. *Journal of Sound and Vibration*, 219(5):859–879, 1999.

[23] L. M. B. C. Campos and J. M. G. S. Oliveira. On the acoustic modes in a cylindrical nozzle with an arbitrary impedance distribution. In *Proceedings of the 23rd ICAS Congress*, 2002.

[24] L. M. B. C. Campos and J. M. G. S. Oliveira. On the optimization of non-uniform acoustic liners on annular nozzles. To be published, 2003.

[25] S. D. Savkar. Radiation of cylindrical duct acoustic modes with flow mismatch. *Journal of Sound and Vibration*, 42:363–386, 3 1975.

[26] W. Koch. Attenuation of sound in multi-element acoustically lined rectangular ducts in the absence of mean flow. *Journal of Sound and Vibration*, 52(4):459–496, 1977.

[27] S. W. Rienstra. Acoustic radiation from a semi-infinite duct in a uniform subsonic mean flow. *Journal of Sound and Vibration*, 94(2):267–288, 1984.

[28] R. Martinez. Diffracting open-ended pipe treated as a lifting surface. *AIAA Journal*, 26(4):396–404, 1988.

[29] A. Snakovska, H. Idczak, and B. Bogusz. Modal analysis of the acoustic field radiated from an unflanged cylindrical duct – theory and measurement. *Acustica*, 82:201–206, 1996.

[30] A. Snakowska and H. Idczak. Prediction of multitone sound radiation from a circular duct. *Acustica*, 83:955–962, 1997.

[31] P. Joseph and C. L. Morfey. Multimode radiation from an unflanged, semi-infinite circular duct. *Journal of the Acoustical Society of America*, 105(5):2590–2600, May 1999.

[32] S. T. Hocter. Exact and approximate directivity patterns of the sound radiated from cylindrical duct. *Journal of Sound and Vibration*, 227(2):397–407, 1999.

[33] S. J. Horowitz, R. K. Sigman, and B. T. Zinn. An iterative method for predicting turbofan inlet acoustics. *AIAA Journal*, 20(12):1693–1699, December 1982.

[34] H. D. Meyer. Effect of inlet reflections on fan noise radiation. *AIAA Journal*, 34(9):1771–1777, September 1996.

[35] R. M. Munt. Acoustic radiation from a semi-infinite circular duct in an uniform subsonic mean flow. *Journal of Fluid Mechanics*, 83:609–640, 1977.

[36] M. Abramowitz and I. A. Stegun. *Handbook of Mathematical Functions*. Dover, 1965.

[37] E. T. Whittaker and G. N. Watson. *A Course of Modern Analysis*. Cambridge University Press, 1927.